Chapter 9
Mechanical Properties of Fluids

Liquids and gases can flow and are therefore, called fluids.
The fluid does not have any resistance to change of its shape. Thus, the shape of a fluid is governed by the shape of its container.

Basic difference between Liquids and Gases

A liquid is incompressible and has a free surface of its own. A gas is compressible and it expands to occupy all the space available to it. Gas has no free surface.

Pressure

The normal force(F) exerted by a fluid on an area A is called pressure.

$$\text{Pressure, } P = \frac{F}{A}$$

Pressure is a scalar quantity.
Its SI unit is Nm^{-2} or pascal (Pa)
Dimensional formula is $ML^{-1}T^{-2}$

A common unit of pressure is the atmosphere (atm). It is the pressure exerted by the atmosphere at sea level.

$$1 \text{ atm} = 1.013 \times 10^5 \text{ Pa.}$$

Density

Density ρ for a fluid of mass m occupying volume V is given by

$$\rho = \frac{m}{V}$$

It is a positive scalar quantity.
Its SI unit is $kg\ m^{-3}$.
The dimensions of density are $[ML^{-3}]$.

The density of water at 4^0C (277 K) is 1000 kg m^{-3}.

A liquid is incompressible and its density is therefore, nearly constant at all pressures. Gases, on the other hand exhibit a large variation in densities with pressure.

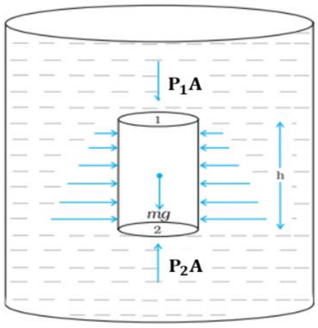
Relative Density

The relative density of a substance is the ratio of its density to the density of water at 4^0C.

$$\text{Relative density} = \frac{\text{Density of substance}}{\text{Density of water at } 4^0C}$$

It is a dimensionless positive scalar quantity.

Variation of Pressure with Depth



A fluid is at rest in a container. Consider a cylindrical element of fluid having area of base A and height h.

In equilibrium, the resultant vertical forces should be balanced.

$$P_2A = P_1A + mg$$
$$P_2A - P_1A = mg$$
$$(P_2 - P_1)A = mg$$

But $m = \rho V$

$V = hA$

$m = \rho hA$

$$(P_2 - P_1)A = \rho hA\, g$$
$$P_2 - P_1 = \rho\, gh$$

If the point 1 at the top of the fluid , which is open to the atmosphere, P_1 may be replaced by atmospheric pressure (P_a) and we replace P_2 by P

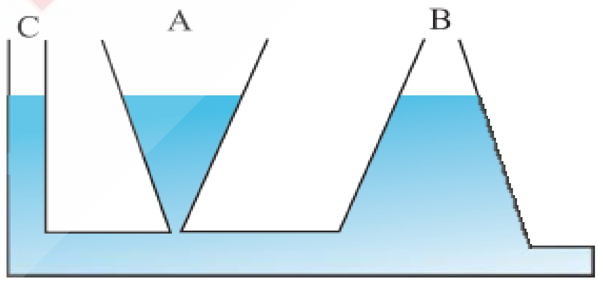
Gauge pressure, $P - P_a = \rho\, gh$

The excess of pressure, $P - P_a$, at depth h is called a gauge pressure at that point.

Absolute Pressure, $P = P_a + \rho\, gh$

Thus, the absolute pressure P, at depth below the surface of a liquid open to the atmosphere is greater than atmospheric pressure by an amount ρgh.

Hydrostatic paradox.



The absolute pressure depends on the height of the fluid column and not on cross sectional or base area or the shape of the container. The liquid pressure is the same at all points at the same horizontal level (same depth). The result is appreciated through the example of **hydrostatic paradox.**

Example

What is the pressure on a swimmer 10 m below the surface of a lake?

$h = 10 \text{ m}$

$\rho = 1000 \text{ kg m}^{-3}$ Take $g = 10 \text{ m s}^{-2}$

$$P = P_a + \rho gh$$
$$= 1.01 \times 10^5 + 1000 \times 10 \times 10$$
$$= 1.01 \times 10^5 + 1 \times 10^5$$
$$= 2.01 \times 10^5 \text{ Pa}$$
$$\approx 2 \text{ atm}$$

(This is a 100% increase in pressure from surface level. At a depth of 1 km the increase in pressure is 100 atm. Submarines are designed to withstand such enormous pressures.)

Atmospheric Pressure

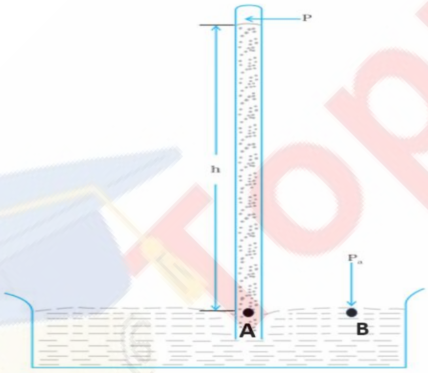
It is the pressure exerted by the atmosphere at sea level.

