# CHAPTER 9
# RAY OPTICS AND OPTICAL INSTRUMENTS

## REFLECTION OF LIGHT BY SPHERICAL MIRRORS

**Laws of reflection**

1. The angle of incidence is equal to the angle of reflection.
2. The incident ray, reflected ray and the normal to the point of incidence all lie in the same plane.

## SOME BASIC TERMS

**1.Pole**

The geometric centre of a spherical mirror is called its pole

**2.Optical centre**

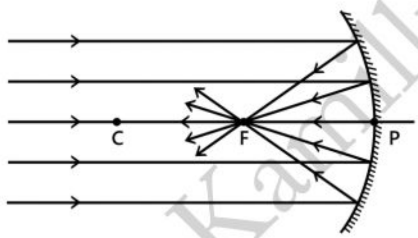
The geometric centre of a lense is called its optic centre.

**3.Principal axis**

The line joining the pole and the centre of curvature of the spherical mirror is known as the principal axis.

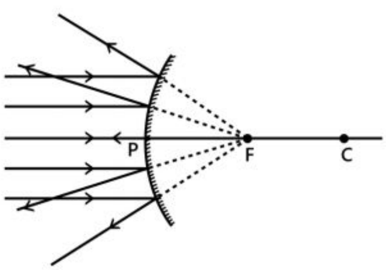
**4.Principal Focus of a concave mirror (F)**

The light rays which are coming parallel to the principal axis,after reflection converge to a point on the principal axis. This point is called the principal focus of a concave mirror.




**5.Principal Focus of a convex mirror (F)**

The light rays which are coming parallel to the principal axis after reflection from the convex mirror appears to diverge from a point on the principal axis, on the other side of the mirror. This point is called the
principal focus of the convex mirror.

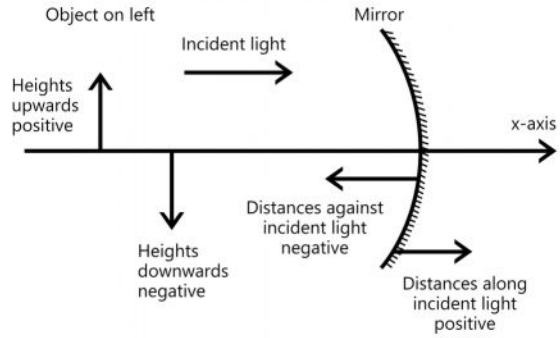



**6. focal length of the mirror (f)**

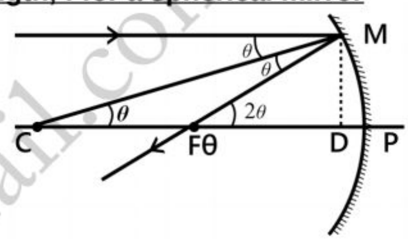
The distance between the focus, F and the pole, P of the mirror is called the focal length of the mirror

**NOTE: Sign convention**

* All distance should measure from the pole




**Relation between Radius of Curvature, R and focal length, f for a spherical mirror**




Let C be the centre of curvature of the mirror. Consider a ray parallel to the principal axis striking the mirror at M. Then CM will be perpendicular to the mirror at M. Let θ be the angle of incidence, and MD be the perpendicular from M on the principal axis. Then,

$$<MCP=\theta \quad \text{and} \quad <MFP=2\theta$$

Now, $\tan\theta=\frac{MD}{CD}$ and $\tan 2\theta=\frac{MD}{FD}$

For small θ, which is true for paraxial rays, $\tan\theta \approx \theta$ and $\tan 2\theta \approx 2\theta$

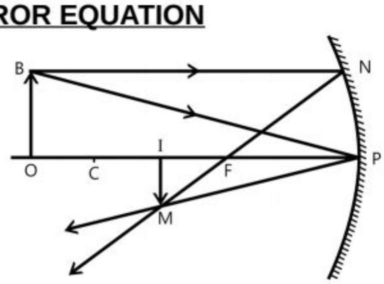
Therefore, the above equation becomes

$$\theta=\frac{MD}{CD} \quad \text{and} \quad 2\theta=\frac{MD}{FD}$$

=> $\frac{MD}{FD}=2\frac{MD}{CD}$ => $FD=\frac{CD}{2}$

=> $\boxed{f=\frac{R}{2}}$ ..................(1)

**THE MIRROR EQUATION**



Let OB – Object
I M – Image
OP – Object distance (u)
I P – Image distance (v)
FP – Focal length (f)

The two right triangle Δ OBP and Δ IMP are similar.

Therefore, $\frac{IM}{OB}=\frac{IP}{OP}$ ............(2)

Now from similar triangles, Δ IFM and Δ FNP

$$\frac{IM}{NP}=\frac{IF}{FP} \quad ............(3)$$

But $NP=OB$

Therefore, (3) => $\frac{IM}{OB}=\frac{IF}{FP}$ ...........(4)

Comparing (2) and (4) $\frac{IP}{OP}=\frac{IF}{FP}=\frac{IP-FP}{FP}$

=> $\frac{IP}{OP}=\frac{IP-FP}{FP}$ ...............(5)

By applying cartesian sign convention,

$IP=-v$ , $OP=-u$ , $FP=-f$

Therefore, (5) => $\frac{-v}{-u}=\frac{-v+f}{-f}$

=> $vf=uv-uf$ ................(6)

dividing throughout by uvf

(6) => $\frac{1}{u}=\frac{1}{f}-\frac{1}{v}$

=> $\boxed{\frac{1}{f}=\frac{1}{u}+\frac{1}{v}}$ This is called **mirror equation.**

**Linear magnification (m)**

It is the ratio of the height of the image to the height of the object.

$$m=\frac{\text{height of the image}}{\text{height of the object}}=\frac{h_i}{h_o}$$

By applying sign convention,

$$\frac{-h_i}{h_o}=\frac{-v}{-u} \quad => \quad \frac{h_i}{h_o}=\frac{-v}{u}$$

=> $\boxed{m=\frac{-v}{u}}$

**Note**

- When 'm' is positive, the image is erect and virtual
- When 'm' is negative, the image is inverted and real
- When m > 1 , image is enlarged
- When m < 1 , image is diminished

**Problem 1**

The radius of curvature of a concave mirror is 40 cm. Find its focal length.

