The relation *f* is defined by $f(x)=\begin{cases}x^2, & 0\le x\le 3\\ 3x, & 3\le x\le 10\end{cases}$

The relation *g*is defined by $g(x)=\begin{cases}x^2, & 0\le x\le 2\\ 3x, & 2\le x\le 10\end{cases}$

Show that *f* is a function and *g* is not a function.

Answer :

The relation *f*is defined as $f(x)=\begin{cases}x^2, & 0\le x\le 3\\ 3x, & 3\le x\le 10\end{cases}$

It is observed that for

$0 \le x < 3$, $f(x) = x^2$

$3 < x \le 10$, $f(x) = 3x$

Also, at $x = 3$, $f(x) = 3^2 = 9$ or $f(x) = 3 \times 3 = 9$

i.e., at $x = 3$, $f(x) = 9$

Therefore, for $0 \le x \le 10$, the images of $f(x)$ are unique.

Thus, the given relation is a function.

The relation *g*is defined as $g(x)=\begin{cases}x^2, & 0\le x\le 2\\ 3x, & 2\le x\le 10\end{cases}$

It can be observed that for $x= 2$, $g(x) = 2^2 = 4$ and $g(x) = 3 \times 2 = 6$

Hence, element 2 of the domain of the relation *g*corresponds to two different images i.e., 4 and 6. Hence, this relation is not a function.

Q2 :

If $f(x) = x^2$, find $\dfrac{f(1.1)-f(1)}{(1.1-1)}$.

Answer :

$$f(x)=x^2$$

$$\therefore \frac{f(1.1)-f(1)}{(1.1-1)}=\frac{(1.1)^2-(1)^2}{(1.1-1)}=\frac{1.21-1}{0.1}=\frac{0.21}{0.1}=2.1$$

Q3 :

Find the domain of the function $f(x)=\dfrac{x^2+2x+1}{x^2-8x+12}$

Answer :

The given function is $f(x)=\dfrac{x^2+2x+1}{x^2-8x+12}$.

$$f(x)=\frac{x^2+2x+1}{x^2-8x+12}=\frac{x^2+2x+1}{(x-6)(x-2)}$$

It can be seen that function *f*is defined for all real numbers except at *x*= 6 and *x*= 2.

Hence, the domain of *f*is **R** â€“ {2, 6}.

Q4 :

Find the domain and the range of the real function *f*defined by $f(x)=\sqrt{(x-1)}$ **.**

Answer :

The given real function is $f(x)=\sqrt{x-1}$.

It can be seen that $\sqrt{x-1}$ is defined for (*x*â€“ 1) ≥0.

i.e., $f(x)=\sqrt{(x-1)}$ is defined for *x* ≥1.

Therefore, the domain of *f*is the set of all real numbers greater than or equal to 1 i.e., the domain of *f*= [1, ∞).

As *x* ≥1 ⇒(*x* â€“ 1) ≥0 ⇒ $\sqrt{x-1}\geq 0$

Therefore, the range of *f*is the set of all real numbers greater than or equal to 0 i.e., the range of *f*= [0, ∞).

Q5 :

Find the domain and the range of the real function *f*defined by *f*(*x*) = |*x*- 1|.

Answer :

The given real function is*f*(*x*) = |*x*- 1|.

It is clear that |*x*- 1| is defined for all real numbers.

∴Domain of *f*= **R**

Also, for $x \in \mathbf{R}$, $|x-1|$ assumes all real numbers.

Hence, the range of *f* is the set of all non-negative real numbers.

Q6 :

Let $f=\left\{\left(x, \frac{x^2}{1+x^2}\right): x \in \mathbf{R}\right\}$ **be a function from R into R. Determine the range of *f*.**

Answer :

$$f=\left\{\left(x, \frac{x^2}{1+x^2}\right): x \in \mathbf{R}\right\}$$

$$=\left\{(0,\ 0),\ \left(\pm 0.5,\ \frac{1}{5}\right),\ \left(\pm 1,\ \frac{1}{2}\right),\ \left(\pm 1.5,\ \frac{9}{13}\right),\ \left(\pm 2,\ \frac{4}{5}\right),\ \left(3,\ \frac{9}{10}\right),\ \left(4,\ \frac{16}{17}\right),\ \ldots\right\}$$

The range of *f* is the set of all second elements. It can be observed that all these elements are greater than or equal to 0 but less than 1.

[Denominator is greater numerator]

Thus, range of *f* = [0, 1)

Q7 :

Let *f*, *g*: R Ã¢â€ ’ R be defined, respectively by *f*(*x*) = *x* + 1, *g*(*x*) = 2*x* â€“ 3. Find *f* + *g*, *f* â€“ *g* and $\frac{f}{g}$ **.**

Answer :

f, *g*: **R** Ã¢â€ ’ **R** is defined as *f*(*x*) = *x* + 1, *g*(*x*) = 2*x* â€“ 3

(*f* + *g*) (*x*) = *f*(*x*) + *g*(*x*) = (*x* + 1) + (2*x* â€“ 3) = 3*x* â€“ 2

∴(*f* + *g*) (*x*) = 3*x* â€“ 2

(*f* â€“ *g*) (*x*) = *f*(*x*) â€“ *g*(*x*) = (*x* + 1) â€“ (2*x* â€“ 3) = *x* + 1 â€“ 2*x* + 3 = â€“ *x* + 4

∴ (*f* â€“ *g*) (*x*) = â€“*x* + 4

$$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)},\ g(x)\neq 0,\ x\in\mathbf{R}$$

$$\therefore \left(\frac{f}{g}\right)(x)=\frac{x+1}{2x-3},\ 2x-3\neq 0 \text{ or } 2x\neq 3$$

$$\therefore \left(\frac{f}{g}\right)(x)=\frac{x+1}{2x-3},\ x\neq \frac{3}{2}$$

Q8 :

Let *f* = {(1, 1), (2, 3), (0, -1), (-1, -3)} be a function from Zto Zdefined by *f*(*x*) = *ax*+ *b*, for some integers *a*, *b*. Determine *a*, *b*.

