CHAPTER 6
ELECTROMAGNETIC INDUCTION

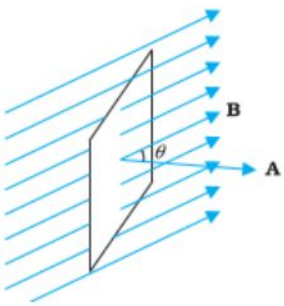
INTRODUCTION

- Electricity and magnetism were considered separate and unrelated phenomena for a long time.
- In the early decades of the nineteenth century, experiments by Oersted found that a current carrying conductor produces a magnetic field
- **Is the converse effect possible? Can moving magnets produce electric currents?**
  YES: 1830- Michael Faraday(England) and Joseph henry(USA)

MAGNETIC FLUX

Magnetic flux through a plane of area A placed in a uniform magnetic field B can be written as

$$\Phi_B = \bar{B}.\bar{A} = BA\cos\theta$$



Unit: weber (Wb) or Tm^2

ELECTROMAGNETIC INDUCTION

Whenever the magnetic flux linked with a coil changes, an emf is induced in the coil. This process
is called electromagnetic induction.

FARADAY'S LAW OF ELCTROMAGNETIC INDUCTION

The magnitude of the induced emf is equal to the rate
of change of magnetic flux.

$$e = \frac{d\phi}{dt}$$

If there are 'N' turns $\quad e = N\frac{d\phi}{dt}$

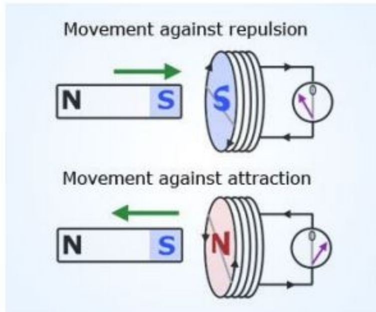
LENZ'S LAW

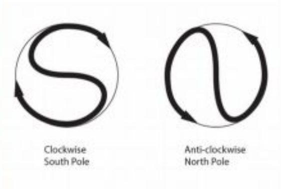
The direction of the induced e.m.f. is always such as
to oppose the rate of change of magnetic flux.

$$e = -\frac{d\phi}{dt}$$

If there are 'N' turns $\quad e = -N\frac{d\phi}{dt}$

The negative sign indicates that the induced emf opposes the change of flux.
Lenz's law gives the direction of induced e.m.f.





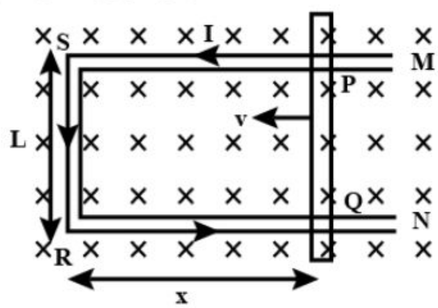
Lenz's law and Law of conservation of Energy

If the polarity of the coil is against Lenz's law, then the South-pole due to the induced current will face the approaching North-pole of the magnet and hence the bar magnet gains kinetic energy without expending any energy. This violates the law of conservation of energy and hence cannot happen.

If the polarity is as per Lenz's law, the bar magnet experiences a repulsive force due to the induced current and hence a person has to do work in moving the magnet. The energy spent by the person is dissipated by Joule heating produced by the induced current.

MOTIONAL EMF

The emf induced by the motion of a conductor in a magnetic field is called motional emf.
Expression for Motional emf



The magnetic flux Φ_B through the loop PQRS is,

$\Phi_B = BLx$

Since 'x' is changing with time, the rate of change of flux Φ B will induce an emf given by

$$e = -\frac{d\phi}{dt} \quad => \quad e = -\frac{d}{dt}(BLx)$$

$$=> \quad e = -\frac{d}{dt}(BLx) \quad => \quad e = -BL\frac{dx}{dt}$$

$$=> \quad \boxed{e = -BLv}$$

SELF INDUCTION AND MUTUAL INDUCTION

SELF INDUCTION

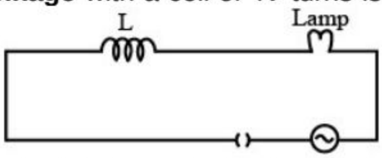
An emf can be induced in a coil by a flux change produced by the same coil is called self induction