**Solution**

$$f=\frac{R}{2}=\frac{40}{2}=20\,cm$$

Since the mirror is concave, $f=-20\,cm$

**Problem 2 (Example 9.3 NCERT)**

An object is placed at (i) 10 cm, (ii) 5 cm in front of a concave mirror of radius of curvature 15 cm. Find the position, nature, and magnification of the image in each case.

**Solution**

The focal length f = –15/2 cm = –7.5 cm

**(i)** We know $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$ => $\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$

=> $\frac{1}{v}=\frac{u-f}{uf}$ => $v=\frac{uf}{u-f}$

By applying sign convention u = –10 cm, f = -7.5 cm

$$v=\frac{(-10)\,x\,(-7.5)}{-10+7.5}=\frac{75}{-2.5}=-30\,cm$$

The image is 30 cm from the mirror on the same side as the object.

Also, the magnification

$$m=\frac{-v}{u}=\frac{-(-30)}{-10}=-3$$

The image is magnified, real and inverted.

**(ii)** We know $v=\frac{uf}{u-f}$

By applying sign convention u = –5 cm, f = -7.5 cm

$$v=\frac{(-5)\,x\,(-7.5)}{-5+7.5}=\frac{37.5}{2.5}=15\,cm$$

This image is formed at 15 cm behind the mirror. It is a virtual image.

Also, the magnification $m=\frac{-v}{u}=\frac{-(15)}{-5}=3$

The image is magnified, virtual and erect.

***Problem 3 ( For more reading only)***

*Why is the sign convention used in the derivation of the mirror equation and yet again used when it is applied in numerical problems? Won't the whole idea of sign convention be eliminated if it is used twice?*

***Solution***

*See Appendix 1*

## REFRACTION

Refraction is the bending of light when it passes obliquely from one transparent medium into another.

- When light travels from a rarer medium to a denser medium, it bends towards the normal(Fig.a).
- When light travels from a denser medium to a rarer medium,it bends away from the normal(Fig.b).

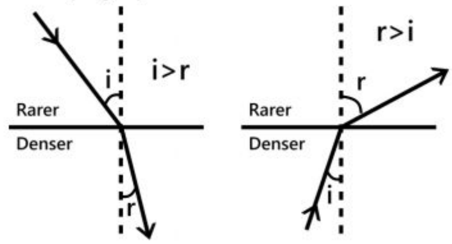



fig (a) fig (b)

### Laws of refraction

1. The incident ray, the refracted ray and the normal at the point of incidence are all lie in the same plane.
2. The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant for a given pair of media and for the given colour of light used. This constant is known as the **refractive index** of second medium w.r.t. the first medium.

   Ie, $$\boxed{\frac{\sin i}{\sin r}=\frac{n_2}{n_1}=n_{21}}$$

### NOTE

- If the first medium is air, then $\frac{\sin i}{\sin r}$ is known as absolute refractive index of the second medium.

  i.e, $\frac{\sin i}{\sin r}=n$ where 'n' is the refractive index of the second medium.
- Refractive index of a medium is defined as the ratio of velocity of light vacuum to the velocity of light in the medium.

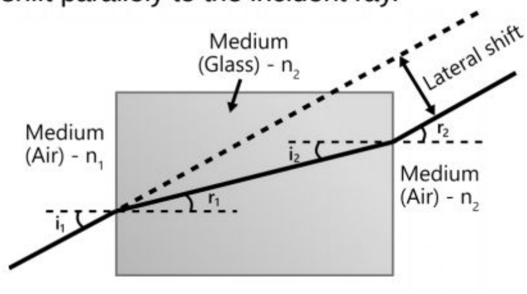
$$n=\frac{c}{v}$$

| Substances | Absolute refractive index |
|---|---|
| Air | 1 |
| Water | 1.33 |
| Glass | 1.5 |
| Diamond | 2.42 |

## SOME APPLICATIONS OF REFRACTION

### (1) Lateral shift of a ray refracted through a parallel-sided slab.

For a parallel-sided slab The angle of incidence = angle of emergence, but the emergent ray shift parallely to the incident ray.




At the air- glass interface, we can write the Snell's law:

$$\frac{\sin i_1}{\sin r_1}=\frac{n_2}{n_1} \quad \text{................(1)}$$

At the glass-air interface, we can write the Snell's law:

$$\frac{\sin i_2}{\sin r_2}=\frac{n_1}{n_2} \quad \text{................(2)}$$

Taking reciprocal,

$$\frac{\sin r_2}{\sin i_2}=\frac{n_2}{n_1} \quad \text{...............(3)}$$

Comparing (3) and (1)

$$\frac{\sin i_1}{\sin r_1}=\frac{\sin r_2}{\sin i_2}$$

=> $\sin i_1=\sin r_2$ => $\boxed{i_1=r_2}$

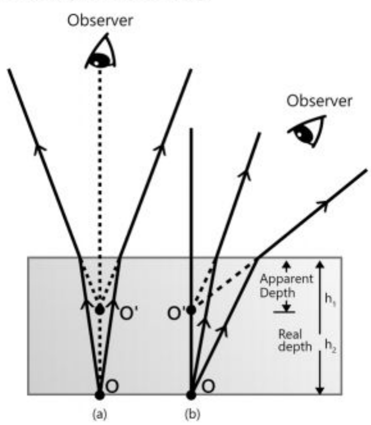
i.e, angle of incidence = angle of emergence

Or the incident ray and the emergent ray are parallel.

