CHAPTER 7
ALTERNATING CURRENT

AC VOLTAGE AND AC CURRENT

The voltage which varies sinusoidally with time is called **ac voltage**.

When such voltage applied across a load the current flowing through the load also varies sinusoidally with time. Such current is called the **ac current.**

ADVANTAGES OF AC VOLTAGE

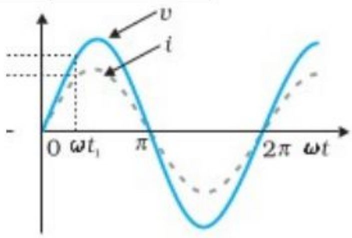
Most of the electrical devices use ac voltage.

- It can be easily and efficiently step up and step down by means of transformers.
- Electrical energy can be transmitted economically over long distances.

REPRESENTATION OF AC VOLTAGE AND CURRENT

Since AC voltage varies sinusoidally ,voltage can be represented by $v = v_m \sin \omega t$

Graphical representation

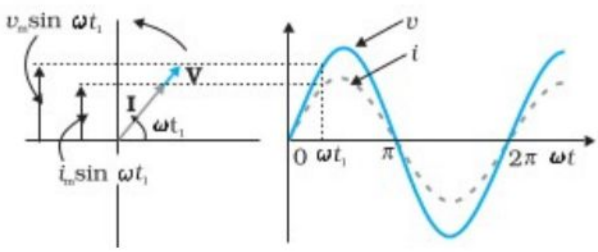


Phasor Diagram

A sinusoidal alternating voltage (or current) can be represented by a vector which rotates about the origin in anti-clockwise direction at a constant angular velocity ω. Such a rotating vector is called a **phasor**.

A phasor is drawn in such a way that

- The length of the phasor equals the peak value v_m (or i_m) of the alternating voltage (or current)
- Its angular velocity ω is equal to the angular frequency of the alternating voltage (or current)
- The vertical component of phasor represent the sinusoidally varying voltage (or current)



AC VOLTAGE APPLIED TO A RESISTOR

An AC source produce a sinusoidally varying pd across terminals.

Ie, $v = v_m \sin \omega t$ ....(1)

To find the value of current through the resistor, apply KVL to the loop.

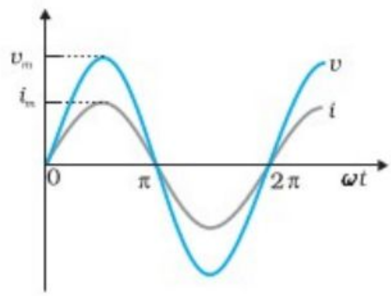
Ie, $iR = v_m \sin \omega t$ => $i = \frac{v_m}{R} \sin \omega t$

=> $i = i_m \sin \omega t$ ....(2)

where $i_m = \frac{v_m}{R}$ ....(3)

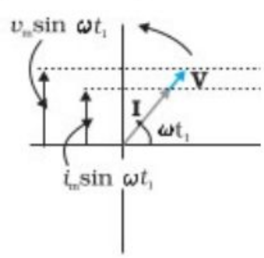
Note: On Comparing (1) and (2) , In a pure resistive circuit voltage and current are in phase

Plot of Voltage and current with time in a pure resistive circuit



In a pure resistor, the voltage and current are in phase. The minima, zero and maxima occur at the same respective times.

Phasor diagram for a pure resistive circuit



NOTE

- Since AC voltage and current are sinusoidally varying, the average value of voltage and current over one complete cycle is zero.

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- The average current is zero does not mean the average power consumed is zero and there is no dissipation of electrical energy. Joule heating is given by 'i^2R' depends on 'i^2' (which is always +ve whether i is -ve or +ve)

Power Dissipated in a Resistor

The instantaneous power dissipated in a resistor is given by

$$p = i^2 R = i_m^2 R \sin^2 \omega t$$

The average value of power over a cycle is

$$\bar{p} = \langle i^2 R \rangle = \langle i_m^2 R \sin^2 \omega t \rangle$$

=> $\bar{p} = \langle i_m^2 R \frac{(1-\cos 2\omega t)}{2} \rangle$

=> $\bar{p} = \langle \frac{i_m^2 R}{2} (1-\cos 2\omega t) \rangle$

=> $\bar{p} = \langle \frac{i_m^2 R}{2} - \frac{i_m^2 R}{2} \cos 2\omega t \rangle$