The **flux** through a coil is proportional to the current. $\phi_B \alpha I$

Therefore the **flux linkage** with a coil of 'N' turns is $N\phi_B$ and

$N\phi_B \alpha I$

=> $N\phi_B = L I$



Where the constant 'L' is called the **Self Inductance** of the coil

EMF INDUCED IN A CONDUCTOR

We know $e = -N\frac{d\phi}{dt} = -L\frac{dI}{dt}$

ie, emf induced in an inductor $\boxed{e = -L\frac{dI}{dt}}$

NOTE:

1. If $\frac{dI}{dt} = 1\,A/s$, $\boxed{e = L}$ (Numerically)

Thus the self inductance of a coil is defined as the induced emf developed in the coil when the rate of change of current through it is unity.

2. The induced emf in a coil opposes the growth or decay of current through it. Therefore, it is called **back emf.**

3. Unit : henry (H)

EXPRESSION FOR SELF INDUCTANCE OF A COIL

Consider a solenoid total no. Of turns N, number of turns per unit length 'n' and length l.

Magnetic flux linked with the solenoid,

$$N\phi_B = N B A$$

But the Magnetic field inside the solenoid,

$$B = \mu_0 n I$$

=> $$N\phi_B = nl\,\mu_0 n I A$$

$$N\phi_B = (\mu_0 n^2 A l) I$$

Comparing this with $N\phi_B = L I$

=> $$\boxed{L = \mu_0 n^2 A l}$$

Problem 1

A solenoid of length 'l' metre has self-inductance 'L' henry. If number of turns are doubled (without any change in its length), what is its new self inductance?

Ans) 4 times

Problem 2

Current in a circuit falls from 5A to 1A in 0.1 second. If an average emf of 200 volts is induced, find the self inductance of the coil.

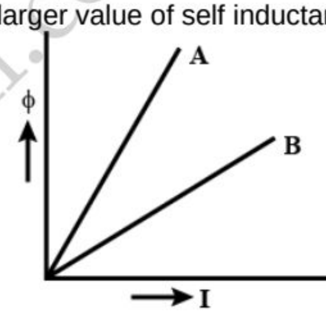
Ans) $e = -L\frac{dI}{dt}$

$$\frac{dI}{dt} = \frac{4}{0.1} = 40$$

$$L = \frac{e}{\frac{dI}{dt}} = \frac{200}{40} = 5H$$

Problem

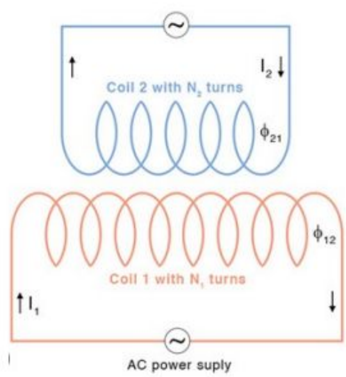
A plot of magnetic flux(ϕ) versus current(I) is shown in the figure for the two inductors A and B. Which of the two has larger value of self inductance?



Ans) Slope $\frac{\phi}{I}$ is proportional to L. A has higher L

MUTUAL INDUCTION

In the two neighbouring coil, when the current flowing through a coil changes an e.m.f. is induced in the neighbouring coil. This is called mutual induction .



The **flux linkage** through the second coil is proportional to the current in the first coil

$$N_2 \phi_2 \alpha I_1$$

=> $$N_2 \phi_2 = M I_1$$

Where the constant 'M' is called the **Mutual Inductance** of the coil

EMF INDUCED IN THE SECOND COIL

We know $e = -N_2 \frac{d\phi_2}{dt} = -M \frac{dI_1}{dt}$

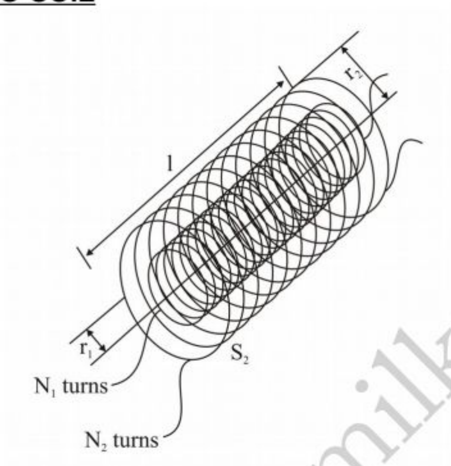
ie, emf induced in an inductor $\boxed{e = -M \frac{dI_1}{dt}}$