The pressure of the atmosphere at any point is equal to the weight of a column of air of unit cross sectional area extending from that point to the top of the atmosphere.

$$1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$

Mercury barometer

Mercury barometer is used to measure Atmospheric Pressure. Italian scientist Evangelista Torricelli devised mercury barometer.



(The space above the mercury column in the tube contains only mercury vapour whose pressure P is so small that it may be neglected.)

The pressure inside the column at point A =The pressure at point B, which is at the same level.

Pressure at B = P_a (atmospheric pressure)

Pressure at A = ρgh

$$P_a = \rho gh$$

where ρ is the density of mercury and h is the height of the mercury column in the tube.

At sea level h= 76 cm and is equivalent to 1 atm.

The unit of the pressure is the pascal (Pa). It is the same as N m^{-2}. Other common units of pressure are

1 atm = 1.01×10^5 Pa

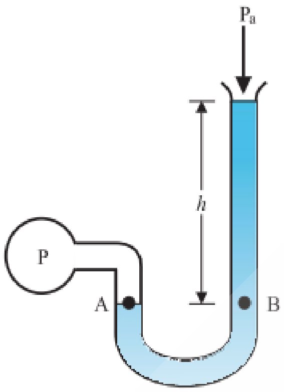
1 bar = 10^5 Pa

1 torr = 133 Pa = 0.133 kPa

1 mm of Hg = 1 torr = 133 Pa

Open-tube manometer

An open-tube manometer is a used for measuring Guage pressure or pressure differences.



It consists of a U-tube containing a suitable liquid i.e. a low density liquid (such as oil) for measuring small pressure differences and a high density liquid (such as mercury) for large pressure differences.

One end of the tube is open to the atmosphere and other end is connected to the system whose pressure we want to measure .

The pressure at A = pressure at point B

$$P = P_a + \rho gh$$

$$P - P_a = \rho gh$$

The gauge pressure is proportional to manometer height h.

Example

The density of the atmosphere at sea level is 1.29 kg/m3. Assume that it does not change with altitude. Then how high would the atmosphere extend

$$P_a = \rho gh$$

$$h = \frac{P_a}{\rho g}$$

$$h = \frac{1.01\times10^5}{1.29 \times 9.8}$$

$$h = 7989 \text{ m} \approx 8 \text{ km}$$

Example

At a depth of 1000 m in an ocean (a) what is the absolute pressure? (b) What is the gauge pressure? (c) Find the force acting on the window of area 20 cm × 20 cm of a submarine at this depth, the interior of which is maintained at sea-level atmospheric pressure.
(The density of sea water is $1.03 \times 10^3 \text{ kg m}^{-3}$, $g = 10 \text{m s}^{-2}$)

$h = 1000 \text{ m}$, $\rho = 1.03 \times 10^3 \text{ kg m}^{-3}$

(a) Absolute pressure, $P = P_a + \rho g h$

$$= 1.01 \times 10^5 + 1.03 \times 10^3 \times 10 \times 1000$$
$$= 1.01 \times 10^5 + 103 \times 10^5$$
$$= 104.01 \times 10^5 \text{ Pa}$$
$$\approx 104 \text{ atm}$$

(b) Gauge pressure , $P - P_a = \rho g h$

$$= 1.03 \times 10^3 \times 10 \times 1000$$
$$= 103 \times 10^5 \text{ Pa}$$
$$\approx 103 \text{ atm}$$

(c)The pressure outside the submarine is $P = P_a + \rho g h$ and the pressure inside it is P_a.

Hence, the net pressure acting on the window is gauge pressure, $\rho g h$.

Since the area of the window is $A = 0.04 \text{ m}^2$, the force acting on it is

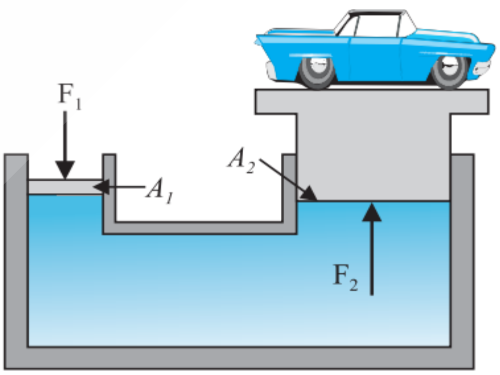
$$F = \text{Gauge Pressure x A}$$
$$= 103 \times 10^5 \times 0.04$$
$$= 4.12 \times 10 \text{ N}$$

Pascal's law for transmission of fluid pressure

Whenever external pressure is applied on any part of a fluid contained in a vessel, it is transmitted undiminished and equally in all directions.

Applications of Pascal's law

1.Hydraulic lift



The pressure on smaller piston

$$P = \frac{F_1}{A_1} \text{-------------} (1)$$

This pressure is transmitted equally to the larger cylinder with a larger piston of area A_2 producing an upward force F_2.

$$P = \frac{F_2}{A_2} \text{-------------} (2)$$

From eq(1) and (2) $\frac{F_1}{A_1} = \frac{F_2}{A_2}$

$$\mathbf{F_2 = F_1 \frac{A_2}{A_1}}$$

Thus, the applied force has been increased by a factor of $\frac{A_2}{A_1}$ and this factor is the mechanical advantage of the device.