=> $\bar{p} = \langle \frac{i_m^2 R}{2} \rangle - \langle \frac{i_m^2 R}{2} \cos 2\omega t \rangle$

=> $\bar{p} = \frac{i_m^2 R}{2} - \frac{i_m^2 R}{2} \langle \cos 2\omega t \rangle$

=> $\bar{p} = \frac{i_m^2 R}{2} - \frac{i_m^2 R}{2} x\, 0$

=> $\boxed{\bar{p} = \frac{i_m^2 R}{2}}$ ....(4)

NOTE

- To express ac power in the same form as dc power ($P=I^2R$), a special value of current is defined and used. It is called, **root mean square (rms) current** and is denoted by I_{rms} .
- Therefore $\boxed{I_{rms} = \frac{i_m}{\sqrt{2}}}$ and $\boxed{V_{rms} = \frac{v_m}{\sqrt{2}}}$
- The house hold line voltage of 220 V is an rms value with a peak voltage of $v_m = \sqrt{2} V_{rms} = 311 V$
- In fact the I_{rms} is the equivalent dc current that would produce the same average power loss as the alternating current.

Example 7.1 NCERT

A light bulb is rated at 100W for a 220 V supply. Find
(a) the resistance of the bulb
(b) the peak voltage of the source
(c) the rms current through the bulb.

Solution

(a) $R = \frac{V^2}{P} = \frac{(220\,V)^2}{100\,W} = 484\,\Omega$

(b) $v_m = \sqrt{2} V_{rms} = \sqrt{2}$ x $220\,V = 311\,V$

(c) $I_{rms} = \frac{p}{V_{rms}} = \frac{100\,W}{220\,V} = 0.45\,A$

AC VOLTAGE APPLIED TO AN INDUCTOR

$\boxed{v = v_m \sin \omega t}$ ....(1)

Apply KVL to the loop

$$v = L \frac{di}{dt}$$

=> $v_m \sin \omega t = L \frac{di}{dt}$

=> $\frac{di}{dt} = \frac{v_m \sin \omega t}{L}$ => $\int \frac{di}{dt} dt = \frac{v_m}{L} \int \sin \omega t\, dt$

=> $i = \frac{v_m}{L\omega}(-\cos \omega t)$

=> $\boxed{i = i_m \sin\left(\omega t - \frac{\pi}{2}\right)}$ ....(2)

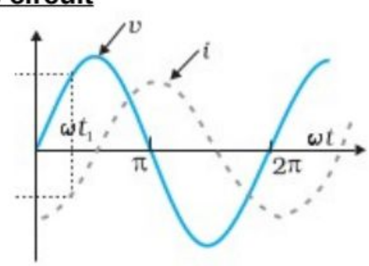
where, $\boxed{i_m = \frac{v_m}{L\omega}}$ is the amplitude of the current .

The quantity $L\omega$ is analogous to the resistance and is called **inductive reactance**, denoted by 'X_L'.

Ie, $\boxed{i_m = \frac{V_m}{X_L}}$

Note: On Comparing (1) and (2) , In a pure inductive circuit current lags behind the voltage by an angle $\frac{\pi}{2}$

Plot of Voltage and current with time in a pure inductive circuit



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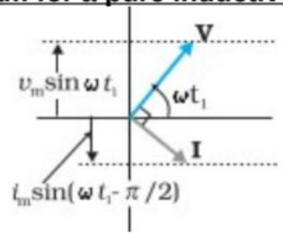
Phasor diagram for a pure inductive circuit



Power Dissipated in a Resistor

The instantaneous power dissipated in an inductor is given by

$$p = iv = i_m \sin(\omega t - \frac{\pi}{2}) v_m \sin \omega t$$

=> $p = -i_m v_m \cos \omega t \sin \omega t$

=> $p = \frac{-i_m v_m}{2} \sin 2\omega t$

So the average over a complete cycle is

$$\bar{p} = \langle \frac{-i_m v_m}{2} \sin 2\omega t \rangle$$

=> $\bar{p} = \frac{-i_m v_m}{2} \langle \sin 2\omega t \rangle = 0$

Thus, the average power supplied to an inductor over one complete cycle is zero

Example 7.2 NCERT

A pure inductor of 25.0 mH is connected to a source of220 V. Find the inductive reactance and rms current in the circuit if the frequency of the source is 50 Hz. Solution The inductive reactance, = 7.85Ω