Answer :

f = {(1, 1), (2, 3), (0, -1), (-1, -3)}

f(*x*) = *ax*+ *b*

(1, 1) ∈ *f*

⇒ *f*(1) = 1

⇒ *a* x 1 + *b*= 1

⇒ *a*+ *b*= 1

(0, -1) ∈ *f*

⇒ *f*(0) = -1

⇒ *a* x 0 + *b*= -1

⇒ *b*= -1

On substituting *b*= -1 in *a*+ *b*= 1, we obtain *a*+ (-1) = 1 ⇒ *a*= 1 + 1 = 2.

Thus, the respective values of *a*and *b*are 2 and -1.

Q9 :

Let R be a relation from N to N defined by R = {(*a*, *b*): *a*, *b* ∈ N and *a* = b^2}. Are the following true?

(i) (*a*, *a*) ∈ R, for all *a* ∈ N
(ii) (*a*, *b*) ∈ R, implies (*b*, *a*) ∈ R

(iii) (*a*, *b*) ∈ R, (*b*, *c*) ∈ R implies (*a*, *c*) ∈ R.

Justify your answer in each case.

Answer :

R = {(a, b): a, b ∈ **N** and $a = b^2$}

(i) It can be seen that 2 ∈ **N**;however, $2 \neq 2^2 = 4$.

Therefore, the statement "(a, a) ∈ R, for all a ∈ **N**" is not true.

(ii) It can be seen that (9, 3) ∈ **N** because 9, 3 ∈ **N** and $9 = 3^2$.

Now, $3 \neq 9^2 = 81$; therefore, (3, 9) ∠"° **N**

Therefore, the statement "(a, b) ∈ R, implies (b, a) ∈ R" is not true.

(iii) It can be seen that (16, 4) ∈ R, (4, 2) ∈ R because 16, 4, 2 ∈ **N** and $16 = 4^2$ and $4 = 2^2$.

Now, $16 \neq 2^2 = 4$; therefore, (16, 2) ∠"° **N**

Therefore, the statement "(a, b) ∈ R, (b, c) ∈ R implies (a, c) ∈ R" is not true.

Q10 :

Let A = {1, 2, 3, 4}, B = {1, 5, 9, 11, 15, 16} and *f* = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}. Are the following true?

(i) *f*is a relation from A to B (ii) *f*is a function from A to B.

Justify your answer in each case.

Answer :

A = {1, 2, 3, 4} and B = {1, 5, 9, 11, 15, 16}

∴A x B = {(1, 1), (1, 5), (1, 9), (1, 11), (1, 15), (1, 16), (2, 1), (2, 5), (2, 9), (2, 11), (2, 15), (2, 16), (3, 1), (3, 5), (3, 9), (3, 11), (3, 15), (3, 16), (4, 1), (4, 5), (4, 9), (4, 11), (4, 15), (4, 16)}

It is given that f = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}

(i) A relation from a non-empty set A to a non-empty set B is a subset of the Cartesian product A x B.

It is observed that *f*is a subset of A x B.

Thus, *f*is a relation from A to B.

(ii) Since the same first element i.e., 2 corresponds to two different images i.e., 9 and 11, relation f is not a function.

Q11 :

Let *f*be the subset of Z x Zdefined by *f* = {(*ab*, *a*+ *b*): *a*, *b* ∈ Z}. Is *f*a function from Zto Z: justify your answer.

Answer :

The relation *f*is defined as f = {(ab, a+ b): a, b ∈ **Z**}

We know that a relation *f*from a set A to a set B is said to be a function if every element of set A has unique images in set B.

Since 2, 6, -2, -6 ∈ **Z**, (2 x 6, 2 + 6), (-2 x -6, -2 + (-6)) ∈ f

i.e., (12, 8), (12, -8) ∈ f

It can be seen that the same first element i.e., 12 corresponds to two different images i.e., 8 and -8. Thus, relation *f* is not a function.

Q12 :

Let A = {9, 10, 11, 12, 13} and let *f*: A → N be defined by *f*(*n*) = the highest prime factor of *n*. Find the range of *f*.

Answer :

A = {9, 10, 11, 12, 13}

f: A → **N** is defined as

f(*n*) = The highest prime factor of *n*

Prime factor of 9 = 3

Prime factors of 10 = 2, 5

Prime factor of 11 = 11

Prime factors of 12 = 2, 3

Prime factor of 13 = 13

∴ *f*(9) = The highest prime factor of 9 = 3

f(10) = The highest prime factor of 10 = 5

f(11) = The highest prime factor of 11 = 11

f(12) = The highest prime factor of 12 = 3

f(13) = The highest prime factor of 13 = 13

The range of *f* is the set of all *f*(*n*), where *n* ∈ A.

∴ Range of *f* = {3, 5, 11, 13}