Q1 :

How many words, with or without meaning, each of 2 vowels and 3 consonants can be formed from the letters of the word DAUGHTER?

Answer :

In the word DAUGHTER, there are 3 vowels namely, A, U, and E, and 5 consonants namely, D, G, H, T, and R.

Number of ways of selecting 2 vowels out of 3 vowels = $^3C_2 = 3$.

Number of ways of selecting 3 consonants out of 5 consonants = $^5C_3 = 10$

Therefore, number of combinations of 2 vowels and 3 consonants = 3 × 10 = 30

Each of these 30 combinations of 2 vowels and 3 consonants can be arranged among themselves in 5! ways.

Hence, required number of different words = 30 × 5! = 3600

Q2 :

How many words, with or without meaning, can be formed using all the letters of the word EQUATION at a time so that the vowels and consonants occur together?

Answer :

In the word EQUATION, there are 5 vowels, namely, A, E, I, O, and U, and 3 consonants, namely, Q, T, and N.

Since all the vowels and consonants have to occur together, both (AEIOU) and (QTN) can be assumed as single objects. Then, the permutations of these 2 objects taken all at a time are counted. This number would be $^2P_2 = 2!$

Corresponding to each of these permutations, there are 5! permutations of the five vowels taken all at a time and 3! permutations of the 3 consonants taken all at a time.

Hence, by multiplication principle, required number of words = 2! × 5! × 3!

= 1440

Q3 :

A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of:

(i) exactly 3 girls? (ii) atleast 3 girls? (iii) atmost 3 girls?

Answer :

A committee of 7 has to be formed from 9 boys and 4 girls.

i. Since exactly 3 girls are to be there in every committee, each committee must consist of (7 â€“ 3) = 4 boys only.

Thus, in this case, required number of ways = $^4C_3 \times {}^9C_4 = \frac{4!}{3!1!} \times \frac{9!}{4!5!}$

$$= 4 \times \frac{9 \times 8 \times 7 \times 6 \times 5!}{4 \times 3 \times 2 \times 1 \times 5!}$$
$$= 504$$

(ii) Since at least 3 girls are to be there in every committee, the committee can consist of

(a) 3 girls and 4 boys or (b) 4 girls and 3 boys

3 girls and 4 boys can be selected in $^4C_3 \times {}^9C_4$ ways.

4 girls and 3 boys can be selected in $^4C_4 \times {}^9C_3$ ways.

Therefore, in this case, required number of ways = $^4C_3 \times {}^9C_4 + {}^4C_4 \times {}^9C_3$

$$= 504 + 84 = 588$$

(iii) Since atmost 3 girls are to be there in every committee, the committee can consist of

(a) 3 girls and 4 boys (b) 2 girls and 5 boys

(c) 1 girl and 6 boys (d) No girl and 7 boys

3 girls and 4 boys can be selected in $^4C_3 \times {}^9C_4$ ways.

2 girls and 5 boys can be selected in $^4C_2 \times {}^9C_5$ ways.

1 girl and 6 boys can be selected in $^4C_1 \times {}^9C_6$ ways.

No girl and 7 boys can be selected in $^4C_0 \times {}^9C_7$ ways.

Therefore, in this case, required number of ways

$$= {}^4C_3 \times {}^9C_4 + {}^4C_2 \times {}^9C_5 + {}^4C_1 \times {}^9C_6 + {}^4C_0 \times {}^9C_7$$

$$= \frac{4!}{3!1!} \times \frac{9!}{4!5!} + \frac{4!}{2!2!} \times \frac{9!}{5!4!} + \frac{4!}{1!3!} \times \frac{9!}{6!3!} + \frac{4!}{0!4!} \times \frac{9!}{7!2!}$$

$$= 504 + 756 + 336 + 36$$

$$= 1632$$

Q4 :

If the different permutations of all the letter of the word EXAMINATION are listed as in a dictionary, how many words are there in this list before the first word starting with E?

Answer :

In the given word EXAMINATION, there are 11 letters out of which, A, I, and N appear 2 times and all the other letters appear only once.

The words that will be listed before the words starting with E in a dictionary will be the words that start with A only.

Therefore, to get the number of words starting with A, the letter A is fixed at the extreme left position, and then the remaining 10 letters taken all at a time are rearranged.

Since there are 2 Is and 2 Ns in the remaining 10 letters,

Number of words starting with A = $\frac{10!}{2!2!} = 907200$

Thus, the required numbers of words is 907200.

Q5 :

How many 6-digit numbers can be formed from the digits 0, 1, 3, 5, 7 and 9 which are divisible by 10 and no digit is repeated?

Answer :

A number is divisible by 10 if its units digits is 0.

Therefore, 0 is fixed at the units place.

Therefore, there will be as many ways as there are ways of filling 5 vacant places □□□□□ [0] in succession by the remaining 5 digits (i.e., 1, 3, 5, 7 and 9).