Solution

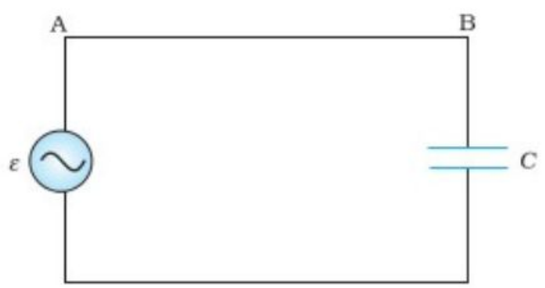
The inductive reactance, $X_L = L\omega = L(2\pi f)$

$$X_L = 25 x 10^{-3} x 2 x 3.14 x 50 = 7.85\Omega$$

The rms current in the circuit is

$$I_{rms} = \frac{V_{rms}}{X_L} = \frac{220 V}{7.85 \Omega} = 28 A$$

AC VOLTAGE APPLIED TO A CAPACITOR



$$v = v_m \sin \omega t \quad ....(1)$$

Apply KVL to the loop

$$v = \frac{q}{C}$$

=> $v_m \sin \omega t = \frac{q}{C}$ => $q = v_m C \sin \omega t$

=> $i = \frac{dq}{dt} = \frac{d}{dt}(v_m C \sin \omega t)$

=> $i = v_m C \omega \cos \omega t$

=> $i = v_m C \omega \sin(\omega t + \frac{\pi}{2})$

=> $i = \frac{v_m}{1/c\omega} \sin(\omega t + \frac{\pi}{2})$

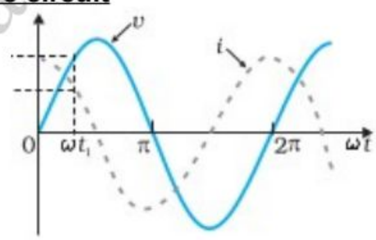
=> $i = i_m \sin(\omega t + \frac{\pi}{2})$ ....(2)

where, $i_m = \frac{v_m}{1/C\omega}$ is the amplitude of the current . The quantity $1/C\omega$ is analogous to the resistance and is called **capacitive reactance**, denoted by 'X_C'.

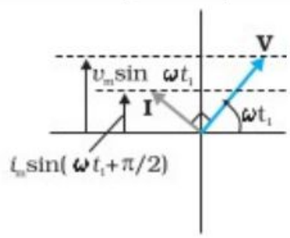
Ie, $i_m = \frac{V_m}{X_C}$

Note: On Comparing (1) and (2) , In a pure inductive circuit the current is π/2 ahead of voltage.

Plot of Voltage and current with time in a pure capacitive circuit



Phasor diagram for a pure capacitive circuit



Power Dissipated in a capacitor

The instantaneous power dissipated in an inductor is given by

$$p = iv = i_m \sin(\omega t + \frac{\pi}{2}) v_m \sin \omega t$$

=> $p = i_m \cos \omega t \, v_m \sin \omega t$

=> $p = i_m v_m \sin \omega t \cos \omega t$

=> $p = \frac{i_m v_m}{2} \sin 2\omega t$

So the average over a complete cycle is

$$\bar{p} = \langle \frac{i_m v_m}{2} \sin 2\omega t \rangle$$

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=> $\overline{p}=\frac{i_m v_m}{2}\langle \sin 2\omega t\rangle=0$

Thus, the average power supplied to a capacitor over one complete cycle is zero

Example 7.3 NCERT

A lamp is connected in series with a capacitor. Predict your observations for dc and ac connections. What happens in each case if the capacitance of the capacitor is reduced?

Solution

When a dc source is connected to a capacitor, the capacitor gets charged and after charging no current flows in the circuit and the lamp will not glow. There will be no change even if C is reduced.

With ac source, the capacitor offers capacitative reactance (1/Cω) and the current flows in the circuit. Consequently, the lamp will shine. Reducing C will increase reactance and the lamp will shine less brightly than before.

Example 7.4 NCERT

A 15.0 μF capacitor is connected to a 220 V, 50 Hz source. Find the capacitive reactance and the current (rms and peak) in the circuit. If the frequency is doubled, what happens to the capacitive reactance and the current?