### (2) Apparent depth

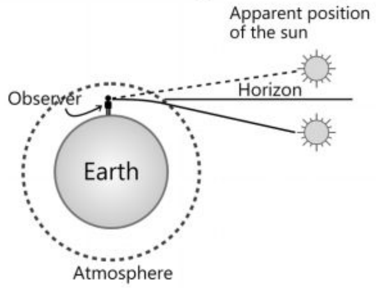
If an object in a denser medium is viewed from a rarer medium the image appears to be raised towards the surface.

Figure shows apparent depth for (a) Normal viewing (b) Oblique viewing



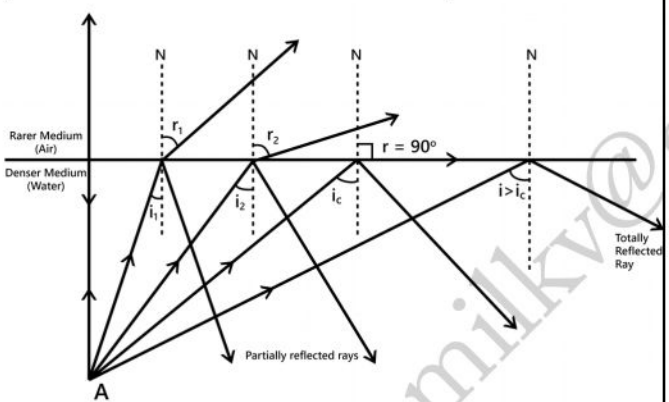
It can be shown that $$n_{21} = \frac{Real\ depth}{Apparent\ depth}$$

### (3) Early sunrise and delayed sunset




As we go up, the density of air in the atmosphere continuously decreases. Therefore the light coming from the sun is travelling from a rarer medium to denser medium. Therefore it bends towards the normal. Thus we see the sun at an apparent position raised above the horizon. This is the reason for early sunrise and delayed sunset.

## TOTAL INTERNAL REFLECTION




When light travels from an optically denser medium to a rarer medium at the interface, it is partly reflected back into the same medium and partly refracted to the second medium. This reflection is called the **internal reflection**.

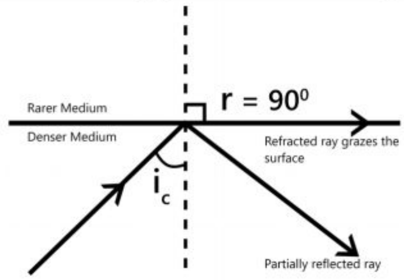
When light travel from a denser medium to a rarer medium, the refracted ray bends away from the normal. As the angle of incidence increases, the angle of refraction also increases. For a particular angle of incidence the angle of refraction becomes $90^0$. The angle of incidence in the denser medium for which angle of refraction $90^0$ is called the **critical angle($i_C$)**. If the angle of incidence in the denser medium is greater than critical angle, the ray gets totally reflects back to the same medium. This phenomenon is called **total internal reflection.**

### NOTE: Conditions for total internal reflection

(i) Light should travel from denser to rarer medium.

(ii) Angle of incidence should be greater than the critical angle.

### Refractive Index (n) and Critical Angle( $i_C$)



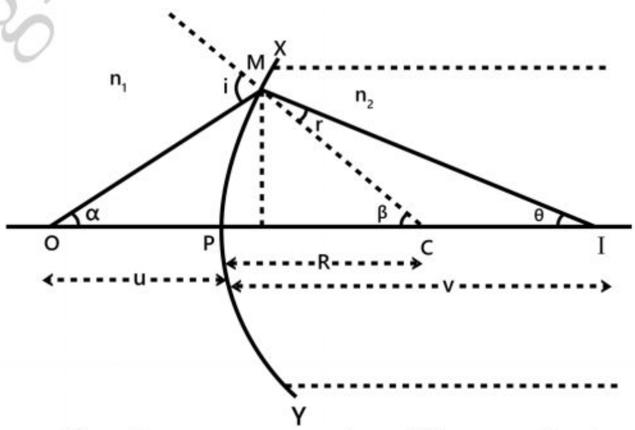

From Snell's law $\frac{\sin i_c}{\sin 90} = \frac{n_2}{n_1}$ => $\sin i_c = \frac{n_2}{n_1}$

If the rarer medium is air and the refractive index of denser medium is n. Then, $\sin i_c = \frac{1}{n}$

=> $$n = \frac{1}{\sin i_c}$$

## REFRACTION AT SPHERICAL SURFACES AND BY LENSES

### REFRACTION AT A SPHERICAL SURFACE (CURVED SURFACE FORMULA)




Consider a convex surface XY separating two transparent media of refractive indices $n_1$ and $n_2$. Let O be a point object in medium $n_1$ on the principal axis of the convex surface. I is the image of O formed on the other side of the convex surface. ( $n_2 > n_1$)

we take the aperture (or the lateral size) of the surface to be small compared to other distances involved, so that small angle approximation can be made.

We know that for small angles,

$$\tan\theta \approx \theta$$

From Δ OMP, $\tan\alpha \approx \alpha = \frac{PM}{PO}$

From Δ PCM, $\tan\beta \approx \beta = \frac{PM}{PC}$

From Δ PMI, $\tan\theta \approx \theta = \frac{PM}{PI}$

From triangle OMC, Exterior angle = sum of interior angles. Thus,

$$i = \alpha + \beta = \frac{PM}{PO} + \frac{PM}{PC} \quad ..............(1)$$

From triangle OMC,

$$\beta = r + \theta$$

=> $$r = \beta - \theta = \frac{PM}{PC} - \frac{PM}{PI} \quad ...............(2)$$

By Snell's law,

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1} \quad ................(3)$$

If i and r are small $\sin i = i$ , $\sin r = r$

Therefore (3)=> $\frac{i}{r} = \frac{n_2}{n_1}$

=> $$n_1 i = n_2 r$$

=> $$n_1\left(\frac{PM}{PO} + \frac{PM}{PC}\right) = n_2\left(\frac{PM}{PC} - \frac{PM}{PI}\right)$$

=> $$n_1\frac{PM}{PO} + n_1\frac{PM}{PC} = n_2\frac{PM}{PC} - n_2\frac{PM}{PI}$$

=> $$\frac{n_1}{PO} + \frac{n_1}{PC} = \frac{n_2}{PC} - \frac{n_2}{PI}$$

=> $$\frac{n_1}{PO} + \frac{n_2}{PI} = \frac{n_2 - n_1}{PC} \quad ...............(4)$$

Apply Cartesian sign convention ,

$PO = -u$ , $PI = v$ , $PC = R$

ie, $$\boxed{\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}} \quad ..............(5)$$

This is eqn for refraction at spherical surfaces.