NOTE:

1. If $\frac{dI_1}{dt} = 1\, A/s$, $\boxed{e = M}$ (Numerically)

Thus the Mutual inductance of two coils is defined as the induced e.m.f developed in the secondary coil, when the rate of change of current through the primary is unity

EXPRESSION FOR THE MUTUAL INDUCTANCE OF TWO COIL



For the inner solenoid S 1

Radius - r_1

Number of turns per unit length - n_1

Total number of turns – N_1

Area of the coil – A_1

For the inner solenoid S 2

Radius - r_2

Number of turns per unit length - n_2

Total number of turns – N_2

Area of the coil – A_2

Flux linkage with solenoid S 1

$$N_1 \phi_1 = M_{12} I_2$$

Flux linkage with solenoid S 2

$$N_2 \phi_2 = M_{21} I_1$$

EXPRESSION FOR M 12

We know $N_1 \phi_1 = M_{12} I_2$

=> $n_1 l B_2 A_1 = M_{12} I_2$

=> $n_1 l \mu_0 n_2 I_2 \pi r_1^2 = M_{12} I_2$

=> $\mu_0 n_1 n_2 \pi r_1^2 l I_2 = M_{12} I_2$

=> $M_{12} = \mu_0 n_1 n_2 \pi r_1^2 l$

EXPRESSION FOR M 21

We know $N_2 \phi_2 = M_{21} I_1$

=> $n_2 l B_1 A_1 = M_{21} I_1$

=> $n_1 l \mu_0 n_1 I_1 \pi r_1^2 = M_{21} I_1$

=> $\mu_0 n_1 n_2 \pi r_1^2 l I_1 = M_{21} I_1$

=> $M_{21} = \mu_0 n_1 n_2 \pi r_1^2 l$

NOTE

$$M_{12} = M_{21} = M$$

ENERGY STORED IN AN INDUCTOR

When current through a coil increases, back e.m.f. is developed. This e.m.f. opposes the growth of current through the circuit. Therefore some work has to be done by the voltage source to establish a current in the circuit. This work done is stored as the energy of the inductor.

Workdone in dt time , $dw = P\, dt$

=> $$dw = e\, I\, dt = L \frac{dI}{dt} I\, dt$$

=> $$dw = L\, I\, dI$$

Therefore the total work done in increasing the current from '0' to ' I ' is given by,

=> $$w = \int_0^I L\, I\, dI$$

=> $$w = L \int_0^I I\, dI = L \left[\frac{I^2}{2} \right]_0^I$$

=> $$w = \frac{1}{2} L I^2$$

Therefore the energy stored in an inductor

$$\boxed{U = \frac{1}{2} L I^2}$$

NOTE: Energy stored in an inductor is in the form of a magnetic potential energy

Example 6.9 NCERT

Obtain the expression for the magnetic energy stored in a solenoid in terms of magnetic field B, area A and length l of the solenoid.

Soln) $U = \frac{1}{2} L I^2$

=> $U = \frac{1}{2} \mu_0 n^2 A l \left(\frac{B}{\mu_0 n} \right)^2$

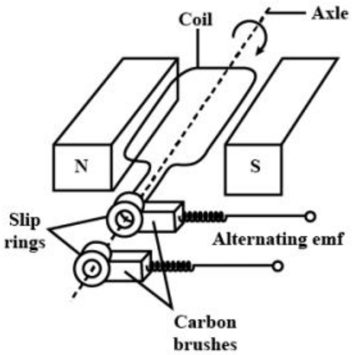
=> $U = \frac{1}{2\mu_0} B^2 A l$

COMBINED EFFECT OF SELF INDUCTANCE AND MUTUAL INDUCTANCE OF COIL

When currents are flowing simultaneously through two coils we get $e_1 = -L\frac{dI_1}{dt} - M_{12}\frac{dI_2}{dt}$

AC GENERATOR

Construction



The basic elements of an ac generator are
(i) Field magnet
(ii) Armature
(iii) Slip rings
(iv) Brushes

Principle

A.C. generator works on the principle of electro magnetic induction. In a generator mechanical energy is converted in to electric energy

When the armature coil rotates between the pole pieces of field magnet, magnetic flux linked with the armature continuously changes an emf induced in the coil.

