Chapter 8
Mechanical Properties of Solids

A solid has definite shape and size. In order to change (or deform) the shape or size of a body, a force is required.

Elasticity

The property of a body, by virtue of which it tends to regain its original size and shape when the applied force is removed, is known as elasticity and such substances are called elastic .

Eg: Steel, Rubber

Steel is more elastic than rubber.

Plasticity

Some substances have no tendency to regain their previous shape on the removal of deforming force and they get permanently deformed. Such substances are called plastic and this property is called plasticity.

Eg:Putty and mud

Stress and Strain

When a force is applied on body, it is deformed to a small or large extent depending upon the nature of the material of the body and the magnitude of the deforming force. When a body is subjected to a deforming force, a restoring force is developed in the body. This restoring force is equal in magnitude but opposite in direction to the applied force.

Stress

The restoring force per unit area is known as stress.

If F is the force applied and A is the area of cross section of the body,

$$\text{Stress} = \frac{F}{A}$$

The SI unit of stress is N m^{-2} or pascal (Pa)

Dimensional formula of stress is [$ML^{-1}T^{-2}$]

Strain

Strain is defined as the **fractional change in dimension.**

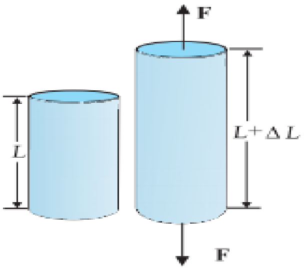
$$\text{Strain} = \frac{\text{Change in dimension}}{\text{Original dimension}}$$

Strain has no unit and dimension.

There are three ways in which a solid may change its dimensions when an external force acts on it. As a result there are three types of stress and strain.

1. Longitudinal Stress and Longitudinal Strain
2. Shearing Stress and Shearing Strain
3. Hydraulic Stress and Hydraulic Strain (Volume Strain)

1.Longitudinal Stress and Longitudinal Strain



Longitudinal stress is defined as the restoring force per unit area when force is applied normal to the cross-sectional area of a cylinder.

$$\text{Longitudinal stress} = \frac{F}{A}$$

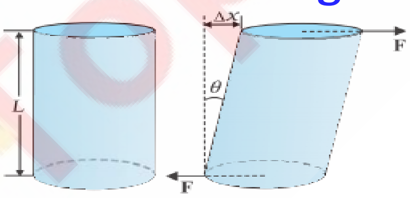
If the cylinder is stretched the stress is called **tensile stress** and If the cylinder is compressed it is called **compressive stress**.

Longitudinal strain is defined as the ratio of change in length(ΔL) to original length(L) of the body .

$$\text{Longitudinal strain} = \frac{\text{Change in length}}{\text{Original length}}$$

$$\text{Longitudinal strain} = \frac{\Delta L}{L}$$

2.Shearing Stress and Shearing Strain



Shearing stress is defined as the restoring force per unit area when a tangential force is applied on the cylinder.

$$\text{Shearing stress} = \frac{F}{A}$$

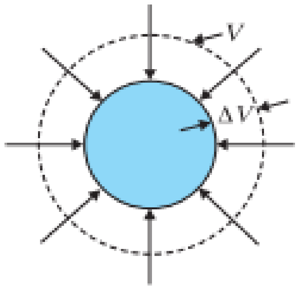
Shearing strain is defined as the ratio of relative displacement of the faces Δx to the length of the cylinder L

$$\text{Shearing strain} = \frac{\Delta x}{L} = \tan\theta$$

Usually θ is very small, tan θ ≈ θ

$$\text{Shearing strain} = \theta$$

3.Hydraulic Stress and Hydraulic strain (Volume Strain)



When a solid sphere placed in the fluid, the force applied by the fluid acts in perpendicular direction at each point of the surface.

The restoring force per unit area of solid sphere, placed in the fluid is called hydraulic stress.

$$\textbf{Hydraulic stress} = \frac{F}{A} = -P \text{ (pressure)}$$

The negative sign indicates that when pressure increases, the volume decreases.

