CHAPTER 3
CURRENT ELECTRICITY

ELECTRIC CURRENT (I)

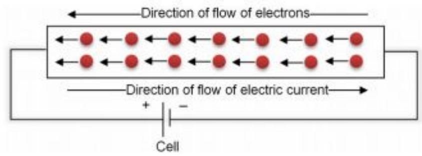
The electric current in a conductor is defined as the rate of flow of charges through a given cross-sectional area.

$$I = \frac{dQ}{dt}$$

- **Unit :** C/s -> ampere (A)

Thus the current through a wire is 1A if 1 C charge is flowing per second through any cross-section of the wire .

Direction of electric current



By convention direction of electric current is the direction of positive charges. In a conductor it is opposite to the direction of electron flow.

Note : Electric current is a scalar quantity

CURRENT DENSITY (J)

It is the current flowing through unit area of cross section of a conductor . $J = \frac{I}{A}$

- Unit : A/m^2

THERMAL VELOCITY OF ELECTRON

The velocity of an electron in the absence of an electric field in a conductor is the thermal velocity of the electron.

The average thermal velocity of the electrons in a conductor is zero , since they are moving randomly inside the conductor.

Let $u_1, u_2, u_3, ...u_n$ be the thermal velocities of electrons in a conductor.

Then, $\frac{u_1+u_2+u_3+.......+u_n}{n} = 0$

DRIFT VELOCITY (V_d)

When the ends of a conductor are connected to a battery (means applying electric field) the electrons move opposite to the electric field and get an average velocity. **The average velocity acquired by an electron in the presence of an external electric field is called drift velocity.**

RELAXATION TIME (τ)

Relaxation time is the average time interval between two successive collisions.

EXPRESSION FOR DRIFT VELOCITY

Let $u_1, u_2, u_3, ...u_n$ be the thermal velocities of electrons in a conductor.

Suppose a potential difference is set across the conductor by connecting a battery, an electric field is created in the conductor. This electric field exerts a force on the electrons, producing a current. The electric field accelerates the electrons with $a = \frac{-eE}{m}$ while the electrons accelerate, it collide with positive ions and scatter the electrons and change the direction of motion. Thus, we have zigzag paths of electrons. In addition to the zigzag motion due to the collisions, the electrons move slowly along the conductor to the positive terminal.

Let

$v_1 = u_1 + at_1, v_2 = u_2 + at_2, ...v_n = u_n + at_n$ be the velocity acquired by the electrons between two collisions. Therefore,

$$V_d = \frac{v_1+v_2+v_3+.......+v_n}{n}$$

$$V_d = \frac{u_1+at_1+u_2+at_2+.........u_n+at_n}{n}$$

$$V_d = \frac{u_1+u_2+u_3+.......+u_n}{n} + a\left(\frac{t_1+t_2+t_3+.......+t_n}{n}\right)$$

$$V_d = a\tau$$

$$V_d = \frac{-eE}{m}\tau$$

where

$\tau = \frac{t_1+t_2+t_3+.......+t_n}{n}$, relaxation time.

DRIFT VELOCITY AND CURRENT

Let l – length of the conductor

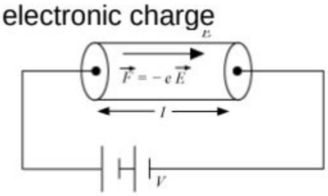
n – free electron density

V_d - drift velocity

V – potential difference across the conductor.

A – area of cross-section of the conductor

e – electronic charge



The total charge in the conductor of the volume , A l = n A l e

Time to cross the entire charge through a cross-section , $t=\frac{l}{V_d}$

Therefore the current , $I=\frac{q}{t}=\frac{nAle}{\frac{l}{V_d}}$

=> $\boxed{I=neAV_d}$

DRIFT VELOCITY AND CURRENT DENSITY

$$\boxed{J=\frac{I}{A}=neV_d}$$

DRIFT VELOCITY AND MOBILITY

The drift velocity of electron in a conductor is directly proportional to the electric field inside the conductor.

$\boxed{V_d \alpha E}$ Or $\boxed{V_d=\mu E}$ Or $\boxed{\mu=\frac{V_d}{E}}$

Where , μ is the mobility. ie , mobility is the drift velocity per electric field.

Mobility and current : $\boxed{I=neA\mu E}$

Problem 1 :

A current of 5 A is passing through a metallic wire of cross-sectional area 4 x 10 $^{-6}$ m^2 . If the density of the charge carrier in the wire is 5 x 10 ^{26} m $^{-3}$. Find the drift speed of the electron.