We know the flux linked the coil

$$\Phi = NBA\cos\theta = NBA\cos\omega t$$

According to Farady's law $e = -\frac{d\phi}{dt}$

=> $e = -\frac{d}{dt}(NBA\cos\omega t)$

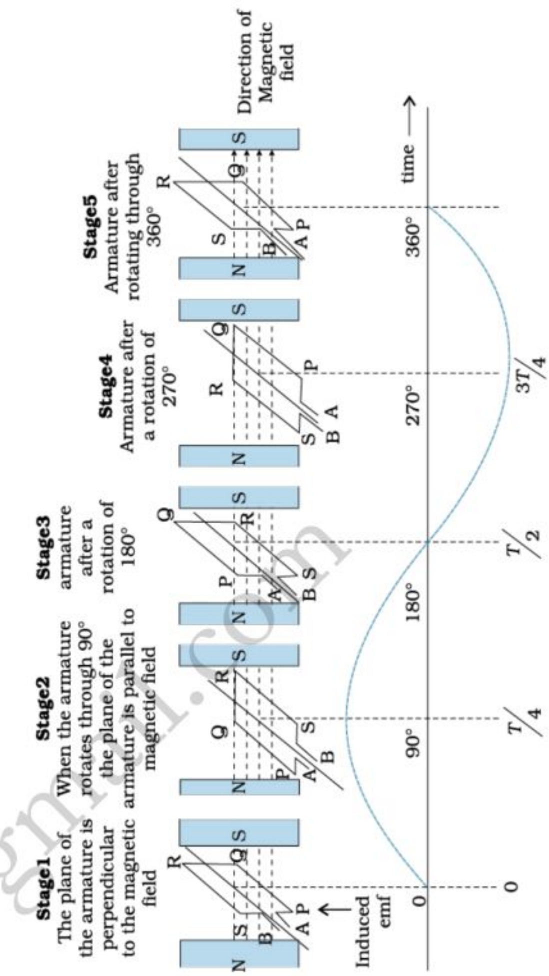
=> $e = NBA\,\omega\sin\omega t$

=> $e = e_0\sin\omega t$

Where $e_0 = NBA\,\omega$

ie, a sinusoidal emf generates in the armature

An alternating emf is generated by a loop of wire rotating in a magnetic field.



PREVIOUS YEAR QUESTIONS

1.a)	Coefficient of mutual inductance of two coils is 1 H. Current in one of the coils is increased from 4 A to 5 A in 1 ms. What average emf will be induced in the other coil ? (a) 1000 V (b) 2000 V (c) 100 V (d) 200 V	1
2.a)	State Faraday's law of electromagnetic induction.	1
b)	How does the magnetic energy stored in an inductor and electrostatic energy stored in a capacitor related to their respective field strengths ?	3
3.	The direction of induced emf in a conductor placed in a varying magnetic field is given (a) Faraday's Law (b) Lenz's Law	1

	(c) Maxwell's law (d) None of these	
4.	Current in a circuit falls from 5.0 A to 0.0 A in 0.1 s. If an average emf of 200 V is induced, calculate the self-inductance of the circuit.	2
5.	State Faraday's laws of electromagnetic induction.	2
6.	State the principle of an ac generator and explain its working with the help of a labelled diagram. Obtain the expression for the emf induced.	4
7.	State Faraday's laws of electromagnetic induction.	2
8. a) b) c)	Lenz's law gives the polarity of the induced e.m.f. in a conductor in a clear and concise fashion. Lenz's law is a consequence of law of conservation of .... State Lenz's law. Of which of the following the e.m.f. induced in a coil does not depend on : (i) number of turns in the coil. (ii) resistance of the coil. (iii) rate of change of magnetic flux.	
19.a) b)	State the principle of a.c. generator. Obtain the expression for the emf generated by an a.c. generator.	1 2
20.	What is self-induction and define the expression for self-inductance of a solenoid.	3
21.	Which of the following gives the polarity of the induced emf ? (i)Biot-Savart law (ii)Lenz's law (iii)Ampere's circuital law (iv)Fleming's right-hand rule	1
22.a) b)	What is the working principle of an a.c generator ? With the help of a diagram, explain theory an working of an a.c. generator.	1 3