Volume strain(hydraulic strain) is defined as the ratio of change in volume (ΔV) to the original volume (V).

$$\text{Volume strain} = \frac{\textbf{Change in volume}}{\textbf{Original volume}}$$

$$\text{Volume strain} = \frac{\Delta V}{V}$$

Hooke's Law

For small deformations the stress is directly proportional to strain. This is known as Hooke's law.

$$\text{Stress} \propto \text{Strain}$$

$$\text{stress} = k \times \text{strain}$$

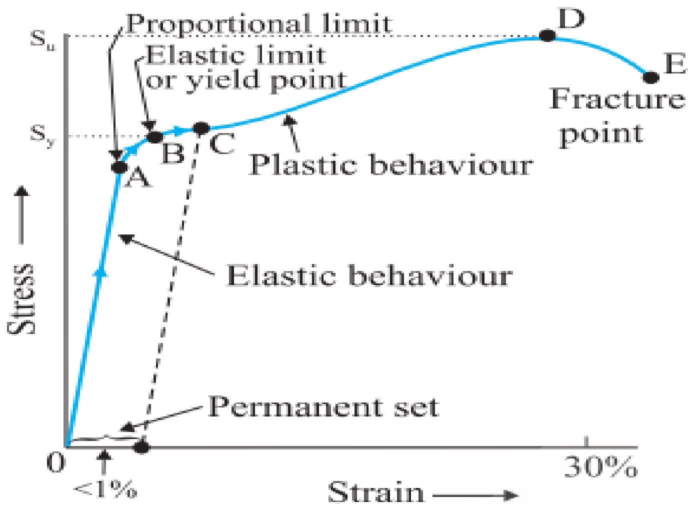
$$\frac{\text{Stress}}{\text{strain}} = k$$

where k is a constant and is known as **Modulus of Elasticity.**

- The SI unit of modulus of elasticity is N m^{-2} or pascal (Pa)
  (same as that of stress,since strain is unitless)
- Dimensional formula is $[\,ML^{-1}T^{-2}\,]$

Stress-Strain Curve

A typical stress-strain curve for a metal is as shown in figure:



In the region from O to A

The curve is linear. In this region, stress is proportional to strain i.e, Hooke's law is obeyed.

In the region from A to B

Stress and strain are not proportional, i.e, Hooke's law is not obeyed. Nevertheless, the body is still elastic.

The point B in the curve is known as yield point or elastic limit.

The stress corresponding to yield point is known as yield strength (S_y) of the material.

In the region from B to D

Beyond the point B ,the strain increases rapidly even for a small change in the stress. When the load is removed, at some point C between B and D, the body does not regain its original dimension. The material is said to have a permanent set. The material shows plastic behaviour in this region. **The point D on the graph is the ultimate tensile strength (S_u) of the material.**

In the region from D to E

Beyond this point D, additional strain is produced even by a reduced applied force and fracture occurs at point E.
The point E is called **Fracture Point.**
If the ultimate strength and fracture points **D and E are close,** the material is said to be **brittle.**
If **D and E are far apart,** the material is said to be **ductile.**

Elastomers

Substances like tissue of aorta, rubber etc. which can be stretched to cause large strains are called elastomers.

- Eventhough elastic region is very large, the material does not obey Hoke's law for most of the regions.
- There is no well defined plastic region.

Elastic Moduli

The ratio of stress and strain, called modulus of elasticity. Depending upon the types of stress and strain there are three moduli of elasticity.