Solution

The capacitive reactance

$$X_C=1/C\omega=\frac{1}{2\pi f C}$$

$$X_C=\frac{1}{2\,x\,3.14\,x\,50\,x\,15\,x\,10^{-6}}=212\,\Omega$$

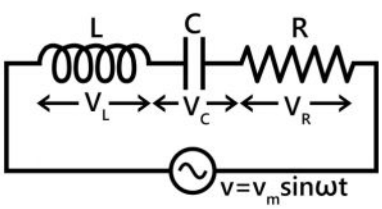
$$I_{rms}=\frac{V_{rms}}{X_C}=\frac{220}{212}=1.04\,A$$

$$i_m=\sqrt{2}\,I_{rms}=\sqrt{2}\,x\,1.04=1.47\,A$$

This current oscillates between +1.47A and –1.47 A, and is ahead of the voltage by π/2.

If the frequency is doubled, the capacitive reactance is halved and consequently, the current is doubled.

AC VOLTAGE APPLIED TO A SERIES LCR CIRCUIT

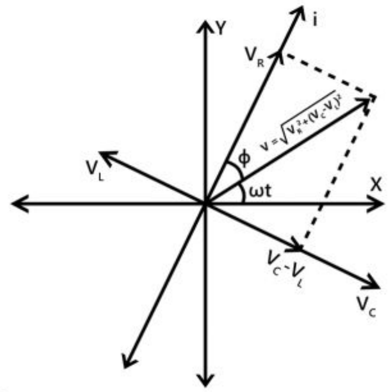


$$v=L\frac{di}{dt}+iR+\frac{q}{c} \quad ....(1)$$

We want to determine the instantaneous current i and its phase relationship to the applied alternating voltage v. we use Phasor- diagram solution. (This can be treated also by analytical solution, but not needed)

I. Phasor- diagram solution (Suppose $V_C > V_L$)

Consider an a.c. voltage $v=v_m \sin\omega t$ applied to a series LCR circuit .



From the phasor diagram, we can see that current i leads the resultant voltage by a phase angle φ.

Therefore $\boxed{i=i_m \sin(\omega t+\phi)}$

If we assume V L > V C , we will obtain

$$\boxed{i=i_m \sin(\omega t-\phi)}$$

Combining the above expressions,

$$\boxed{i=i_m \sin(\omega t\pm\phi)}$$

Impedence (Z)

Impedance (Z) means the total resistance offered by L, C and R towards AC.

From the phasor diagram we have,

$$v=\sqrt{V_R^2+(V_C-V_L)^2}$$

At the maximum value of v and i

$$v=v_m \quad , \quad V_R=i_m R \quad ,$$

$$V_C=i_m X_C \quad , \quad V_L=i_m X_L$$

Therefore,

$$v_m=\sqrt{(i_m R)^2+(i_m X_C-i_m X_L)^2}$$

=> $v_m=\sqrt{i_m^2 R^2+i_m^2 (X_C-X_L)^2}$

=> $v_m=i_m\sqrt{R^2+(X_C-X_L)^2}$

=> $\frac{v_m}{i_m}=\sqrt{R^2+(X_C-X_L)^2}$ But $\frac{v_m}{i_m}=Z$

=> $\boxed{Z=\sqrt{R^2+(X_C-X_L)^2}}$

This is the the impedance of LCR circuit.

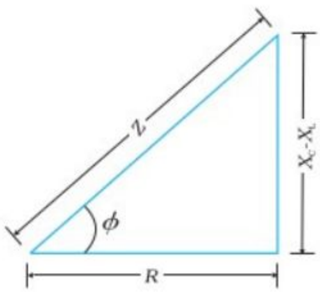
Where, $X_C = \frac{1}{c\omega}$, is the capacitative reactance,

$X_L = L\omega$, is the inductive reactance and

R , is the ohmic resistance

Impedance triangle

It is a right angled triangle, whose base represents the ohmic resistance,altitude represents the net reactance (X_C – X_L) and the hypotenuse represents the impedance (Z) of LCR circuit.