Example

Two syringes of different cross sections (without needles) filled with water are connected with a tightly fitted rubber tube filled with water. Diameters of the smaller piston and larger piston are 1.0 cm and 3.0 cm respectively. (a) Find the force exerted on the larger piston when a force of 10 N is applied to the smaller piston. (b) If the smaller piston is pushed in through 6.0 cm, how much does the larger piston move out?

$$F_2 = F_1 \frac{A_2}{A_1}$$

$$F_2 = 10 \text{ x} \frac{\pi \text{ x } (1.5 \text{ x } 10^{-2})^2}{\pi \text{ x } (0.5 \text{ x } 10^{-2})^2}$$

$$= 10 \text{ x} 9$$

$$= 90 \text{ N}$$

(b) Volume covered by the smaller piston is equal to volume moved by the larger piston.

$$L_1 A_1 = L_2 A_2$$

$$L_2 = L_1 \frac{A_1}{A_2}$$

$$= 6 \text{ x} 10^{-2} \text{ x} \frac{\pi \text{ x } (0.5 \text{ x } 10^{-2})^2}{\pi \text{ x } (1.5 \text{ x } 10^{-2})^2} = 0.54 \text{m}$$

$$= 6 \text{ x} 10^{-2} \text{ x} 0.111$$

$$= 0.67 \text{ x} 10^{-2} \text{m}$$

$$= 0.67 \text{cm}$$

Example

In a car lift compressed air exerts a force F_1 on a small piston having a radius of 5.0 cm. This pressure is transmitted to a second piston of radius 15 cm . If the mass of the car to be lifted is 1350 kg, calculate F_1. What is the pressure necessary to accomplish this task? ($g = 9.8\ ms^{-2}$).

$$F_1 = F_2\left(\frac{A_1}{A_2}\right)$$

$$F_2 = mg = 1350 \times 9.8$$
$$=13230N$$

$$F_1 = 13230 \times \frac{\pi \times (5 \times 10^{-2})^2}{\pi \times (15 \times 10^{-2})^2}$$
$$= 13230 \times \frac{25}{225}$$
$$=1470\ N$$

The air pressure that will produce this force is

$$P = \frac{F_1}{A_1}$$
$$P = \frac{1470}{3\cdot14 \times (5 \times 10^{-2})^2}$$
$$=1.9 \times 10^5\ Pa$$

2.Hydraulic brakes

When we apply a force on the pedal with our foot the master piston moves inside the master cylinder, and the pressure caused is transmitted through the brake oil to act on a piston of larger area. A large force acts on the piston and is pushed down expanding the brake shoes against brake lining. In this way a small force on the pedal produces a large retarding force on the wheel.

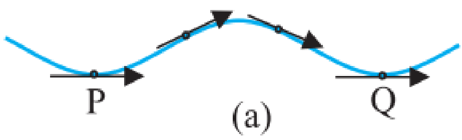
The pressure set up by pressing pedal is transmitted equally to all cylinders attached to the four wheels so that the braking effort is equal on all wheels.

Streamline Flow (Steady Flow)

The study of the fluids in motion is known as fluid dynamics.

The flow of the fluid is said to be steady if at any given point, the velocity of each passing fluid particle remains constant in time.

The velocity of a particular particle may change as it moves from one point to another.



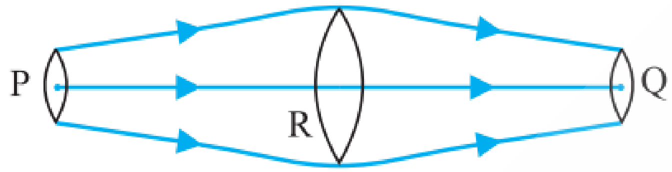
(a)

The path taken by a fluid particle under a steady flow is a streamline.

Streamline is defined as a curve whose tangent at any point is in the direction of the fluid velocity at that point.

No two streamlines can cross, for if they do, an oncoming fluid particle can go either one way or the other and the flow would not be steady.

Equation of Continuity



Consider a region of streamline flow of a fluid. The points P, R and Q are planes perpendicular to the direction of fluid flow . The area of cross-sections at these points are A_P , A_R , A_Q and speeds of fluid particles are v_P, v_R and v_Q.

The mass of fluid crossing at P in a small interval of time $\Delta t = \rho_P A_P v_P \Delta t$

The mass of fluid crossing at Q in a small interval of time $\Delta t = \rho_Q A_Q v_Q \Delta t$

The mass of fluid crossing at R in a small interval of time $\Delta t = \rho_R A_R v_R \Delta t$

The mass of liquid flowing out = The mass of liquid flowing in

$$\rho_P A_P v_P \Delta t = \rho_Q A_Q v_Q \Delta t = \rho_R A_R v_R \Delta t$$

If the fluid is incompressible $\rho_P = \rho_Q = \rho_R$

$$\mathbf{A_P v_P = A_Q v_Q = A_R v_R}$$

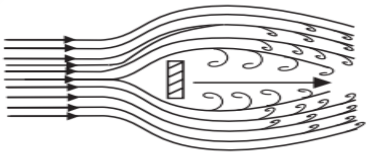
$$\mathbf{Av = constant}$$

This is called the equation of continuity and it is a statement of conservation of mass in flow of incompressible fluids.