**Note**

This formula is general for both concave and convex spherical surfaces.

**Problem 4 (Example 9.6 NCERT)**

Light from a point source in air falls on a spherical glass surface (n = 1.5 and radius of curvature = 20 cm). The distance of the light source from the glass surface is 100 cm. At what position the image is formed?

**Solution**

$n_1 = 1$ , $n_2 = 1.5$ , $R = 20\,cm$ , $u = -100\,cm$ , $v = ?$

We know eqn for refraction at spherical surface

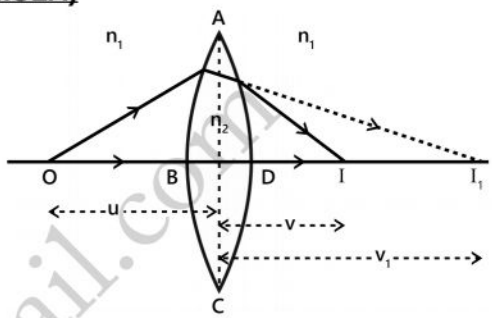
$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

$$\frac{n_2}{v} = \frac{n_1}{u} + \frac{n_2 - n_1}{R} \quad => \quad v = \frac{n_2}{\left(\frac{n_1}{u} + \frac{n_2 - n_1}{R}\right)}$$

$$v = \frac{1.5}{\left(\frac{-1}{100} + \frac{0.5}{20}\right)} = \frac{1.5}{(-0.01 + 0.025)} = +100\,cm$$

The image is formed at a distance of 100 cm from the glass surface, in the direction of incident light.

## REFRACTION BY A LENS ( LENS MAKER'S FORMULA)




Consider a point object placed on the principal axis of a convex lens. The image formation has two steps:

i. The first refracting surface forms the image $I_1$ of the object O.
ii. The image formed by the first refracting surface acts as the virtual object for the second refracting surface and the final image is formed at I.

We have the curved surface formula,

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

<u>For refraction at the surface ABC</u>

- Light ray travels from $n_1$ to $n_2$.
- O is the object and $I_1$ is the image
- The radius of curvature of ABC be $R_1$
- $u \rightarrow u$ , $v \rightarrow v_1$ , $R \rightarrow R_1$

Therefore , for this surface spherical surface formula becomes

$$\frac{n_2}{v_1} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} \quad ..............(1)$$

<u>For refraction at the surface ADC</u>

- Light ray travels from $n_2$ to $n_1$.
- $I_1$ is the object ( It act as a virtual object for the second surface ) and I is the image.
- $u \rightarrow v_1$ , $v \rightarrow v$ , $R \rightarrow R_2$

Therefore , for this surface spherical surface formula becomes

$$\frac{n_1}{v}-\frac{n_2}{v_1}=\frac{n_1-n_2}{R_2} \quad ...............(2)$$

=> $$\frac{n_1}{v}-\frac{n_2}{v_1}=\frac{-(n_2-n_1)}{R_2} \quad ...............(3)$$

(1) + (3) =>

$$\frac{n_2}{v_1}-\frac{n_1}{u}+\frac{n_1}{v}-\frac{n_2}{v_1}=\frac{n_2-n_1}{R_1}-\frac{(n_2-n_1)}{R_2}$$

=> $$\frac{n_1}{v}-\frac{n_1}{u}=(n_2-n_1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad ..........(4)$$

=> $$n_1\left(\frac{1}{v}-\frac{1}{u}\right)=(n_2-n_1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

=> $$\frac{1}{v}-\frac{1}{u}=\left(\frac{n_2}{n_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

=> $$\frac{1}{v}-\frac{1}{u}=(n_{21}-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad ............. (5)$$

If the object is at infinity, image formed at the principal focus. Ie, If $u=\infty$ , $v=f$

Therefore (5)=> $$\frac{1}{f}-\frac{1}{\infty}=(n_{21}-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

=> $$\boxed{\frac{1}{f}=(n_{21}-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)} \quad ...............(6)$$

This is **Lens maker's formula**.

**Thin Lens Formula**

From eqn (5) and eqn (6)

$$\boxed{\frac{1}{f}=\frac{1}{v}-\frac{1}{u}} \quad ...............(7)$$

This is **Thin lens formula.**

**NOTE**

The formula is valid for both convex as well as concave lenses and for both real and virtual images.

**linear magnification of a lens**

Magnification (m) produced by a lens is defined as the ratio of the size of the image to that of the object.

$$\boxed{m=\frac{h_i}{h_o}}$$

We can prove that $\boxed{m=\frac{v}{u}}$

***Problem 5 ( For more reading only)***

*why we are not using sign convention while substitute u,v,$R_1$,$R_2$ in the spherical surface formula in the lens maker's derivation?*

***Solution***

*See appendix 2*

***Problem 6 ( For more reading only)***

*How the image formed by the first surface acts as a virtual object for the second surface?*

***Solution***

*See appendix 3*

**POWER OF A LENS (P)**

Power of a lens is the reciprocal of focal length expressed in metre.

$$\boxed{P=\frac{1}{f}}$$

Unit : Dioptre ( D) $\quad 1D=1m^{-1}$

**Problem 7 ( Example 9.8 NCERT)**

If f = 0.5 m for a glass lens, what is the power of the lens?