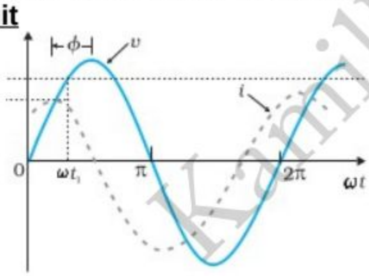
From the impedance triangle,

$$\tan\phi = \frac{X_C - X_L}{R} \quad \Rightarrow \quad \phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

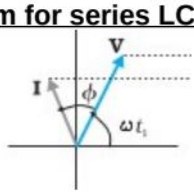
If $X_C > X_L$, Ø is positive, ie, the current leads the voltage.

If $X_C < X_L$, then Ø is negative ie, the current lags behind the voltage.

Plot of Voltage and current with time in a series LCR circuit



Phasor diagram for series LCR circuit



Resonance

For an LCR circuit driven with voltage of amplitude v_m and frequency ω , we found that the current amplitude is given by

$$i_m = \frac{v_m}{Z} = \frac{v_m}{\sqrt{R^2 + (X_C - X_L)^2}}$$

with $X_c = \frac{1}{C\omega}$ and $X_L = L\omega$

So if ω is varied, then at a particular frequency ω_0 , $X_C = X_L$, and the impedance is minimum Z = R . This frequency is called the **resonant frequency**.

Expression for resonant frequency

At resonance $X_c = X_L$ => $\frac{1}{C\omega_0} = L\omega_0$

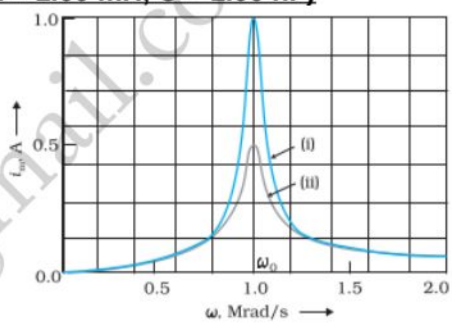
=> $\omega_0^2 = \frac{1}{LC}$ => $\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$

=> $2\pi f_0 = \frac{1}{\sqrt{LC}}$ => $\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$

NOTE

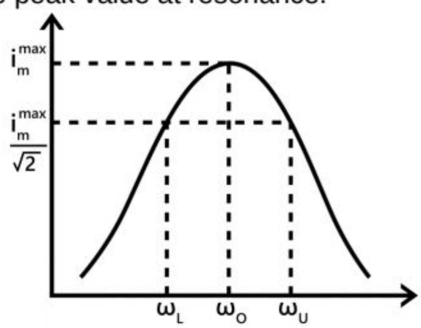
At resonant frequency, the current amplitude is maximum and is given by $i_m^{max} = \frac{v_m}{R}$

Variation of i_m with ω (for (i)R =100 Ω,(ii) R = 200 Ω, L = 1.00 mH, C = 1.00 nF)



Band width(β) of the resonance curve

Band width is defined as the difference of frequencies between on either side of resonance frequency for which the value of current is $\frac{1}{\sqrt{2}}$ times the peak value at resonance.



$$\beta = \omega_U - \omega_L$$

Where, ω_U -> Upper cut off frequency

ω_L -> Lower cut off frequency

Sharpness of the resonance curve

Sharpness of LCR circuit is defined as " the ratio of resonant frequency to the band width"

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$$S = \frac{\omega_0}{\omega_U - \omega_L}$$

The resonance curve is sharp if the band width is small.

Quality factor or Q–factor

$$Q = \frac{Voltage\ across\ L\ or\ C\ at\ resonance}{Applied\ voltage}$$

=> $Q = \frac{i_m X_L}{i_m Z} = \frac{i_m X_L}{i_m R} = \frac{X_L}{R}$

=> $\boxed{Q = \frac{L\omega_o}{R}}$

=> $Q = \frac{L}{R} x \frac{1}{\sqrt{LC}}$

=> $\boxed{Q = \frac{1}{R} x \sqrt{\frac{L}{C}}}$

Average power consumed by a series LCR circuit

The instantaneous power is then written as

$p = vi$

=> $p = v_m i_m \sin\omega t \sin(\omega t + \phi)$

=> $p = v_m i_m \sin\omega t [\sin\omega t \cos\phi + \cos\omega t \sin\phi]$

=>

$p = v_m i_m \sin^2\omega t \cos\phi + v_m i_m \sin\omega t \cos\omega t \sin\phi$

Then the average power consumed over a cycle,

=>

$\overline{p} = \langle v_m i_m \sin^2\omega t \cos\phi + v_m i_m \sin\omega t \cos\omega t \sin\phi \rangle$

Here $\langle \sin^2\omega t \rangle = \frac{1}{2}$ and $\langle \sin\omega t \cos\omega t \rangle = 0$

Substituting these values, we obtain average power over a cycle.