Thus, at narrower portions where the streamlines are closely spaced, velocity increases and its vice versa.

Turbulent Flow

Steady flow is achieved at low flow speeds. Beyond a limiting value, called critical speed, the flow of fluid loses steadiness and becomes turbulent.



A jet of air striking a flat plate placed perpendicular to it is an example of turbulent flow.

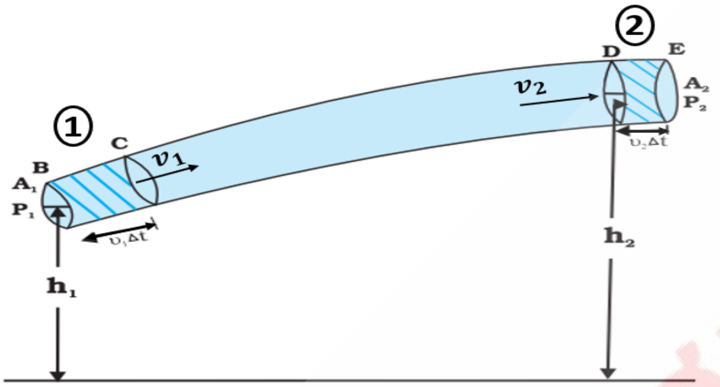
Bernoulli's Principle

Bernoulli's principle states that as we move along a streamline, the sum of the pressure , the kinetic energy per unit volume and the potential energy per unit volume remains a constant.

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

The equation is basically the **conservation of energy** **applied to non viscous fluid motion in steady state.**

Proof



Consider the flow of an ideal fluid in a pipe of varying cross section, from region **(1)** to region **(2)**. The fluid in the two region is displaced a length of $v_1\ \Delta t$ and $v_2\ \Delta t$ in time Δt.

The work done on the fluid at left end (BC) is

$$W_1 = F_1\Delta x_1 = P_1A_1\ (v_1\Delta t) \quad | \quad (A_1v_1\Delta t = \Delta V\ \text{,volume})$$

$$W_1 = P_1\Delta V.$$

The work done by the fluid at the end (DE) is

$$W_2 = F_2\Delta x_2 = P_2A_2\ (v_2\Delta t)$$

$$W_2 = P_2\Delta V.$$

(or)The work done on the fluid at the end (DE) is

$$W_2 = -P_2\Delta V.$$

The total work done on the fluid is

$$W_1 + W_2 = P_1\Delta V - P_2\Delta V$$

$$\mathbf{W_1 + W_2 = (P_1 - P_2)\Delta V} \text{-------------}\mathbf{(1)}$$

Part of this work goes into changing the kinetic energy of the fluid, and part goes into changing the gravitational potential energy.

The change in its kinetic energy is

$$\Delta K = \frac{1}{2}m\left(v_2^2 - v_1^2\right) \text{ -------------}(2)$$

The change in gravitational potential energy is

$$\Delta U = mg(h_2 - h_1) \text{ ----------------}(3)$$

By work – energy theorem

$$W_1 + W_2 = \Delta K + \Delta U$$

Substituting from eq(1),(2) and (3)

$$(\mathbf{P_1 - P_2})\mathbf{\Delta V} = \frac{1}{2}\mathbf{m}\left(\mathbf{v_2^2 - v_1^2}\right) + \mathbf{mg(h_2\text{-}h_1)}\text{------------}(4)$$

Divide each term by ΔV to obtain ,

$m = \rho\Delta V$
$\rho = \frac{m}{\Delta V}$

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(h_2\text{-}h_1)$$

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho g h_2 - \rho g h_1$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$\mathbf{P + \frac{1}{2}\rho v^2 + \rho g h = constant}\text{----------}(5)$$

This is Bernoulli's theorem

When a fluid is at rest i.e. its velocity is zero everywhere, Bernoulli's equation becomes

$$\mathbf{P_1 + \rho g h_1 = P_2 + \rho g h_2}$$

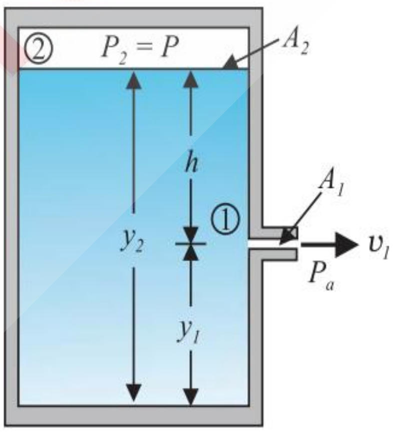
Note:-Bernoulli's theorem is applicable only to the streamline flow of non viscous and incompressible fluids.