1. **Young's Modulus(Y)**
2. **Shear Modulus or Rigidity Modulus** (G)
3. **Bulk modulus(B)**

1.Young's Modulus(Y)

The ratio of longitudinal stress to longitudinal strain is defined as Young's modulus of the material .

$$Y=\frac{\text{longitudinal stress}}{\text{longitudinal strain}}$$

$$Y=\frac{\frac{F}{A}}{\frac{\Delta L}{L}}$$

$$Y=\frac{FL}{A\ \Delta L}$$

If $F=mg$ and $A=\pi r^2$

$$Y=\frac{mgL}{\pi r^2\ \Delta L}$$

- SI unit of Young's modulus is N m^{-2} or Pa.
- For metals Young's moduli are large.
- Steel is more elastic than rubber as the Young's modulus of steel is large.
- Wood, bone, concrete and glass have rather small Young's moduli.

Why steel is preferred in heavy-duty machines and in structural designs?

Young's modulus of steel is greater than that of copper, brass and aluminium. It means that steel is more elastic than copper, brass and aluminium. It is for this reason that steel is preferred in heavy-duty machines and in structural designs.

Example

A structural steel rod has a radius of 10 mm and a length of 1.0 m. A 100 kN force stretches it along its length. Calculate (a) stress, (b) elongation, and (c) strain on the rod. Young's modulus, of structural steel is 2.0×10^{11} N m^{-2}

(a)

$$\text{stress} = \frac{F}{A} = \frac{F}{\pi r^2}$$

$$= \frac{100\ x\ 10^3}{3.14\ x\ (10\ x\ 10^{-3})^2} = \frac{100\ x\ 10^3}{3.14\ x\ 10^{-4}}$$

$$= \mathbf{3.18\ x10^8\ N m^{-2}}$$

(b)

$$Y = \frac{FL}{A\ \Delta L}$$

$$\Delta L = \frac{(\frac{F}{A})L}{Y} = \frac{3.18\ x10^8\ x\ 1}{2 \times 10^{11}}$$

$$= 1.59\ x10^{-3}\ \text{m}$$

$$= \mathbf{1.59\ mm}$$

(c)

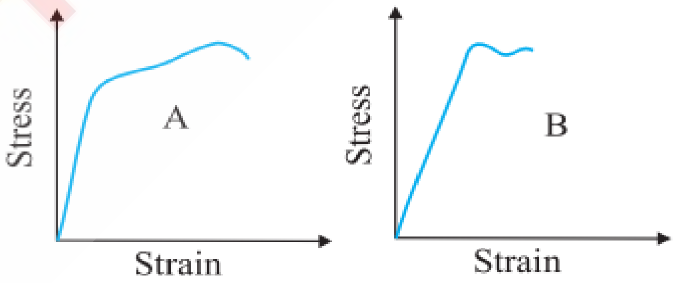
$$\text{Strain} = \frac{\Delta L}{L}$$

$$= \frac{1.59\ x10^{-3}}{1}$$

$$= \mathbf{1.59\ x10^{-3}\ m}$$

Example

The stress-strain graphs for materials A and B are shown in Figure.



The graphs are drawn to the same scale.

(a) Which of the materials has the greater Young's modulus?
(b) Which of the two is the stronger material?
(c) Which of the two materials is more ductile?

(a) Young's modulus $Y = \frac{\text{stress}}{\text{strain}}$ = slpoe of the graph.

Slope of graph for material A is greater than that of B.

So materials A has the greater Young's modulus.

(b) Strength of a material is determined by the amount of stress required to cause fracture.

The fracture point is greater for material A.

So material A is stronger than B

(c) The fracture point is far apart for material A than B.

So material A is more ductile than B.

2.Shear Modulus or Rigidity Modulus(G)

The ratio of shearing stress to the corresponding shearing strain is called the shear modulus or Rigidity modulus of the material .

$$G = \frac{\text{Shearing stress}}{\text{Shearing strain}}$$

$$G = \frac{\frac{F}{A}}{\frac{\Delta x}{L}} = \frac{\frac{F}{A}}{\theta}$$

$$\mathbf{G = \frac{F}{A\,\theta}}$$

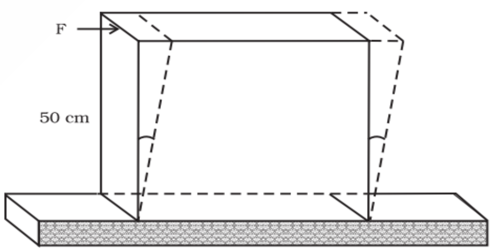
SI unit of shear modulus is N m^{-2} or Pa.

