CHAPTER 10
WAVE OPTICS

TOPICS TO BE COVERED

WAVEFRONT

A wavefrront is a set of points in a medium which are at the same phase of vibration.

The speed with which the wave front moves outwards from the source is called the speed of the wave.

The light ray and the energy of the wave travels in a direction perpendicular to the wavefront.

TYPES OF WAVEFRONTS

(i) Spherical wavefront

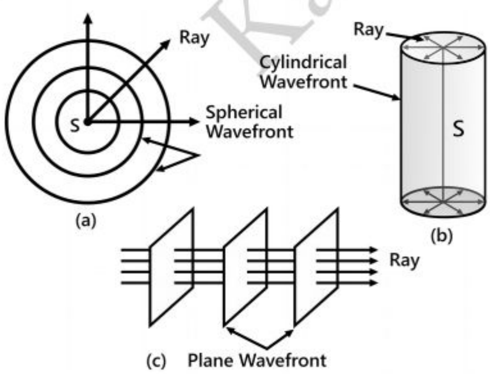
A point source of light emits a spherical wavefront (Fig .a)

(ii) Cylindrical wavefront

A linear source of light produces cylindrical wave front (Fig.b)

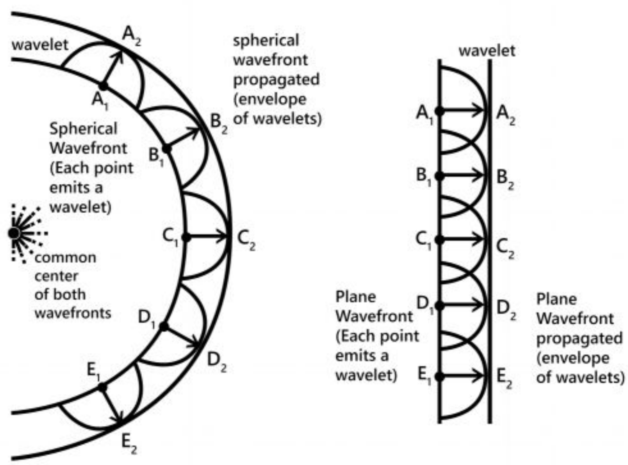
(iii) Plane wavefront

If the source is at infinity, the wavefront is planar (Fig.c)

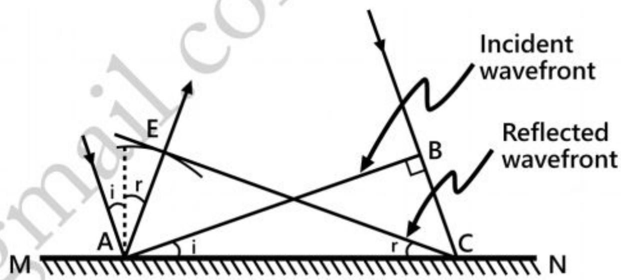


HUYGEN'S PRINCIPLE

- Each point on a wavefront is the source of secondary wavelets (small waves) which spread in all directions with the speed of the wave.
- The new wavefront at any instant is the envelop of these wavelets in the forward direction.



LAW OF REFLECTION USING HUYGENS PRINCIPLE



Consider a plane wave AB incident at an angle, i on a reflecting surface MN. If v represents the speed of the wave in the medium and 't represents the time taken by the wavefront to advance from the point B to C then the distance

$$BC = vt$$

In order the construct the reflected wavefront we draw a sphere of radius vt from the point A as shown in Fig.

$$AE = BC = vt$$

In ΔEAC and ΔBAC ,

$AE = BC = vt$ and $\angle AEC = \angle ABC = 90^0$
and the side AC is common to both triangle. So these triangles are congruent.

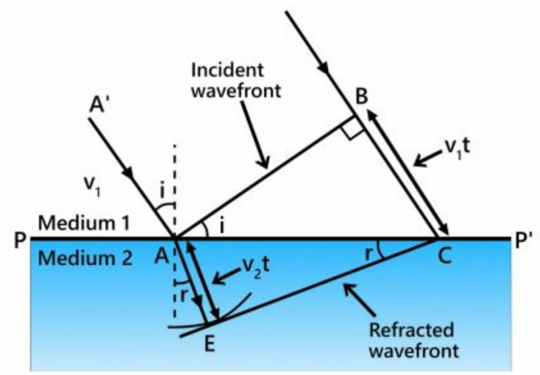
Therefore the angles i and r would be equal

ie, $\boxed{i = r}$

This is the law of reflection

LAW OF REFRACTION USING HUYGENS PRINCIPLE (SNELL'S LAW)

Consider a wave front incident on the surface separating two media at an angle 'i'. It is refracted through an angle'r'. Let t be the time taken by the wavefront to travel the distance BC.