**Solution**

$$P=\frac{1}{f}=\frac{1}{0.5}=2D$$

**Problem 8 ( Example 9.8 NCERT)**

The radii of curvature of the faces of a double convex lens are 10 cm and 15 cm. Its focal length is 12 cm. What is the refractive index of glass?

**Solution**

f = +12 cm, $R_1$ = +10 cm, $R_2$ = –15 cm, $n_{air}$=1

$$\frac{1}{f}=(n_{21}-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

=> $$\frac{1}{12}=(n-1)\left(\frac{1}{10}+\frac{1}{15}\right)$$

=> $$\frac{1}{12}=(n-1)(0.167)$$

=> $$n=\frac{1}{(12x\,0.167)}+1=1.5$$

**Problem 9 ( Example 9.8 NCERT)**

A convex lens has 20 cm focal length in air. What is focal length in water? (Refractive index of air-water = 1.33, refractive index for air-glass = 1.5.)

**Solution**

For a glass lens in air, $n_2$ = 1.5, $n_1$ = 1, f = +20 cm. Hence, the lens formula gives

$$\frac{1}{20}=(0.5)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad ..............(1)$$

For the same glass lens in water, $n_2$ = 1.5, $n_1$ =

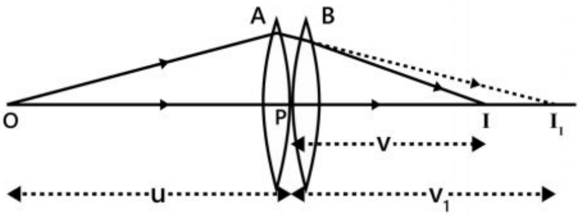
1.33, f = ?

$$\frac{1}{f}=(\frac{1.5}{1.33}-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad ..............(2)$$

(1)/(2)=> $\frac{f}{20}=\frac{0.5}{0.128}$

=> $f=\frac{0.5\,x\,20}{0.128}=78.1\,cm$

**COMBINATION OF LENSES IN CONTACT**




- For the first lens, the object is at 'O' and image is at $I_1$

$$u \rightarrow u \quad , \quad v \rightarrow v_1 \quad , \quad f \rightarrow f_1$$

So the lens eqn becomes

$$\frac{1}{f_1}=\frac{1}{v_1}-\frac{1}{u} \quad ..................(1)$$

- For the second lens object is $I_1$ and image is at I

$$u \rightarrow v_1 \quad , \quad v \rightarrow v \quad , \quad f \rightarrow f_2$$

So the lens eqn becomes

$$\frac{1}{f_2}=\frac{1}{v}-\frac{1}{v_1} \quad ..................(2)$$

(1) + (2) => $\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{v}-\frac{1}{u}$ ..................(3)

If the combination of the two lenses is replaced by a single lens of focal length f such that the image of the same object is formed at the same position. Then we have

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u} \quad ..................(4)$$

Comparing (3) and (4), the

**Effective focal length**

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2} \quad ................(5)$$

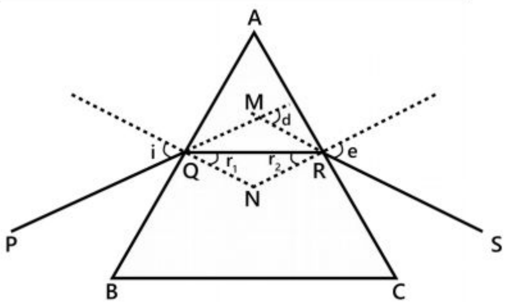
**Effective Power**

$$P=P_1+P_2 \quad ................(6)$$

**Effective Magnification**

$$m=m_1\,x\,m_2 \quad ................(7)$$

**REFRACTION THROUGH A PRISM**




**Angle of deviation, (d)**

The angle between the emergent ray RS and the direction of the incident ray PQ is called the angle of deviation, d.

**Angle of minimum deviation (D)**

The angle of deviation for which the refracted ray inside the prism becomes parallel to its base is called angle of minimum deviation.

**Prism Formula ( Equation for refractive index)**

In the quadrilateral A Q N R ,

$$<Q+<R=180^0$$

Therefore $<A+<N=180^0$ ..............(1)

From Δ Q N R,

$$r_1+r_2+<N=180^0 \quad ...............(2)$$

From (1) and (2)

$$r_1+r_2=A \quad ................(3)$$

from Δ Q R M , exterior angle = sum of opposite interior angles

$$(i-r_1)+(e-r_2)=d$$

=> $(i+e)-(r_1+r_2)=d$ .................(4)

sub (3) in (4)

(4)=> $(i+e)-A=d$

=> $(i+e)=A+d$ .................(5)

If we increase the angle of incidence, the angle of deviation decreases, reaches a minimum value and then increases.

At the minimum deviation D , the refracted ray inside the prism becomes parallel to its base and

$$r_1=r_2=r \quad i=e \quad \text{and} \quad d=D$$

At the minimum deviation position ,

(3) => $r+r=A$ => $2r=A$

=> $r=\frac{A}{2}$ ................(6)

(4)=> $i+i=A+D$ => $2i=A+D$

=> $i=\frac{A+D}{2}$ .................(7)

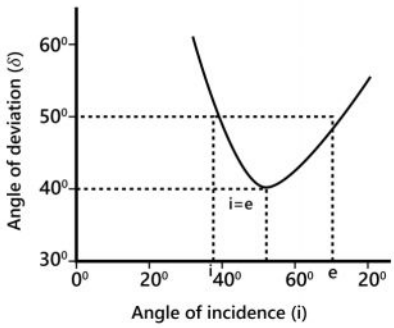
By Snell's law refractive index of the material of the prism , $n=\frac{sini}{sinr}$ ..................(8)

Substitute (6) and (7) in (8)

(8) => $$n=\frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

**i – d Curve**

It is the graph between angle of incidence (i) and angle of deviation (δ).