Shear modulus is generally less than Young's modulus.

For most materials G ≈ Y/3

Example

A square lead slab of side 50 cm and thickness 10 cm is subject to a shearing force (on its narrow face) of 9.0×10^4 N. The lower edge is riveted to the floor. How much will the upper edge be displaced? Given shear modulus, G =$5.6 \times 10^9 \text{Nm}^{-2}$



$$\text{Stress} = \frac{F}{A} = \frac{9.0 \times 10^4}{0.5 \times 0.1} = \frac{9.0 \times 10^4}{0.05}$$

$$= 1.80 \times 10^6 \text{ N } m^{-2}$$

$$G = \frac{\text{stress}}{\frac{\Delta x}{L}}$$

$$\Delta x = \frac{stress \ x \ L}{G}$$

$$= \frac{1.80 \times 10^6 \ x \ 0.5}{5.6 \times 10^9} = 1.6 \times 10^{-4} \text{ m} = 0.16 \text{ mm}$$

3.Bulk Modulus(B)

The ratio of hydraulic stress to the corresponding hydraulic strain is called bulk modulus.

$$B = \frac{\text{Hydraulic stress}}{\text{Hydraulic strain}}$$

$$B = \frac{\frac{F}{A}}{\frac{\Delta V}{V}} = \frac{-P}{\frac{\Delta V}{V}}$$

$$\mathbf{B = \frac{-PV}{\Delta V}}$$

SI unit of Bulk modulus is N m^{-2} or Pa.

The negative sign indicates that when pressure increases, the volume decreases. That is, if p is positive, ΔV is negative. Thus for a system in equilibrium, the value of bulk modulus B is always positive.

Compressibility(k)

The reciprocal of the bulk modulus is called compressibility.

$$k = \frac{1}{B}$$

$$\mathbf{k = \frac{-1}{P} \frac{\Delta V}{V}}$$

- The bulk moduli for solids are much larger than for liquids, which are again much larger than the bulk modulus for gases (air).
- Thus solids are least compressible whereas gases are most compressible.

Example

The average depth of Indian Ocean is about 3000 m. Calculate the fractional compression, $\Delta V/V$, of water at the bottom of the ocean, given that the bulk modulus of water is 2.2×10^9 N m^{-2}. (Take g = 10 m s^{-2})

$$B = \frac{-P}{\frac{\Delta V}{V}}$$

$$\frac{\Delta V}{V} = \frac{P}{B}$$

$$P = h\rho g = 3000 \text{ x}1000 \text{ x}10 = 3\text{x}10^7 \text{ N m}^{-2}$$

$$\frac{\Delta V}{V} = \frac{3\text{x}10^7}{2.2 \times 10^9} = 1.36 \times 10^{-2}$$

Poisson's ratio

When a material is stretched in one direction, it tends to compress in the direction perpendicular to that of force application and vice versa.
For example, a rubber band tends to become thinner when stretched.

The strain in the direction of applied force is called longitudinal strain.

$$\text{Longitudinal strain} = \frac{\text{Change in length}}{\text{Original length}}$$

$$= \frac{\Delta L}{L}$$

The strain perpendicular to the direction of applied force is called lateral strain.

$$\text{Lateral Strain} = \frac{\text{Change in diameter}}{\text{Original diameter}}$$

$$= \frac{\Delta d}{d}$$

The ratio of lateral strain to longitudinal strain is called Poisson's ratio.

$$\text{Poisson's Ratio } \sigma = \frac{\text{Lateral Strain}}{\text{Longitudnal Strain}}$$

$$\sigma = \frac{\frac{\Delta d}{d}}{\frac{\Delta L}{L}}$$

$$\boldsymbol{\sigma = \frac{\Delta d}{\Delta L} \text{ x} \frac{L}{d}}$$

Poisson's ratio has no unit and dimension.