AB is an incident wavefront incident at an angle , i . It refracts at the interface at an angle ,r .Let v_1 and v_2 be the speed of light in medium 1 and medium 2, respectively. ($v_1 > v_2$).

Let 't' be the time taken by the wavefront to travel the distance BC. Thus,

$$BC = v_1 t$$

In order the construct the refracted wavefront we draw a sphere of radius v_2t from the point A as shown in Fig.

From ΔABC , $\sin i = \frac{v_1 t}{AC}$ ....(1)

From ΔACE , $\sin r = \frac{v_2 t}{AC}$ ....(2)

$\frac{(1)}{(2)}$ => $\frac{\sin i}{\sin r} = \frac{v_1}{v_2}$ ....(3)

If c represents the speed of light in vacuum. Then, the refractive index of the first medium

$$n_1 = \frac{c}{v_1}$$

And the refractive index of the second medium

$$n_2 = \frac{c}{v_2}$$

Therefore $\frac{v_1}{v_2} = \frac{n_2}{n_1}$

Therefore $\boxed{\frac{\sin i}{\sin r} = \frac{n_2}{n_1}}$ ....(4)

This is Snell's law.

NOTE

If λ_1 and λ_2 denote the wavelengths of light in medium 1 and medium 2, respectively and if the distance BC is equal to λ_1 then the distance AE will be equal to λ_2 (because if the crest from B has reached C in time t , then the crest from A should have also reached E in time t); thus,

$$\frac{\lambda_1}{\lambda_2} = \frac{BC}{AE} = \frac{v_1}{v_2}$$

Therefore, $\frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2} = \nu_1 = \nu_2$

ie, when a wave gets refracted into a denser medium ($v_1 > v_2$) the speed of propagation and the wavelength decrease but the frequency $\nu = \frac{v}{\lambda}$ remains the same.

PRINCIPLE OF SUPERPOSITION OF WAVES AND INTERFERENCE OF LIGHT

SUPERPOSITION PRINCIPLE

It states that when two or more waves travelling at a particular point in a medium, the resultant displacement 'y' is the vector sum of the displacements of each of the waves

ie, $\boxed{y = y_1 + y_2 + y_3 + \ldots\ldots\ldots\ldots y_n}$

Coherent Sources of light

Two sources are said to be coherent if the waves emitted from them have the same frequency and constant phase difference.

INTERFERENCE

The modification in the distribution of light energy when waves from more than one coherent sources superpose each other.

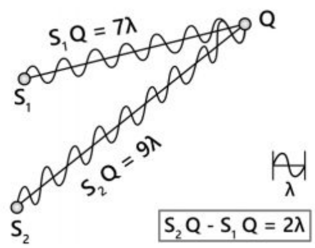
(i) Constructive interference

Constructive interference takes place when the crest of one wave falls on the crest of another wave such that the amplitude is maximum. The waves are in phase

Condition for Constructive interference

The path difference between the superposing coherent waves reaching at point is the integral multiple of the wavelength, λ . ie,

Path difference, $\boxed{\delta = n\lambda}$ where $n = 1, 2, 3 \ldots.$



We know. Intensity of a wave $I \alpha a^2$

During constructive interference

$$I = I_{max} \alpha (a_1 + a_2)^2$$

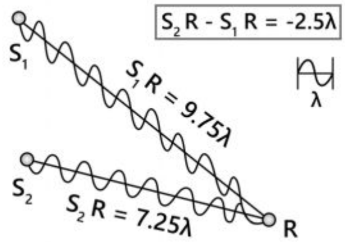
(ii) Destructive interference

In destructive interference the crest of one wave falls on the trough of another wave such that the amplitude is minimum. The phase of these waves are out of phase

Condition for destructive interference

The path difference between the superposing coherent waves reaching at point is an odd multiple of λ/2

Path difference, $\delta=(n+\frac{1}{2})\lambda$



During destructive interference,

$$I=I_{min}\alpha(a_1-a_2)^2$$

Relation between path difference and phase difference

Path difference of λ corresponds to a phase difference of 2π.

$$Path\,difference\,\lambda\rightarrow Phase\,difference\,2\pi$$

$$Path\,difference, x\rightarrow Phase\,difference, \frac{2\pi}{\lambda}x$$

Conditions for sustained interference

- The sources must be coherent
- The coherent sources must be narrow and very close to each other.
- The screen must be comparatively at a large distance from the coherent source.