Solution

I = 5 A , A = 4 x 10 $^{-6}$ m^2 , n=5 x 10 ^{26} m $^{-3}$

$$V_d=\frac{I}{neA}=\frac{5}{5x10^{26}x1.6x10^{-19}x4x10^{-6}}$$

=> $V_d=1.5x10^{-2}m/s=1.5cm/s$

Problem 2

The electron drift speed is estimated to be only a few mm s $^{-1}$ for currents in the range of a few amperes? How then is current established almost the instant a circuit is closed?

Solution

Establishment of a current does not have to wait for electrons from one end of the conductor travelling to the other end. Electric field is established throughout the circuit, almost instantly (with the speed of light) causing at every point a local electron drift.

OHM'S LAW

Ohm's law states that **'at constant temperature the current flowing through a conductor is directly proportional to potential difference between the ends of the conductor'**

$\boxed{I\alpha V}$ Or $\boxed{I=\frac{1}{R}V}$ Or $\boxed{V=IR}$

Where , R is a constant called **resistance** of the conductor , and which depends on

- **Nature of the material** of the conductor
- Directly proportional to the **length of the conductor,** ' l '
- Inversely proportional to the **Area of cross-section ,** 'A'
- Directly proportional to the **temperature**

At constant temperature,

$\boxed{R\alpha\frac{l}{A}}$ or $\boxed{R=\rho\left(\frac{l}{A}\right)}$

Where , $'\rho'$ is called the **resistivity** of the material , which **depends** only on **nature of material and temperature** and is **independent** of the **size of the material**.

$\boxed{\rho=R\frac{A}{l}}$ Unit : Ω m

NOTE

If $A=1m^2, l=1m, \rho=R$

ie, Resistivity is numerically equal to the resistance of the conductor having length 1 m and area of cross-section 1 m^2

ALTERNATE FORMS OF OHM'S LAW

We know $V=IR$

=> $V=I\rho\frac{l}{A}$ => $El=I\rho\frac{l}{A}$ => $\boxed{E=J\rho}$

(or) $\boxed{J=\frac{E}{\rho}=\sigma E}$ Where , $\frac{1}{\rho}=\sigma$ is called the **conductivity** of the material

Unit : Ω^{-1} m^{-1}

Conductance is the reciprocal resistance Unit : Ω^{-1} (or) mho (or) siemens

LIMITATIONS OF OHM'S LAW

Although Ohm's law has been found valid over a large class of materials, there do exist materials and devices used in electric circuits where the proportionality of V and I does not hold.

Eg: Diode, GaAS etc.

RESISTIVITY AND RELAXATION TIME

We know $R=\rho\left(\frac{l}{A}\right)$ ....(1)

Also from Ohm's law $R=\frac{V}{I}$

=> $R=\frac{El}{I}$ => $R=\frac{El}{neAV_d}=\frac{El}{neA\frac{eE}{m}\tau}$

=> $$R=\frac{m}{ne^2\tau}\left(\frac{l}{A}\right)$$ ....(2)

Comparing (1) and (2)

$$\boxed{\rho=\frac{m}{ne^2\tau}}$$

TEMPERATURE DEPENDENCE OF RESISTANCE

When temperature increases, resistances of materials change accordingly

$$\boxed{R_2=R_1[1+\alpha(T_2-T_1)]}$$

Where R_2 ---> Resistance at temp T_2

R_1 ---> Resistance at temp T_1

α ---> Temperature coefficient of resistance

$$\boxed{\alpha=\frac{R_2-R_1}{R_1(T_2-T_1)}}$$

NOTES

- **For metals :** As temperature increases resistance also increases ($T_2>T_1$, $R_2>R_1$) ie, α is positive (or **metals have positive temperature coefficient of resistance**)
- **For semi conductors :** As temperature incre- ases resistance decreases ($T_2>T_1$, $R_2<R_1$) ie, α is negative (or **semi conductors have negative temperature coefficient of resistance**)
- **For alloys like manganine , Eureka :** As temperature increases resistance remains the same ($T_2>T_1$, $R_2=R_1$) ie, α =0 (or **these alloys have zero tempe rature coefficient of resistance**)
  materials like constantan (Eureka) and manganin are used to make standard resistances since their α =0

TEMPERATURE DEPENDENCE OF RESISTIVITY

$$\boxed{\rho_2=\rho_1[1+\alpha(T_2-T_1)]}$$

And $$\boxed{\alpha=\frac{\rho_2-\rho_1}{\rho_1(T_2-T_1)}}$$

Do Example 3.3 , 3.4 NCERT

Problem 3 (Do yourself)
A silver wire has a resistance of 2.1 Ω at 27.5 ^{0}C and a resistance of 2.7 Ω at 100 ^{0} C. Determine the temp coefficient of resistance of silver
[Ans : 3.94 × 10 $^{-3}$ / °C]

CELLS, INTERNAL RESISTANCE , EMF

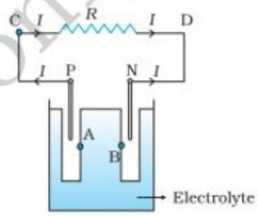
A cell is a single electrical energy source which uses chemical reactions to produce current in the conductors.