Applications of Bernoulli's Principle

1.Speed of Efflux: Torricelli's Law

The word efflux means fluid outflow

Torricelli's law states that the speed of efflux of fluid through a small hole at a depth h of an open tank is equal to the speed of a freely falling body i.e, $\sqrt{2gh}$



Consider a tank containing a liquid of density ρ with a small hole in its side at a height y_1 from the bottom.

According to Bernoulli principle

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Consider regions 1 and 2

According to equation of continuity ,since ($A_2 >> A_1$), $v_2 = 0$.

$$P_a + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P + \rho g y_2$$

$$\frac{1}{2}\rho v_1^2 = \rho g(y_2 - y_1) + P - P_a$$

$$y_2 - y_1 = h$$

$$\frac{1}{2}\rho v_1^2 = \rho g h + P - P_a$$

$$v_1^2 = 2gh + \frac{2(P - P_a)}{\rho}$$

$$\mathbf{v_1 = \sqrt{2gh + \frac{2(P - P_a)}{\rho}}}$$

If the tank is open to the atmosphere, then $P = P_a$

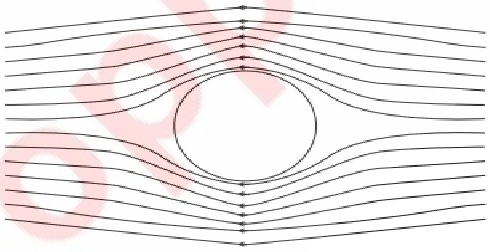
$$\mathbf{v_1 = \sqrt{2gh}}$$

This equation is known as Torricelli's law.

This is the speed of a freely falling body.

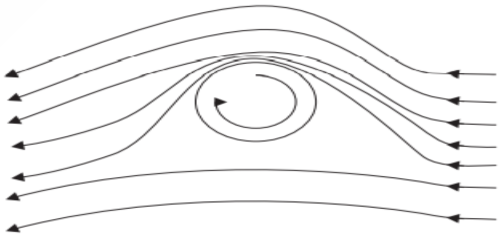
2.Dynamic Lift

(i)Ball moving without spin:



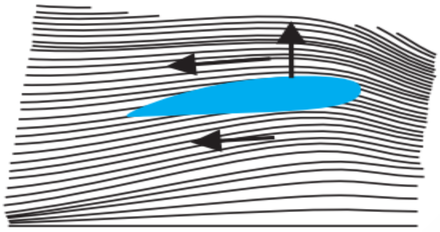
The velocity of fluid (air) above and below the ball at corresponding points is the same resulting in zero pressure difference. The air therefore, exerts no upward or downward force on the ball.

(ii)Ball moving with spin:Magnus Effect



The ball is moving forward and relative to it the air is moving backwards. Therefore, the relative velocity of air above the ball is larger and below it is smaller. This difference in the velocities of air results in the pressure difference between the lower and upper faces and there is a net upward force on the ball. This dynamic lift due to spining is called **Magnus effect.**

(iii)Aerofoil or lift on aircraft wing

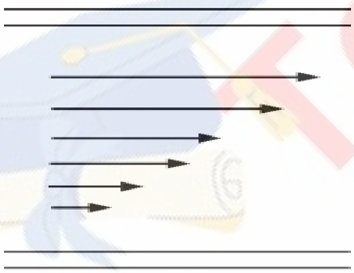


Aerofoil is a solid piece shaped to provide an upward dynamic lift when it moves horizontally through air.

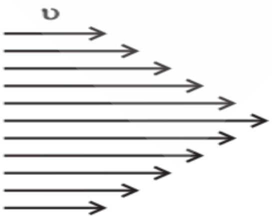
When the aerofoil moves against the wind, the orientation of the wing relative to flow direction causes the streamlines to crowd together above the wing more than those below it. The flow speed on top is higher than that below it. There is an upward force resulting in a dynamic lift of the wings and this balances the weight of the plane.

Viscosity

The internal frictional force that acts when there is relative motion between layers of the liquid is called viscosity.

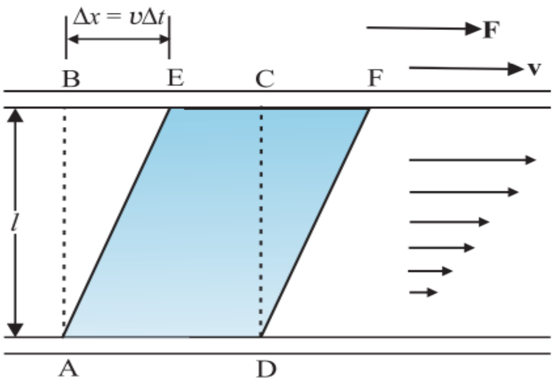


When liquid flows between a fixed and moving glass plates, the layer of the liquid in contact with top surface moves with a velocity v and the layer of the liquid in contact with the fixed surface is stationary. The velocities of layers increase uniformly from bottom to the top layer.