$\overline{p} = v_m i_m \cos\phi\, x \frac{1}{2}$ => $\overline{p} = \frac{v_m}{\sqrt{2}} \frac{i_m}{\sqrt{2}} \cos\phi$

=> $\boxed{\overline{p} = V_{rms} I_{rms} \cos\phi}$

$\cos\phi$ is called power factor and

$\phi = \tan^{-1}(\frac{X_C - X_L}{R})$

Special Cases

Case 1: Pure resistive circuit

The phase angle between voltage and current is

zero and $\cos\phi = 1$

Therefore $\boxed{\overline{p} = V_{rms} I_{rms}}$

Case 2: Pure capacitive or inductive circuit

The phase angle between voltage and current is

$\frac{\pi}{2}$ and $\cos\phi = 0$

Therefore $\boxed{\overline{p} = 0}$

Case 3: LCR circuit at resonance,

At resonance the phase angle between voltage and current is $\phi = \tan^{-1}(\frac{X_C - X_L}{R}) = 0$

=> $\cos\phi = 1$

Therefore $\boxed{\overline{p} = V_{rms} I_{rms}}$

Wattless Current

The current in an AC circuit is said to be wattless current if the power consumed by it is zero. This **wattless current** occurs in a purely inductive or capacitive circuit.

Example 7.8 NCERT

A sinusoidal voltage of peak value 283V and frequency 50 Hz is applied to a series LCR circuit in which R = 3 Ω, L = 25.48 mH, and C = 796 μF. Find (a) The impedance of the circuit

(b) The phase difference between the voltage across the source and the current

(c) The power dissipated in the circuit

(d) The power factor.

Solution

(a) To find the impedance of the circuit, we first calculate X_L and X_C.

$X_L = 2\pi fL$

$X_L = 2 x 3.14 x 50 x 25.48 x 10^{-3} = 8\Omega$

$X_C = \frac{1}{2\pi fC}$

$X_C = \frac{1}{2 x 3.14 x 50 x 796 x 10^{-6}} = 4\Omega$

Therefore, $Z = \sqrt{R^2 + (X_C - X_L)^2}$

$Z = \sqrt{3^2 + (4-8)^2} = 5\Omega$

(b) $\phi = \tan^{-1}(\frac{X_C - X_L}{R})$

$\phi = \tan^{-1}(\frac{4-8}{3}) = -53.1^0$

Since ϕ is negative, the current in the circuit lags the voltage across the source.

(c) $\overline{p} = V_{rms} I_{rms} \cos\phi$

$V_{rms} = \frac{v_m}{\sqrt{2}} = \frac{283}{\sqrt{2}} = 200.11$

$i_m = \frac{v_m}{Z} = \frac{283}{5} = 56.6$

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$$I_{rms}=\frac{i_m}{\sqrt{2}}=\frac{56.6}{\sqrt{2}}=40.02$$

Therefore

$$\overline{p}=200.11\,x\,40.02\,x\cos(-53.1)=4807\,W$$

(d) $\cos\phi=\cos(-53.1)=\cos(53.1)=0.6$

Example 7.9 NCERT

Suppose the frequency of the source in the previous
example can be varied.

(a) What is the frequency of the source at which resonance occurs?

(b) Calculate the impedance, the current,and the power dissipated at the resonant condition.

Solution

(a) $f_0=\frac{1}{2\pi\sqrt{LC}}$

$$f_0=\frac{1}{2\,x\,3.14\sqrt{25.48\,x\,10^{-3}\,x\,766\,x\,10^{-6}}}=35.4\,Hz$$

(b) $Z=R=3\,\Omega$

$$I_{rms}=\frac{v_{rms}}{Z}=\frac{v_{rms}}{R}=\frac{200.11}{3}=66.7\,A$$

$$\overline{p}=V_{rms}I_{rms}$$

$$\overline{p}=200.11\,x\,66.7=13340\,W$$

You can see that in the present case,power dissipated at resonance is more than the power dissipated in Example 7.8.