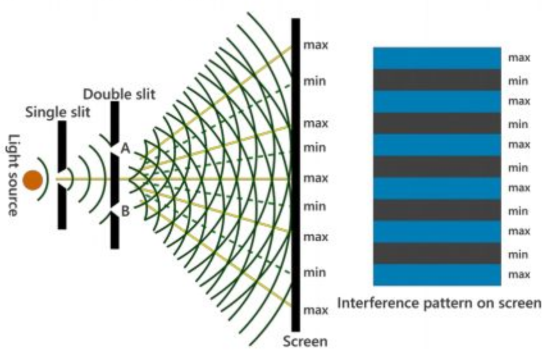
YOUNG'S DOUBLE SLIT EXPERIMENT

(No derivation)

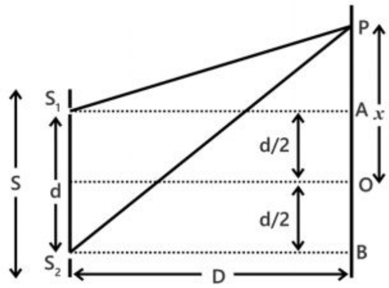
English scientist Thomas Young in 1802 performed this experiment.

It is an experiment to confirm the wave nature of light . The two slits act as coherent sources and the waves from these sources superpose and form alternate dark and bright bands on the screen. These bands are called interference fringes.

Experimental set up



Expression for band width



Band width is the distance between two consecutive bright fringes (or the distance between two consecutive dark fringes)

First we find the path difference S_2P-S_1P

From the triangle Δ S_1AP

$$S_1P^2=S_1A^2+AP^2$$

=> $$S_1P^2=D^2+(x-\frac{d}{2})^2 \quad ....(1)$$

From the triangle Δ S_2BP

$$S_2P^2=S_2B^2+BP^2$$

=> $$S_2P^2=D^2+(x+\frac{d}{2})^2 \quad ....(2)$$

(2)-(1)=> $$S_2P^2-S_1P^2=(x+\frac{d}{2})^2-(x-\frac{d}{2})^2$$

$$= x^2+xd+\frac{d^2}{4}-(x^2-xd+\frac{d^2}{4})$$

=> $S_2P^2-S_1P^2=2xd$

=> $(S_2P-S_1P)(S_2P+S_1P)=2xd \quad ....(3)$

If x and $d\ll D$, $S_2P\approx S_1P=D$

(3)=> $(S_2P-S_1P)2D=2xd$

=> $$S_2P-S_1P=\frac{xd}{D} \quad ....(4)$$

This is the path difference.

Condition for bright fringe, $\frac{xd}{D}=n\lambda$

ie, $$x=x_n=\frac{n\lambda D}{d} \quad ....(5)$$

This is the distance to the n[th] bright band.

Therefore the distance to the (n+1)[th] bright band,

$$x_{n+1}=\frac{(n+1)\lambda D}{d} \quad ....(6)$$

Therefore the band width,

$$\beta=x_{n+1}-x_n=\frac{(n+1)\lambda D}{d}-\frac{n\lambda D}{d}$$

=> $$\boxed{\beta=\frac{\lambda D}{d}} \quad ....(7)$$

NOTE

- The distance between two consecutive dark fringes is also equal to $\beta = \frac{\lambda D}{d}$
- The dark and bright bands are equally spaced.

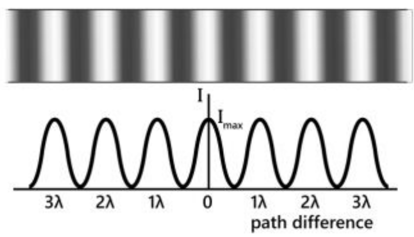
Conditions for getting sustainable interference pattern

- The two sources must be coherent
- The coherent sources must be narrow and very close to each other.
- The screen must be at large distance from the sources.

Methods to increase the fringe width(β)

- Increasing the wavelength of light (λ)
- Increasing the distance between the sources and screen (D)
- By decreasing the distance between the two coherent sources (d).

Intensity distribution in Young's double slit experiment



DIFFRACTION

If we look clearly at the shadow cast by an opaque object, close to the region of geometrical shadow, there are alternate dark and bright regions just like in interference. This is due to diffraction.

Diffraction of light is defined as the bending of light around corners such that it spreads out and illuminates areas where a shadow is expected.

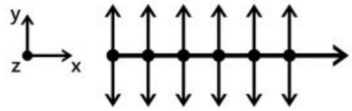
POLARISATION

- The phenomenon of restricting the vibrations of light (electric or magnetic field vector) to a particular direction perpendicular to the direction of propagation of wave is called polarisation of light.
- Polarisation is exhibited by transverse waves only. Thus polarisation proves the light is transverse in nature.

Unpolarised light

An ordinary light which has vibrations of electric field in all directions in a plane perpendicular to the direction of propagation of wave is said to be unpolarised light. All these vibrations could be resolved into parallel and perpendicular components. Fig.a below.