When a fluid is flowing in a pipe or a tube, then velocity of the liquid layer along the axis of the tube is maximum and decreases gradually as we move towards the walls where it becomes zero.

Coefficient of viscosity(η)



Due to viscous force, a portion of liquid, which at some instant has the shape ABCD, take the shape of AEFD after short interval of time (Δt).

$$\text{Shearing stress} = \frac{F}{A}$$

$$\text{Shearing strain} = \frac{\Delta x}{l}$$

$$\text{Strain rate} = \frac{\left(\frac{\Delta x}{l}\right)}{\Delta t} = \frac{\Delta x}{l\,\Delta t} = \frac{v}{l}$$

The coefficient of viscosity(η)for a fluid is defined as the ratio of shearing stress to the strain rate.

$$\eta = \frac{\text{Shearing stress}}{\text{Strain rate}} = \frac{\frac{F}{A}}{\frac{v}{l}}$$

$$\eta = \frac{Fl}{vA}$$

The SI unit of coefficient viscosity is poiseiulle (Pl).
Its other units are N s m^{-2} or Pa s.

The dimensions are $[ML^{-1}T^{-1}]$

Generally thin liquids like water, alcohol etc. are less viscous than thick liquids like coal tar, blood, glycerin etc.

The viscosity of liquids decreases with temperature while it increases in the case of gases.

Stokes' Law

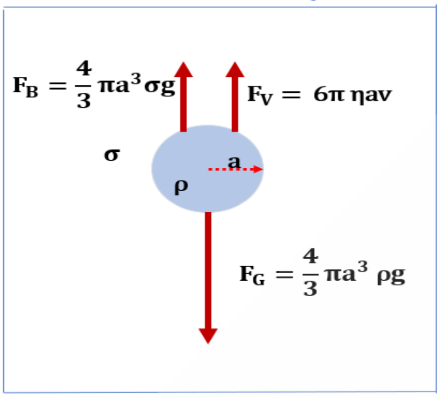
Stokes' law states that the viscous drag force F on a sphere of radius a moving with velocity v through a fluid of coefficient of viscosity η is,

$$F = 6\pi\eta a v$$

Terminal velocity

When an object falls through a viscous medium (raindrop in air), it accelerates initially due to gravity. As the velocity increases, the retarding force also increases. Finally when viscous force plus buoyant force becomes equal to the force due to gravity(weight of the body), the net force and acceleration become zero. The sphere (raindrop) then descends with a constant velocity clalled terminal velocity.

Expression for Terminal velocity



Consider a raindrop in air. The forces acting on the drop are

1. Force due to gravity(weight,mg) acting downwards , $F_G = \frac{4}{3}\pi a^3 \rho g$
2. Buoyant force acting upwards, $F_B = \frac{4}{3}\pi a^3 \sigma g$
3. Viscous force, $F_V = 6\pi\eta a v$

In equilibrium,

$$6\pi\eta a v + \frac{4}{3}\pi a^3 \sigma g = \frac{4}{3}\pi a^3 g\rho$$

$$6\pi\eta a v = \frac{4}{3}\pi a^3 (\rho - \sigma) g$$

Terminal velocity ,

$$v_t = \frac{2a^2(\rho - \sigma)g}{9\eta}$$

So the terminal velocity v_t depends on the square of the radius of the sphere and inversely on the viscosity of the medium.

Example

The terminal velocity of a copper ball of radius 2.0 mm falling through a tank of oil at 20^0C is 6.5 cm s^{-1}. Compute the viscosity of the oil at 20^0C. Density of oil is 1.5×10^3 kg m^{-3}, density of copper is 8.9×10^3 kg m^{-3}.

$v_t = 6.5 \times 10^{-2}\ ms^{-1}$ $\rho = 8.9 \times 10^3$ kg m^{-3}

$a = 2 \times 10^{-3}$ m $\sigma = 1.5 \times 10^3$ kg m^{-3}

$g = 9.8 m\ s^{-2}$,

$$v_t = \frac{2a^2(\rho - \sigma)g}{9\eta}$$

$$\eta = \frac{2a^2(\rho - \sigma)g}{9v_t}$$

$$\eta = \frac{2 \times (2 \times 10^{-3})^2 (8.9 \times 10^3 - 1.5 \times 10^3) \times 9.8}{9 \times 6.5 \times 10^{-2}}$$

$$\eta = 9.9 \times 10^{-1}\ \text{kg m}^{-1}\text{s}^{-1}$$

Reynolds Number

Osborne Reynolds defined a dimensionless number, whose value gives one an approximate idea whether the flow would be turbulent . This number is called the Reynolds number(R_e)

$$R_e = \frac{\rho v d}{\eta}$$

where ρ is the density of the fluid ,v is the speed of fluid, d stands for the dimension of the pipe, and η is the viscosity of the fluid.

$R_e < 1000$ – The flow is streamline or laminar.

$R_e > 2000$ – The flow is turbulent .