As the angle of incidence increases the angle of deviation decreases at first, reaches a minimum value, and then increases. The minimum value of deviation is called angle of minimum deviation.

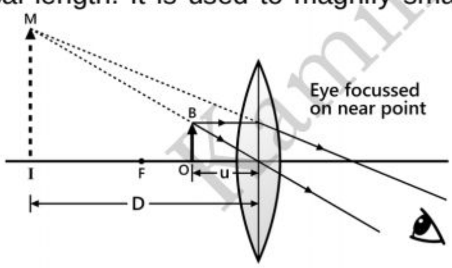
## OPTICAL INSTRUMENTS

### MICROSCOPES

Microscopes are used to get magnified images of near objects.

**(i) SIMPLE MICROSCOPE ( MAGNIFYING LENS )**

A simple microscope is a convex lens of small focal length. It is used to magnify small and nearby objects.




In the simple microscope , convex lens of small focal length is kept close to the eye and the object is placed between the lens and its focus. The image is formed at the near point( Least distance of the distinct vision)

**Magnification of the simple microscope if the image is at near point (m)**

$$m=\frac{v}{u} \quad .............(1)$$

We know , the lens equation $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$ ...........(2)

(2) x v => $\frac{v}{f}=1-\frac{v}{u}$ => $\frac{-D}{f}=1-\frac{-v}{-u}$

$\frac{-D}{f}=1-\frac{v}{u}$ => $\frac{v}{u}=1+\frac{D}{f}$

=> $$m=1+\frac{D}{f} \quad .............(3)$$

ie, lesser the focal length of the convex lens, greater is the magnification.

**Note**

This vision is not very much comfortable for our eye. Our eye will be comfortable when the rays coming to eye lens is parallel rays. Ie, when the image forms at infinity ( So we keep the object in the focus of the convex lens so we get an image at infinity)

**Magnification of the simple microscope if the image is formed at infinity(m)**

In this case we find magnification in terms of the angular magnification.

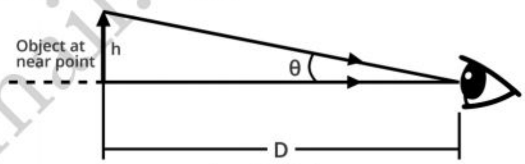



fig.a - Unaided eye.

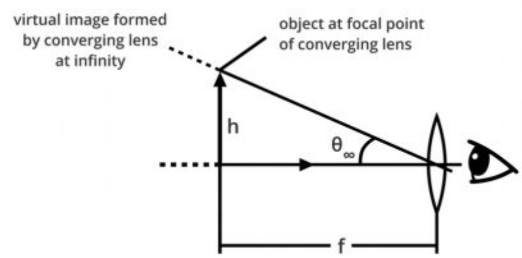



fig.b - Eye aided by convex lens

Since the size of the image on the retina is proportional to the angular size, the magnification

$$m=\frac{\theta_{aided}}{\theta_{un\,aided}} \quad .............(1)$$

where,

$\theta_{unaided}$ is the angular size of the object when it is placed at the near point and

$\theta_{aided}$ is the angular size of the object when it is placed at the focus of the convex lens.

From fig.a $\theta_{un\,aided}=\theta=\frac{h}{D}$ .............(2)

From fig.b $\theta_{aided}=\theta_{\infty}=\frac{h}{f}$ .............(3)

Therefore $m=\frac{\frac{h}{f}}{\frac{h}{D}}$ => $\boxed{m=\frac{D}{f}}$ .............(4)

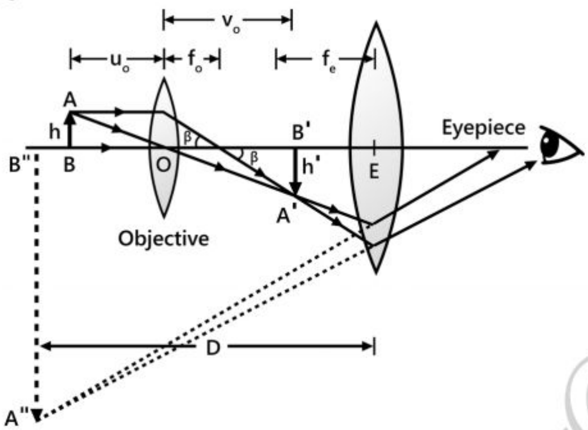
**Note**

This is one less than the magnification when the image is at the near point, but the viewing is more comfortable and the difference in magnification is usually small.

**Limitation of simple microscope**

A simple microscope has a limited maximum magnification (m≤9) for realistic focal lengths. For greater magnifications a compound microscope is used.

**(ii) COMPOUND MICROSCOPE**




Compound microscope consists of two convex lenses objective and eye piece. The focal length and aperture of objective is less than those of eye piece.

When an object is placed beyond the focal length of the objective, a magnified, real and inverted image is formed beyond the '2f' of the objective on the other side.

The distance between the lenses is adjusted so that this image falls within the focal length of the eye piece.

Now the eyepiece acts as a simple microscope and the final image is formed at the least distance of distinct vision.

**Magnification if the final image is at the near point**

The magnification produced by the objective,

$$m_o=\frac{v_o}{u_o}$$

Where,

$v_o$ -> Distance of the image from the objective

$u_o$ -> Distance of the object from the objective

Magnification produced by the eyepiece when the image is formed at the near point,

$$m_e=1+\frac{D}{f_e}$$

Therefore, the magnification produced by the compound microscope,

$$m=m_o x m_e=\frac{v_o}{u_o}\left(1+\frac{D}{f_e}\right)$$

If 'L' is the distance between objective and eyepiece, $v_o \approx L$ since the first image is formed very close to eyepiece.