The 5 vacant places can be filled in 5! ways.

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Hence, required number of 6-digit numbers = 5! = 120

Q6 :

The English alphabet has 5 vowels and 21 consonants. How many words with two different vowels and 2 different consonants can be formed from the alphabet?

Answer :

2 different vowels and 2 different consonants are to be selected from the English alphabet.

Since there are 5 vowels in the English alphabet, number of ways of selecting 2 different vowels from the alphabet = $^5C_2 = \frac{5!}{2!3!} = 10$

Since there are 21 consonants in the English alphabet, number of ways of selecting 2 different consonants from the alphabet $= {}^{21}C_2 = \frac{21!}{2!19!} = 210$

Therefore, number of combinations of 2 different vowels and 2 different consonants = 10 × 210 = 2100

Each of these 2100 combinations has 4 letters, which can be arranged among themselves in 4! ways.

Therefore, required number of words = 2100 × 4! = 50400

Q7 :

In an examination, a question paper consists of 12 questions divided into two parts i.e., Part I and Part II, containing 5 and 7 questions, respectively. A student is required to attempt 8 questions in all, selecting at least 3 from each part. In how many ways can a student select the questions?

Answer :

It is given that the question paper consists of 12 questions divided into two parts â€“ Part **I** and Part **II**, containing 5 and 7 questions, respectively.

A student has to attempt 8 questions, selecting at least 3 from each part.

This can be done as follows.

(a) 3 questions from part **I** and 5 questions from part **II**

(b) 4 questions from part **I** and 4 questions from part **II**

(c) 5 questions from part **I** and 3 questions from part **II**

3 questions from part **I** and 5 questions from part **II** can be selected in $^5C_3 \times {}^7C_5$ ways.

4 questions from part **I** and 4 questions from part **II** can be selected in $^5C_4 \times {}^7C_4$ ways.

5 questions from part **I** and 3 questions from part **II** can be selected in $^5C_5 \times {}^7C_3$ ways.

Thus, required number of ways of selecting questions

$$= {}^5C_3 \times {}^7C_5 + {}^5C_4 \times {}^7C_4 + {}^5C_5 \times {}^7C_3$$
$$= \frac{5!}{2!3!} \times \frac{7!}{2!5!} + \frac{5!}{4!1!} \times \frac{7!}{4!3!} + \frac{5!}{5!0!} \times \frac{7!}{3!4!}$$
$$= 210 + 175 + 35 = 420$$

Q8 :

Determine the number of 5-card combinations out of a deck of 52 cards if each selection of 5 cards has exactly one king.

Answer :

From a deck of 52 cards, 5-card combinations have to be made in such a way that in each selection of 5 cards, there is exactly one king.

In a deck of 52 cards, there are 4 kings.

1 king can be selected out of 4 kings in ${}^4C_1$ ways.

4 cards out of the remaining 48 cards can be selected in ${}^{48}C_4$ ways.

Thus, the required number of 5-card combinations is ${}^4C_1 \times {}^{48}C_4$.

Q9 :

It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?

Answer :

5 men and 4 women are to be seated in a row such that the women occupy the even places.

The 5 men can be seated in 5! ways. For each arrangement, the 4 women can be seated only at the cross marked places (so that women occupy the even places).

$$M \times M \times M \times M \times M$$

Therefore, the women can be seated in 4! ways.

Thus, possible number of arrangements = 4! × 5! = 24 × 120 = 2880

Q10 :

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From a class of 25 students, 10 are to be chosen for an excursion party. There are 3 students who decide that either all of them will join or none of them will join. In how many ways can the excursion party be chosen?

Answer :

From the class of 25 students, 10 are to be chosen for an excursion party.

Since there are 3 students who decide that either all of them will join or none of them will join, there are two cases.

Case I: All the three students join.

Then, the remaining 7 students can be chosen from the remaining 22 students in ${}^{22}C_7$ ways.

Case II: None of the three students join.

Then, 10 students can be chosen from the remaining 22 students in ${}^{22}C_{10}$ ways.

Thus, required number of ways of choosing the excursion party is ${}^{22}C_7 + {}^{22}C_{10}$.

Q11 :

In how many ways can the letters of the word ASSASSINATION be arranged so that all the S's are together?

Answer :

In the given word ASSASSINATION, the letter A appears 3 times, S appears 4 times, I appears 2 times, N appears 2 times, and all the other letters appear only once.

Since all the words have to be arranged in such a way that all the Ss are together, SSSS is treated as a single object for the time being. This single object together with the remaining 9 objects will account for 10 objects.

These 10 objects in which there are 3 As, 2 Is, and 2 Ns can be arranged in $\frac{10!}{3!2!2!}$ ways.

Thus, required number of ways of arranging the letters of the given word

$$= \frac{10!}{3!2!2!} = 151200$$