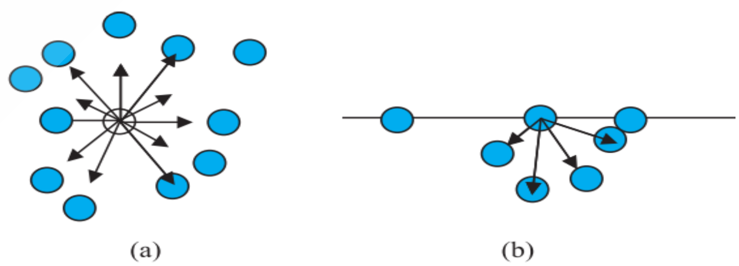
R_e between 1000 and 2000-The flow becomes unsteady .

- **The critical value of Reynolds number at which turbulence sets, is known as critical Reynolds number.** Turbulence dissipates kinetic energy usually in the fm of heat. Racing cars and planes are engineered to precision in order to minimise turbulence.
- Turbulence is sometimes desirable. The blades of a kitchen mixer induce turbulent flow and provide thick milk shakes as well as beat eggs into a uniform texture.

Surface Tension

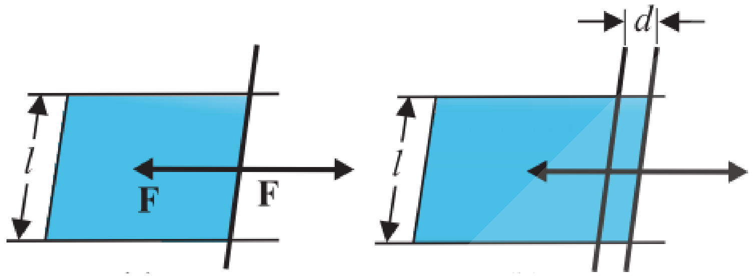
The free surface of a liquid possess some additional energy and it behaves like a stretched elastic membrane. This phenomenon is known as surface tension. Surface tension is concerned with only liquid as gases do not have free surfaces.

Surface Energy



For a molecule well inside a liquid the net force on it is zero. But the molecules on the surface have a net downward pull. So work has to be done against this downward force and this work is stored as energy in suface molecules. Thus, molecules on a liquid surface have some extra energy in comparison to molecules in the interior,which is termed as surface energy. A liquid thus tends to have the least surface area inorder to reduce surface enegy.

Surface Energy and Surface Tension



Consider a horizontal liquid film ending in a movable bar. Due to surface tension the bar is pulled inwards .

Inorder to keep the bar in its original position some work has to be done against this inward full.

W = F x d----------(1)

This work done increases surface energy.

If the surface energy of the film is S per unit area, the extra area is 2d l (film has two sides),

The extra surface energy = S x 2d l ------------(2)

The extra surface energy = work done

S x2dl =Fd

$$S = \frac{F}{2l}$$

This quantity S is the magnitude of surface tension.

Definition of Surface tension

Surface tension is a force per unit length (or surface energy per unit area) acting in the plane of the interface between the plane of the liquid and any other substance. It is the extra energy that the molecules at the interface have as compared to the interior

$$\text{Surface Tension, } S = \frac{\text{Force}}{\text{Length}}$$

The SI Unit is Nm^{-1}

Dimensional formula is MT^{-2}

The value of surface tension depends on temperature.

The surface tension of a liquid decreases with temperature.

Some effects of surface Tension

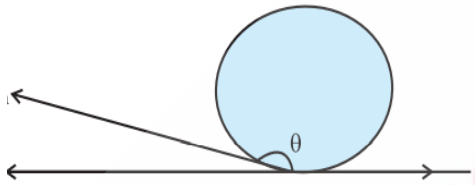
- Oil and water do not mix.
- Water wets you and me but not ducks.
- Mercury does not wet glass but water sticks to it.
- Oil rises up a cotton wick, inspite of gravity.
- Sap and water rise up to the top of the leaves of the tree.
- Hairs of a paint brush do not cling together when dry and even when dipped in water but form a fine tip when taken out of it.

Angle of Contact

The angle between tangent to the liquid surface at the point of contact and solid surface inside the liquid is termed as angle of contact(θ)

The value of θ determines whether a liquid will spread on the surface of a solid or it will form droplets on it.

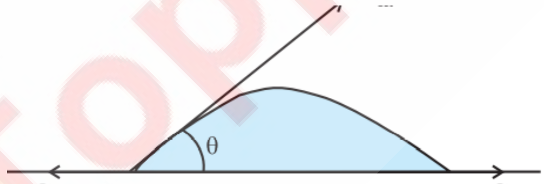
When Angle of contact is Obtuse:



When θ is an obtuse angle(greater than 90) then molecules of liquids are attracted strongly to themselves and weakly to those of solid, and liquid then does not wet the solid.

Eg: Water on a waxy or oily surface, Mercury on any surface.

When Angle of contact is Acute:



When θ is an acute angle (less than 90), the molecules of the liquid are strongly attracted to those of the solid and liquid then wets the solid.

Eg: Water on glass or on plastic, Kerosene oil on virtually anything .

Action Soaps and detergents

Soaps, detergents and dying substances are wetting agents. When they are added the angle of contact becomes small so that these may penetrate well and become effective.