TRANSFORMERS

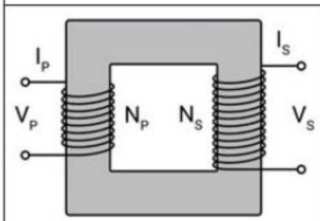
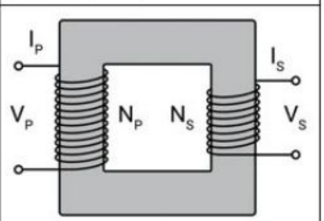
Principle

The principle of transformer is the mutual induction between two coils. That is, when an electric current passing through a coil changes with time, an emf is induced in the neighbouring coil.

Construction

In the simple construction of transformers, there are two coils of high mutual inductance wound over the same transformer core.

Types

Step up transformer	**Step down transformer**
	
$N_s>N_p, V_s>V_p, I_s<I_p$	$N_s<N_p, V_s<V_p, I_s>I_p$

The voltage induced in the secondary, when AC flows through the primary is given by

$$V_S=N_S\frac{d\phi}{dt} \quad .... (1)$$

At the same time, due to self- induction, the back e.m.f. produced in the primary is

$$V_P=N_P\frac{d\phi}{dt} \quad ....(2)$$

$\frac{(1)}{(2)}$ => $\boxed{\frac{V_S}{V_P}=\frac{N_S}{N_P}}$

If the transformer is assumed to be 100% efficient (no energy losses), the power input is equal to the power output.

That is , $\boxed{V_P I_P=V_S I_S}$

NOTE

$$\boxed{\frac{V_S}{V_P}=\frac{N_S}{N_P}=\frac{I_P}{I_S}}$$

Power losses in a transformer

1.Copper Loss

As the current flows through the primary and secondary copper wires, electric energy is wasted in the form of heat.

2.Eddy current Loss

The eddy currents produced in the soft iron core of the transformer produce heating. Thus electric energy is wasted in the form of heat.

3.Magnetic flux leakage

The entire magnetic flux produced by the primary coil may not be available to the secondary coil. Thus some energy is wasted.

4.Hysteresis Loss

Since the soft iron core is subjected to continuous cycles of magnetization, the core gets heated due to hysteresis. Thus some energy is wasted.

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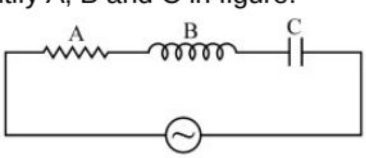
PREVIOUS QUESTIONS

1.	A light bulb of resistance 484 Ω is connected with 220 V ac supply. Find peak value of current through the bulb.	3
2.	Prove that when an alternating voltage is applied to an inductor, the current through it $\frac{\pi}{2}$ lags behind voltage by an angle 2.	3
3.a) b) c)	Working principle of transformer is (i) mutual induction (iii) resonance (iii) motional emf (iv) LC oscillations Write any one difference between step-up and step-down transformer. A power transmission line feeds input power at 3300 V to a step-down transformer with its primary windings having 6000 turns. What should be the number of turns in the secondary in order to get output power at 220 V ?	1 1 3
4.	In pure inductive or capacitive circuit, the power factor (cosϕ) is .... (a) 0 (b) 1 (c) -1	1
5.	A light bulb is rated at 100 W for a 220 V supply. Find the resistance of the bulb	2
6.	Obtain the expression for the current flowing through a resistor when an a.c. voltage is applied to it.	2
7.a) b) c)	State the principle of a transformer Explain the working of a transformer Differentiate between step up transformer and step down transformer.	1 2 2
8.	The household line voltage of ac measured is 220 V, calculate its peak voltage.	2
9.a) b) c)	Write the expression for instantaneous emf of a.c. Identify A, B and C in figure.  Draw the phasor diagram of the above circuit and write the expression for impedance in the circuit, then mention the terms.	
10. a) b)	The instantaneous current and voltage of an a.c circuit are given by i = 10 sin(314t) A and v = 50 sin (314t + π/2) What is the phase difference between voltage and current ? Power dissipation in the circuit ?	
11.a) b)	With the help of a phaser diagram, find the impedance of a series LCR circuit. A series LCR circuit is connected to an ac source of variable frequency as shown in figure below. At what frequency the impedance of this circuit will be minimum ?	

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