INTERNAL RESISTANCE OF A CELL

Internal resistance of a cell is the resistance offered by the electrolyte and electrodes of the cell.

The internal resistance of a cell depends on

1. The distance between electrodes of the cell.
2. Surface area of electrodes dipped in the electrolyte.
3. The nature of electrolyte.



(a)

EMF OF A CELL

The energy supplied by the cell to move one coulomb charge around the circuit including electrolytic solution is called EMF of the cell.

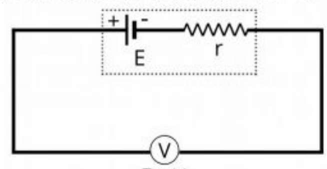


Fig (a)

In the fig(a) , the voltmeter gives the value of the EMF

TERMINAL POTENTIAL DIFFERENCE

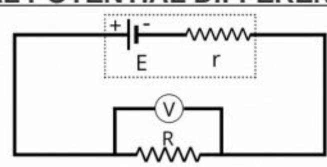


Fig (b)

Effective resistance of the circuit = R + r

Current through the circuit , $I=\frac{E}{R+r}$ ......(1)

Potential difference across the resistor , R = $IR=V$

(1)=> $I(R+r)=E$ => $V+Ir=E$

=> $\boxed{V=E-Ir}$ ....(2)

Where V = IR = Potential difference across the resistor , R = Potential difference

across the terminals of a cell when a current flows through it = The terminal potential difference of the cell

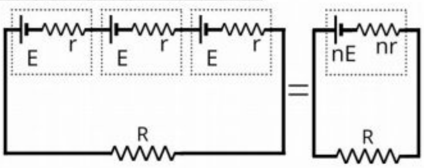
If the circuit is open, ($I=0$ through the cell) and (2) => $V=E$

That is EMF = The p.d between the terminals of a cell, when it is not sending a current

In the fig(b) , the voltmeter gives the value of the terminal pd of the cell

> **Problem 11**
> The storage battery of a car has an emf of 12V. If the internal resistance of the battery is 0.4Ω, what is the maximum current that can be drawn from the battery?
> **[Ans : 30 A]**

COMBINATION OF CELLS



1. SERIES COMBINATION

The emf of the battery is equal to the sum of the individual emfs of the various cell.

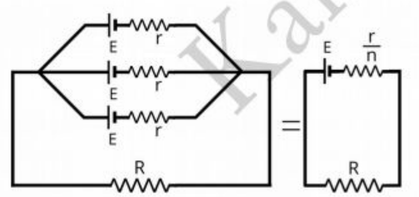
Ie, Total EMF = E + E + E +.........= n E

Total internal resistance = n R

Total resistance = R + n r

The current through each cell is same and the current $I=\frac{nE}{R+nr}$ ....(2)

2. PARALLEL COMBINATION



The emf of the battery is same as the emf of a single cell, E

The effective internal resistance is r^I is given by

$$\frac{1}{r^I}=\frac{1}{r}+\frac{1}{r}+............=\frac{n}{r}$$

=> $r^I=\frac{r}{n}$

Total resistance = $R+\frac{r}{n}$

The current through the circuit,

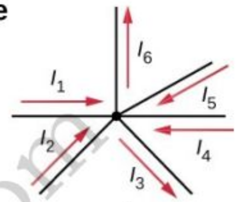
$$I=\frac{E}{R+\frac{r}{n}}=\frac{nE}{nR+r}$$

KIRCHOFF'S LAWS

(1) KIRCHOFF'S JUNCTION RULE OR KIRCHOFF'S CURRENT LAW (KCL)

The algebric sum of the current meeting at any junction is equal to zero.

Note : We adopt the following sign convention
Current entering a junction is taken as positive and current leaving the junction is taken as negative



$$I_1+I_2+I_4+I_5-I_3-I_6=0$$

Or $I_1+I_2+I_4+I_5=I_3+I_6$

The current entering the junction is equal to the current leaving the junction.

Note

Kirchhoff's Current Law (KCL) deals with the conservation of charge entering and leaving a junction.

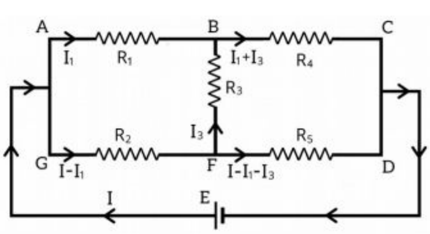
(2) KIRCHOFF'S LOOP RULE OR KIRCHOFF'S VOLTAGE LAW (KVL)

In a closed loop the algebric sum of the voltages plus the algebric sum of the emfs in any closed path is zero

$$\sum IR+\sum emf=0$$

Note : We adopt the following sign convention

A rise in potential is taken as positive and a fall in potential is taken as negative.

