MISCELLANEOUS EXERCISE

Question 1:

Solve the inequality $2 \le 3x - 4 \le 5$

Solution:

$$2 \le 3x - 4 \le 5$$
$$\Rightarrow 2 + 4 \le 3x - 4 + 4 \le 5 + 4$$
$$\Rightarrow 6 \le 3x \le 9$$
$$\Rightarrow 2 \le x \le 3$$

Thus, all the real numbers, x, which are greater than or equal to 2 but less than or equal to 3, are the solutions of the given inequality.

The solution set for the given inequality is $[2,3]$.

Question 2:

Solve the inequality $6 \le -3(2x - 4) < 12$

Solution:

$$6 \le -3(2x - 4) < 12$$
$$\Rightarrow 2 \le -(2x - 4) < 4$$
$$\Rightarrow -2 \ge 2x - 4 > -4$$
$$\Rightarrow 4 - 2 \ge 2x > 4 - 4$$
$$\Rightarrow 2 \ge 2x > 0$$
$$\Rightarrow 1 \ge x > 0$$
$$\Rightarrow 0 < x \le 1$$

Thus, the solution set for the given inequality is $(0,1]$.

Question 3:

Solve the inequality $-3 \le 4 - \frac{7x}{2} \le 18$

Solution:

$$-3 \le 4 - \frac{7x}{2} \le 18$$

$$\Rightarrow -3 - 4 \le -\frac{7x}{2} \le 18 - 4$$

$$\Rightarrow -7 \le -\frac{7x}{2} \le 14$$

$$\Rightarrow 7 \ge \frac{7x}{2} \ge -14$$

$$\Rightarrow 1 \ge \frac{x}{2} \ge -2$$

$$\Rightarrow 2 \ge x \ge -4$$

$$\Rightarrow -4 < x < 2$$

Thus, the solution set for the given inequality is $[-4, 2]$.

Question 4:

Solve the inequality $-15 < \frac{3(x-2)}{5} \le 0$

Solution:

$$-15 < \frac{3(x-2)}{5} \le 0$$

$$\Rightarrow -75 < 3(x-2) \le 0$$

$$\Rightarrow -25 < x - 2 \le 0$$

$$\Rightarrow -25 + 2 < x \le 2$$

$$\Rightarrow -23 < x \le 2$$

Thus, the solution set for the given inequality is $(-23, 2]$

Question5:

Solve the inequality $-12 < 4 - \frac{3x}{-5} \le 2$

Solution:

$$-12 < 4 - \frac{3x}{-5} \le 2$$

$$\Rightarrow -12 - 4 < \frac{3x}{5} \le 2 - 4$$

$$\Rightarrow -16 < \frac{3x}{5} \le -2$$

$$\Rightarrow -80 < 3x \le -10$$

$$\Rightarrow \frac{-80}{3} < x \le \frac{-10}{3}$$

Thus, the solution set for the given inequality is $\left(\frac{-80}{3}, \frac{-10}{3}\right]$.

Question 6:

Solve the inequality $7 \le \frac{(3x+11)}{2} \le 11$

Solution:

$$7 \le \frac{(3x+11)}{2} \le 11$$
$$\Rightarrow 14 \le 3x+11 \le 22$$
$$\Rightarrow 14-11 \le 3x \le 22-11$$
$$\Rightarrow 3 \le 3x \le 11$$
$$\Rightarrow 1 \le x \le \frac{11}{3}$$

Thus, the solution set for the given inequality is $\left[1, \frac{11}{3}\right]$.

Question 7:

Solve the inequalities and represent the solution graphically on number line.

$5x+1>-24, 5x-1<24$

Solution:

$$5x+1>-24$$
$$\Rightarrow 5x>-25$$
$$\Rightarrow x>-5 \quad \ldots(1)$$

$$5x-1<24$$
$$\Rightarrow 5x<25$$
$$\Rightarrow x<5 \quad \ldots(2)$$

From (1) and (2), we get, $-5 < x < 5$

Hence, it can be concluded that the solution set for the given system of inequalities is $(-5,5)$.

The solution of the given system of inequalities can be represented on number line as

Question 8:

Solve the inequalities and represent the solution graphically on number line:

$2(x-1)<x+5, 3(x+2)>2-x$

Solution:

$2(x-1)<x+5$

$\Rightarrow 2x-2<x+5$

$\Rightarrow 2x-x<5+2$

$\Rightarrow x<7 \quad \ldots(1)$

$3(x+2)>2-x$

$\Rightarrow 3x+6>2-x$

$\Rightarrow 3x+x>2-6$

$\Rightarrow 4x>-4$

$\Rightarrow x>-1 \quad \ldots(2)$

From (1) and (2), we get, $-1<x<7$

Hence, it can be concluded that the solution set for the given system of inequalities is $(-1,7)$.