Also, $u_o \approx f_o$ since the object is placed very close to the focus. So

$$\boxed{m=\frac{L}{f_o}\left(1+\frac{D}{f_e}\right)}$$

**Magnification if the final image is at infinity**

$$\boxed{m=\frac{L}{f_o}\left(\frac{D}{f_e}\right)}$$

From the above equation, it is very clear that, to achieve a large magnification of a small object, the objective and eyepiece should have small focal lengths.

**Problem**

A simple microscope has a focal length 4cm, and is used to view an object by a person whose least distance of distinct vision is 25 cm. If he holds the lens close to the eye, what is the position of the object? What is the magnification obtained?

**Solution**

$f=4\,cm$ $\qquad D=25\,cm$

We know the lens equation,

$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$ => $\frac{1}{u}=\frac{1}{v}-\frac{1}{f}$ => $u=\frac{fv}{f-v}$

$v=-D=-25\,cm$

=> $u=\frac{4x-25}{4-(-25)}=-3.45\,cm$

The magnification, $m=1+\frac{D}{f}=1+\frac{25}{4}=7.25$

**Problem**

A compound microscope has a magnification of 30. The focal length of its eyepiece is 5 cm. Assuming the final image to be formed at the least distance of distinct vision. Calculate the magnification produced by the objective.

**Solution**

$m=30$ , $f_e=5\,cm$

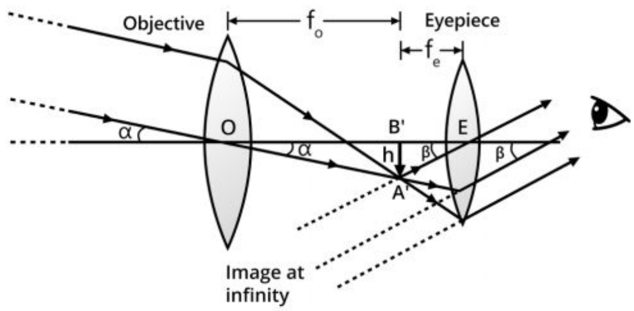
$m=m_o x m_e$ , $m_e=\left(1+\frac{D}{f_e}\right)$

Therefore $m_o = \dfrac{m}{\left(1+\dfrac{D}{f_e}\right)} = \dfrac{30}{\left(1+\dfrac{25}{5}\right)} = 5$

## TELESCOPES

Telescope is used to provide angular magnification of distant objects.

**(i) REFRACTING TYPE TELESCOPE**




In a refracting telescope there are two convex lenses- the objective and eyepiece. The objective has a large focal length and much larger aperture than the eyepiece.

Light from a distant object enters the objective and a real and inverted image is formed at its focus ($F_o$). The eyepiece magnifies this image producing a final inverted image with respect to the object. The most comfortable viewing is when the image produced by the objective is at the focus of the eyepiece so that the final image is produced at the infinity.

**Magnification(m)**

Magnification of a telescope is the ratio of the angle subtended at the observer's eye when looking through the eyepiece to the final image formed at the infinity(angle 'β') to the angle subtended if viewed by the unaided eye (same as the angle subtended at the objective( angle 'α')

$$m = \frac{\beta}{\alpha} \approx \frac{\dfrac{h}{f_e}}{\dfrac{h}{f_o}}$$

=> $$\boxed{m = \frac{f_o}{f_e}}$$

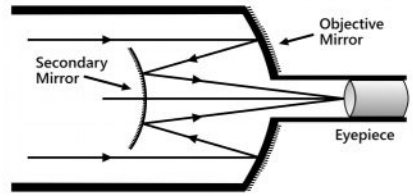
**Tube Length (L)**

The length of the telescope tube tube length is

$$\boxed{L = f_o + f_e}$$

**(ii) REFLECTING TYPE TELESCOPE**

Telescopes with mirror objectives are called reflecting telescopes.



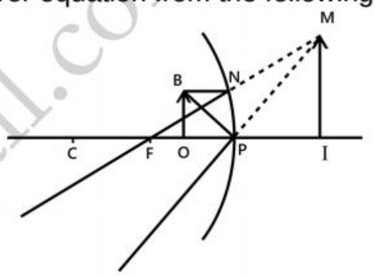

**Advantages**

1. There is no chromatic aberration in a mirror.
2. If a parabolic reflecting surface is chosen, spherical aberration is also removed.
3. Mechanical support is much less since a mirror weighs much less than a lens of equivalent optical quality,

**APPENDIX**

**Appendix 1**

Derive mirror equation from the following diagram




**Solution**

The two right triangle Δ OBP and Δ IMP are similar.

Therefore, $\dfrac{IM}{OB} = \dfrac{IP}{OP}$ ............(i)

Now from similar triangles, Δ IFM and Δ FNP

$\dfrac{IM}{NP} = \dfrac{IF}{FP}$ ............(ii)

But $NP = OB$

Therefore, (ii) => $\dfrac{IM}{OB} = \dfrac{IF}{FP}$ ...........(iii)

Comparing (i) and (iii) $\dfrac{IP}{OP} = \dfrac{IF}{FP} = \dfrac{IP+FP}{FP}$

=> $\dfrac{IP}{OP} = \dfrac{IP+FP}{FP}$ ...............(iv)

By applying cartesian sign convention,

$IP = v$ , $OP = -u$ , $FP = -f$

Therefore, (5) => $\dfrac{v}{-u} = \dfrac{v-f}{-f}$

=> $-vf = -uv + uf$ ................(v)

dividing throughout by uvf

(v) => $\dfrac{-1}{u} = \dfrac{-1}{f} + \dfrac{1}{v}$

=> $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$ This is the **mirror equation.**

**NOTE**

Just compare (5) of first derivation (In the derivation of mirror equation) with (iv) of the present derivation, we get different expression. ie, if we not applied sign convention we get different expression for focal length according to the position of the object. So we just convert distances such as object distance, image distance to their respective co ordinates to get a uniform expression in all the cases and for convex mirror too. Ie, we are not applying sign convention as such, but changing the distances to co ordinates only. Therefore better to say

u = x coordinate of object

v = x coordinate of image, etc.