> **Problem 12 :**
> Apply KVL to loop
> 1) ABFGA2) ABFDEA
>
> 
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>
> **Soln :**
> 1) KVL to loop ABFGA

$$-I_1R_1+I_3R_3+(I-I_1)R_2=0$$

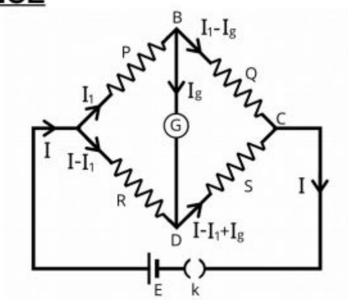
2) KVL to loop ABFDEA

$$-I_1R_1+I_3R_3-(I-I_1-I_3)R_5+E=0$$

WHEATSTONE'S BRIDGE

It is an arrangement to find the resistance of a given wire.

BALANCING CONDITION OF WB AND DETERMINATION OF THE UNKNOWN RESISTANCE



Apply KVL to loop ABDA ,

$$-I_1P-I_gG+(I-I_1)R=0$$

=> $I_1P+I_gG-(I-I_1)R=0$ ....(1)

If $I_g=0$

(1) => $I_1P-(I-I_1)R=0$

=> $I_1P=(I-I_1)R$ ....(2)

Apply KVL to loop BCDB ,

$$-(I_1-I_g)Q+(I-I_1+I_g)S+I_gG=0$$

=> $(I_1-I_g)Q-(I-I_1+I_g)S-I_gG=0$ ....(3)

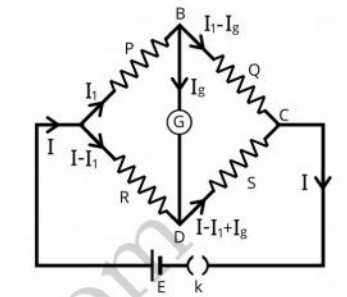
If $I_g=0$

(3) => $I_1Q-(I-I_1)S=0$

=> $I_1Q=(I-I_1)S$ ....(4)

$\frac{(2)}{(4)}$ = > $\boxed{\frac{P}{Q}=\frac{R}{S}}$

PREVIOUS YEAR QUSTIONS

1.	Kirchhoff's rules are very useful for the analysis of complicated electric circuits.	
a)	State Kirchhoff's junction rule and loop rule.	1
b)	Draw circuit diagram of Wheatstone bridge.	1
c)	Obtain the balancing condition of the bridge.	2
2.	Copper is one of the suitable materials to make connecting wires due its low resistivity.	
a)	What do you mean by resistivity?	1
b)	A copper wire is the form of a cylinder and has a resistance R. It is stretched till its thickness reduces by half of its initial size. Find its new resistance in terms of R.	2
c)	What is the new resistivity	1
3.	A Wheatstone bridge is shown in figure. 	
a)	Derive a relation connecting the four resistors for the galvanometer to give zero or null deflection.	4
b)	Name a practical device which uses this principle.	1
4.a)	State Kirchhoff's rules.	2
b)	Use Kirchhoff's rules to obtain conditions for the balance in Wheatstone bridge.	3
5.a)	A battery of emf 10 V and internal resistance 3 Ω is connected to an external resistor. If the current in the circuit is 0.5 A, what is the value of the external resistance ?	2
b)	What is the terminal voltage of the battery when the circuit is closed ?	1
6.	Magnitude of the drift velocity per unit electric field is -------------	1
7.	To analyse electric circuit Kirchhoff's rules are very useful.	
a)	State Kirchhoff's loop rule.	2
b)	Draw the circuit diagram of wheatstone's bridge apparatus	1
8.	Write down the equation for drift velocity acquired by an electron when a potential difference is applied to a conductor.	1

9.	The SI unit of resistance is -------------	1
10.a)	State Kirchhoff's law.	2
11.	Obtain the balancing condition of Wheatstone's bridge with the help of a diagram.	3
12.	Define drift velocity, give its equation.	2
13.a) b)	State Ohm's law. Derive Wheatstone's network principle.	1 3
14.	Show that the resistance of a conductor can be expressed by $R=\frac{m}{ne^2\tau}\left(\frac{l}{A}\right)$ where symbols have their usual meanings.	2
15.a) b)	Kirchoff's voltage law is in accordance with the conservation of (i)mass (ii)energy (iii)momentum (iv)charge With the help of diagram, arrive at the Wheatstone's principle.	1 3