Action of Water proofing agents

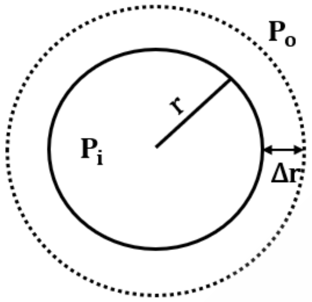
Water proofing agents are added to create a large angle of contact between the water and fibres.

Drops and Bubbles

Why are small drops and bubbles spherical?

Due to surface tension ,liquid surface has the tendency to reduce surface area. For a given volume sphere has minimum surface area. So small drops and bubbles are spherical.
For large drops the effect of gravity predominates that of surface tension and they get flattened.

Excess Pressure inside a spherical drop



Due to surface tension the liquid surface experiences an inward pull and as a result the pressure inside a spherical drop is more than the pressure outside. Due to this excess pressure let the radius of drop increase by Δr

Work done in expansion= Force x Displacement

= Excess pressure x Area x Displacement

$$W = (P_i - P_o) \times 4\pi r^2 \times \Delta r \text{ ------------------(1)}$$

This workdone is equal to the increase in surface energy

Extra Surface energy = Surface tension x Increase in surface area

Increase in surface area of drop $= 4\pi(r+\Delta r)^2 - 4\pi r^2$

$= 4\pi\,(r^2 + 2r\Delta r + \Delta r^2 - r^2)$

$= 8\pi r\Delta r$ (neglecting higher order terms)

Extra surface energy = S x8πrΔr ------------------(2)

The workdone = extra surface energy

$$(P_i - P_o) \times 4\pi r^2 \times \Delta r = 8\pi r\Delta r\, S \text{ --------------(3)}$$

$$(P_i - P_o) = \frac{2S}{r}$$

Excess Pressure Inside a Liquid Bubble

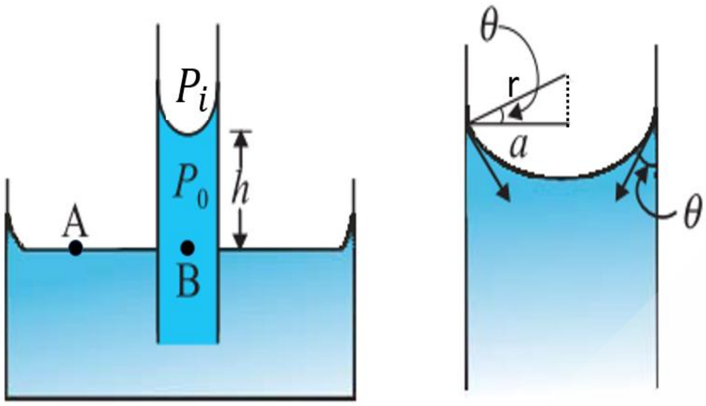
A bubble has two free surfaces.

$$(P_i - P_o) = 2 \times \frac{2S}{r}$$

$$(P_i - P_o) = \frac{4S}{r}$$

Capillary Rise

Due to the pressure difference across a curved liquid-air interface, water rises up in a narrow tube in spite of gravity. This is called capillary rise.



Consider a vertical capillary tube of circular cross section (radius a) inserted into an open vessel of water.

The excess pressure on the concave meniscus

$$(P_i - P_o) = \frac{2S}{r}$$

$$\cos\theta = \frac{a}{r}$$

$$r = \frac{a}{\cos\theta}$$

$$(P_i - P_o) = \frac{2S}{\frac{a}{\cos\theta}}$$

$$(P_i - P_o) = \frac{2S\cos\theta}{a} \text{----------------(1)}$$

Consider two points A and B in the same horizontal level i.e, the points are at the same pressure.

$$\text{Pressure at A} = P_i$$

$$\text{Pressure at B} = P_o + h\rho g$$

$$P_i = P_o + h\rho g$$

$$P_i - P_o = h\rho g \text{----------------(2)}$$

From eq(1) and (2)

$$h\rho g = \frac{2S\cos\theta}{a}$$

$$\mathbf{h = \frac{2S\cos\theta}{\rho g a}}$$

Thus capillary rise is a consequence of surface tension.
Capillary rise is larger, for capillary tube with smaller radius a.

Note:

If the liquid meniscus is convex, as for mercury, angle of contact θ will be obtuse . Then cos θ is negative and hence value of h will be negative. it is clear that the liquid will lower in the capillary and this is called **capillary fall or capillary depression.**

Example

Find the capillary rise when a capillary tube of radius 0.05 cm is dipped vertically in water. Surface tension for water is $0.073Nm^{-1}$ .Density of water is $1000\ kgm^{-3}$.

$$h = \frac{2S\cos\theta}{\rho\ ga}$$

For water-glass angle of contact $\theta = 0$, $\cos 0 = 1$

$$h = \frac{2S}{\rho\ ga}$$

$$h = \frac{2x\ 0.073}{1000\ x\ 9.8\ x\ 0.05\ x10^{-3}}$$

$$h = 2.98 \times 10^{-2}\ m = 2.8\ cm$$