**Appendix 2**

why we are not using sign convention while substitute u,v,$R_1$,$R_2$ in the spherical surface formula in the lens maker's derivation?

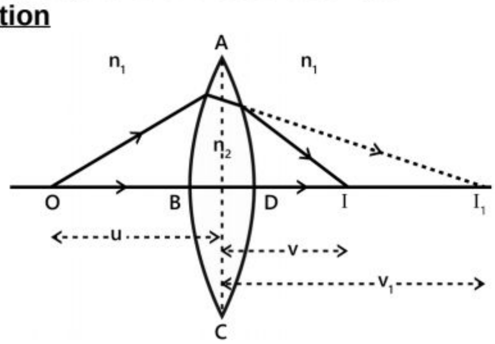
**Solution**

Spherical surface formula is a general formula for any curved surface(We already applied sign convention in the derivation spherical surface formula to use the formula for any spherical surface).But while we are solving numerical problems, which deals specific cases , then we would have apply sign convention. In short while we substitute numbers with a sign in a general formula, we are solving that specific case like a numerical. But, our goal is not to solve for a specific case. Our goal is to solve for a general case. That is the reason we not use a sign convention in this derivation and as a result we end up with a general formula which works for any case(for concave or convex)

**Appendix 3**

How the image formed by the first surface acts as a virtual object for the second surface?

**Solution**




Where is the object for the first surface?

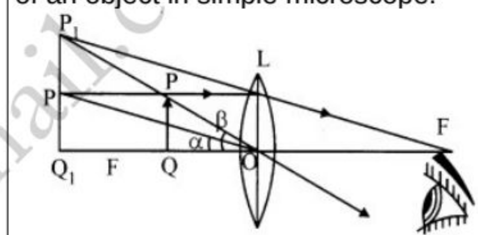
It is where there the incident rays coming to first surface meet, it is at O.

Where is the object for the second surface?

(Try to answer as above)

It is where there the incident rays coming to the second surface meet. That two rays do not meet anywhere, but appears to meet at $I_1$ and hence it act as a virtual object there.

**PREVIOUS QUESTIONS**

| | | |
|---|---|---|
| 1. | Total internal reflection may be observed if<br>(a) light ray is travelling from denser medium to rarer medium<br>(b) Light ray is travelling from rarer medium to denser medium<br>(c) light ray is travelling from any medium to another medium | 1 |
| 2. | The figure shows the image formation of an object in simple microscope.<br> | |
| a) | Find out the object distance and image distance from the figure. | 1 |
| b) | Derive an equation for magnifying power of the simple microscope. | 2 |
| | | |
| 3. | Write Lens maker's formula. | 1 |
| 4. | Using a suitable ray diagram prove that the radius of curvature of a spherical mirror is twice its focal length. | 2 |
| 5. | Using a suitable ray digram derive the relation $\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}$ for refraction at a spherical surface. | 4 |
| 6. | Draw a ray diagram showing the image formation in a refracting telescope when the final image is formed at infinity. Write an equation for the length of the telescope tube in terms of focal length of the objective and eyepiece. | 4 |
| 7. | Draw a diagram showing a ray of light passing through a triangular glass | 5 |

| | | |
|---|---|---|
| | prism. Derive an expression for the refractive index of the material of the prism. | |
| | | |
| 8.a) | If f= 0.5 m, for a glass lens, what is the power of the lens? | 1 |
| b) | The radii of curvature of .the faces of a double convex lens are 10 cm and 15 cm Its focal length is 12 cm. What is the refractive index of glass ? | 2 |
| 9.a) | State Laws of refraction. | 2 |
| b) | Obtain a relation for the total deviation produced for a ray incident on a prism with the help of a ray diagram· | 4 |
| | | |
| 10. | What is total internal reflection ? | 2 |
| 11. | Derive the expression for the refractive index of a prism with the help of a diagram. | 3 |
| 12.a) | With a neat diagram, derive lens makers formula. | 2 |
| b) | The radii of curvature of the faces of a double convex lens are 10 cm and 15 cm. Its focal length is 12 cm. What is the refractive index of glass ? | 2 |
| | | |
| 13. | Draw a diagram showing the ray of light passing through a glass prism. Derive an expression for refractive index. | 4 |
| 14. | A Telescope is used to provide angular magnification of distant objects : | |
| a) | Find the magnifying power of a telescope. | 2 |
| b) | If $f_o$ is the focal length of the object and $f_e$ that of eye piece, what is the length of Telescope tube ? | 1 |
| c) | Write any two advantages of reflecting telescope over refracting telescope. | 2 |
| | | |
| 15.a) | State Snell's law of refraction. | 1 |
| b) | Explain critical angle and total internal reflection. | 3 |
| 16.a) | Derive lens maker's formula. | 3 |
| b) | Draw the image formation in a simple microscope. | 1 |
| c) | Write the value of least distance of distinct vision. | 1 |
| | | |
| 17. | The S.I. unit of power of a lens is......... | 1 |
| 18.a) | Draw the ray diagram of a compound microscope. | 2 |
| b) | Obtain an expression for the magnification produced by a compound microscope. | 2 |
| 19. | Using the equation for a spherical surface $\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}$ derive Lens maker's formula | |
| | | |
| 20. | Two lenses of powers +7 D and -3 D are combined . The focal length of the combination would be ............... (a) -50 cm (b) +50 cm (c) -25 cm (d) +25 cm | |
| 21.a) | Represent the image formation of a compound microscope diagramatically. | 2 |
| b) | A small telescope has an objective of focal length 144 cm and an eye piece of focal length 6 cm. What is the magnifying power of the telescope ? What is the separation between the objective and eye piece? | 2 |
| 22.a) | With the help of a neat diagram obtain the expression for the refractive index of the material of the prism. | 4 |
| b) | Plot the graph showing angle of deviation versus angle of incidence for a triangular prism. | 1 |