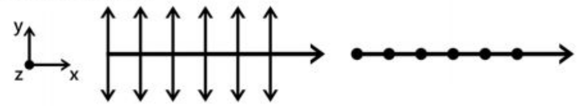
['•' represents the vibrations of the electric field vector perpendicular to the plane of the paper]



fig(a) - unpolarised light

Plane Polarised Light

The light which contains the vibrations of electric field vector in a single plane is called a plane polarised light.



fig(b) - plane polarised light (Vibration of EF in x-y plane)

fig(c) - plane polarised light (Vibration of EF in x-z plane)

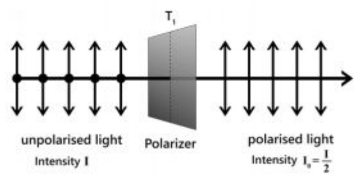
Plane of Vibration and Plane of Polarisation

The plane containing the vibrations of the electric field vector is known as the **plane of vibration** and the plane perpendicular to the plane of vibration of light is known as the **plane of polarisation.**

Polariser

The crystal which produces polarised light is called a polariser.

If the intensity of the unpolarised light is 'I_0' then the intensity of plane polarised light coming through the polariser will be $I = \frac{I_o}{2}$. The other half of intensity is restricted by the polariser.



fig(d) - polarisation using a polarizer

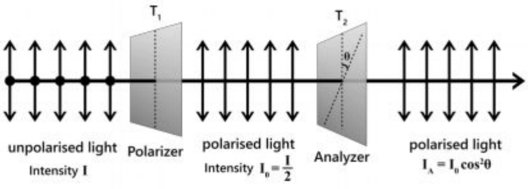
Analyser or Detector

The crystal which is used to check whether the light is polarized or not is called analyser or detector.

Malus' law

When a beam of plane polarised light of intensity 'I_0' is incident on an analyser, the intensity of transmitted light 'I_A' from the analyser varies directly as the square of the cosine of the angle θ between the transmission axis of polariser and analyser.

Demonstration of Malus' law



fig(d) - Malus' law

Intensity of light coming out of the analyser,

$$I_A = I_0 \cos^2\theta$$

Problem

Two polaroids are kept with their transmission axes inclined at 30^0 . Unpolarised light of intensity I falls on the first polaroid. Find out the intensity of light emerging from the second polaroid.

Solution

Let

'I' - intensity of the unpolarised light falling on the first polaroid

' I_0' - the intensity of polarized light emerging from the first polariser

' I_A' intensity of light emerging from the analyser

Then, $I_o = \frac{I}{2}$ and $I_A = I_0 \cos^2\theta$

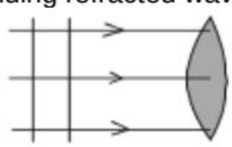
Therefore

$$I_A = I_0 \cos^2\theta = \frac{I}{2}\cos^2 30 = \frac{I}{2}\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3I}{8}$$

Uses of polaroids

1. Polaroids are used as window glasses to control the intensity of incoming light.
2. Polaroids produce polarised lights to be used in liquid crystal display (LCD).
3. Polaroids are used in goggles and cameras to avoid glare of light.

PREVIOUS QUESTIONS

1.	Name the property of light that proves its transverse nature	1
2.	Using Huygens wave theory prove that angle of incidence is equal to angle of reflection.	3
3.	Using Huygens wave theory derive Snell's law.	4
4.	State Malus' law.	2
5.	Using Huygen's principle, explain refraction of a plane wave, with the help of a diagram.	4
6.a) b)	State Huygens principle. Explain the refraction of plane wave using Huygens principle.	2 3
7.	The re-distribution of energy due to the superposition of two or more light waves is called	1
8.a) b)	State Huygens's principle of wave theory. What is coherent source ?	2 1
9.	"The locus of points which have the same phase is called a wave front" the statement is True/False	1
10.a) b)	What are coherent sources ? In Young's double slit experiment, interference pattern is observed at 5 cm from the slits with a fringe width of 1 mm. Calculate the separation between the slits. (λ= 5000 Å)	1 3
11.a) b)	Which of the following phenomena establishes the transverse nature of light waves ? (i) Diffraction (ii) Polarisation (iii) Photoelectric effect (iv) Compton effect Write two uses of Polaroids.	1 1
12.a) b)	State Huygens principle. Using Huygens principle, prove the Snell's law of refraction.	1 3
13.a) b)	Define wavefront. Figure shows a plane wavefront incident on a convex lens, draw the corresponding refracted wavefront. 	
14.	Using Huygens concept of wavefront, derive Snell's law of refraction.	