Elastic Potential Energy in a Stretched String

The workdone to deform a body against the inter atomic force is stored as elastic PE.

For a string, work done for a small elongation dl

$$dW = F.d\,l$$

$$W=\int_0^l F.\,dl$$

$$\text{But} \quad Y= \frac{FL}{A\,l} \;,$$

l is the eleogation of string

$$F = \frac{YA\,l}{L}$$

$$W=\int_0^l \frac{YA\,l}{L}.\,dl$$

$$= \frac{YA}{L}\; x \;\frac{l^2}{2}$$

$$= \frac{YAl^2}{2L}$$

$$= \frac{1}{2}\; x \text{ Y x } (\frac{l}{L})^{\,2} \text{ x A } l$$

$$= \frac{1}{2}\; x \text{ Young's modulus x } (srtain)^{\,2} \text{ x volume}$$

$$= \frac{1}{2}\; x \;\frac{stress}{strain} \text{ x } (srtain)^{\,2} \text{ x volume}$$

$$W = \frac{1}{2} \text{ x stress x strain x volume}$$

This work done is equal to elastic Potential Energy.

Elastic PE $U = \frac{1}{2}$ **x stress x strain x volume**

Energy stored per unit volume

$$u= \frac{\text{Energy}}{volume}$$

$$u= \frac{1}{2} \text{ x stress x strain}$$

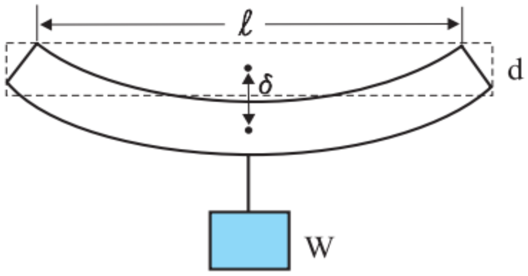
Applications of Elastic Behaviour of Materials

1.Cranes used for lifting and moving heavy loads have a thick metal rope . This is due to the fact that metals have greater youngs modulus.

Also, the elongation of the rope should not exceed the elastic limit. For this thicker rope of radius about 3 cm is recommended. A single wire of this radius would practically be a rigid rod. **So the ropes are always made of a number of thin wires braided together,** like in pigtails, for ease in manufacture, flexibility and strength.

2.The maximum height of a mountain on earth is ~10 km. The height is limited by the elastic properties of rocks.

3.In the construction of bridges and buildings , the beams should not bend too much or break. To reduce the bending for a given load, a material with a large Young's modulus Y is used.



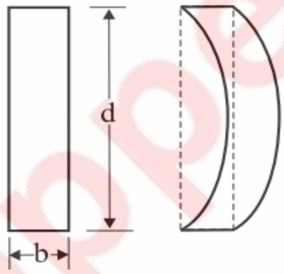
A beam of length l, breadth b, and depth d when loaded at the centre by a load W sags by an amount given by

$$\delta = \frac{W\, l^3}{4bd^3Y}$$

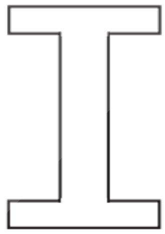
For a given load, the bending reduces when a material with a large Young's modulus Y is used.Bending can also be reduced by increasing thebreadth b, and depth d of the beam.

Buckling

Bending can be effectively reduced by increasing the depth d of the beam. But on increasing the depth, unless the load is exactly at the right place , the deep bar may bend sidewise(as in figure).This is called **buckling.**



To avoid buckling, beams with cross-sectional shape of I is used.



This section provides a large load bearing surface and enough depth to prevent bending. This shape reduces the weight of the beam without sacrificing the strength and hence reduces the cost.