The solution of the given system of inequalities can be represented on number line as

Question 9:

Solve the inequalities and represent the solution graphically on number line:

$3x-7>2(x-6), 6-x>11-2x$

Solution:

$3x-7>2(x-6)$

$\Rightarrow 3x-7>2x-12$

$\Rightarrow 3x-2x>-12+7$

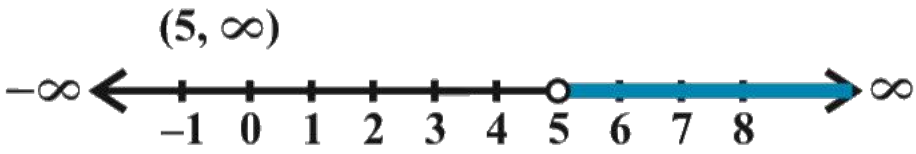
$\Rightarrow x>-5 \quad \ldots(1)$

$$6 - x > 11 - 2x$$
$$\Rightarrow -x + 2x > 11 - 6$$
$$\Rightarrow x > 5 \qquad \ldots(2)$$

From (1) and (2), we get, $-5 < x > 5$

Hence, it can be concluded that the solution set for the given system of inequalities is $(5, \infty)$.

The solution of the given system of inequalities can be represented on number line as



Question 10:

Solve the inequalities and represent the solution graphically on number line:

$5(2x - 7) - 3(2x + 3) \le 0,\ 2x + 19 \le 6x + 47$

Solution:

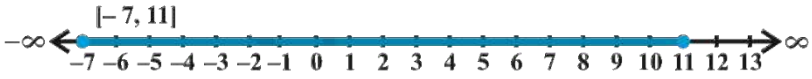
$$5(2x - 7) - 3(2x + 3) \le 0$$
$$\Rightarrow 10x - 35 - 6x - 9 \le 0$$
$$\Rightarrow 4x - 44 \le 0$$
$$\Rightarrow 4x \le 44$$
$$\Rightarrow x \le 11 \qquad \ldots(1)$$

$$2x + 19 \le 6x + 47$$
$$\Rightarrow 19 - 47 \le 6x - 2x$$
$$\Rightarrow -28 \le 4x$$
$$\Rightarrow -7 \le x \qquad \ldots(2)$$

From (1) and (2), we get, $-7 \le x \le 11$

Hence, it can be concluded that the solution set for the given system of inequalities is $[-7, 11]$.

The solution of the given system of inequalities can be represented on number line as



Question 11:

A solution is to be kept between $68^\circ F$ and $77^\circ F$. What is the range in temperature in degree Celsius (C) if the Celsius / Fahrenheit (F) conversion formula is given by $F = \frac{9}{5}C + 32$?

Solution:

Since the solution is to be kept between $68^\circ F$ and $77^\circ F$, $68° < F < 77°$

Putting $F = \frac{9}{5}C + 32$, we obtain

$$68 < \frac{9}{5}C + 32 < 77$$

$$\Rightarrow 68 - 32 < \frac{9}{5}C < 77 - 32$$

$$\Rightarrow 36 < \frac{9}{5}C < 45$$

$$\Rightarrow 36 \times \frac{5}{9} < C < 45 \times \frac{5}{9}$$

$$\Rightarrow 20 < C < 25$$

Thus, the required range of temperature in degree Celsius is between $20^\circ C$ and $25^\circ C$.

Question 12:

A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 litres of the 8% solution, how many litres of the 2% solution will have to be added?

Solution:

Let x litres of 2% boric acid solution is required to be added.

Then, total mixture $= (x + 640)$ litres

This resulting mixture is to be more than 4% but less than 6% boric acid.

Therefore, $2\%x + 8\% \text{ of } 640 > 4\% \text{ of } (x + 640)$ and $2\%x + 8\% \text{ of } 640 < 6\% \text{ of } (x + 640)$

$$2\%x + 8\% \text{ of } 640 > 4\% \text{ of } (x + 640)$$

$$\Rightarrow \frac{2}{100}x + \frac{8}{100}(640) > \frac{4}{100}(x + 640)$$

$$\Rightarrow 2x + 5120 > 4x + 2560$$

$$\Rightarrow 5120 - 2560 > 4x - 2x$$

$$\Rightarrow 2560 > 2x$$

$$\Rightarrow x < 1280 \qquad \ldots(1)$$

$$2\%x + 8\% \text{ of } 640 < 6\% \text{ of } (x + 640)$$

$$\Rightarrow \frac{2}{100}x + \frac{8}{100}(640) < \frac{6}{100}(x + 640)$$

$$\Rightarrow 2x + 5120 < 6x + 2560$$

$$\Rightarrow 5120 - 2560 < 6x - 2x$$

$$\Rightarrow 2560 < 3x$$

$$\Rightarrow x > 320 \qquad \ldots(2)$$

From (1) and (2), we get, $320 < x < 1280$

Thus, the number of litres of 2% of boric acid solution that is to be added will have to be more than 320 litres but less than 1280 litres.

Question 13:

How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

Solution:

Let x litres of water is required to be added.

Then, total mixture $= (1125 + x)$ litres

It is evident that the amount of acid contained in the resulting mixture is 45% of 1125 litres. This resulting mixture will contain more than 25% but less than 30% acid content.

Therefore, $30\% \text{ of } (1125 + x) > 45\% \text{ of } 1125$ and $25\% \text{ of } (1125 + x) < 45\% \text{ of } 1125$

$$30\% \text{ of } (1125 + x) > 45\% \text{ of } 1125$$

$$\Rightarrow \frac{30}{100}(1125 + x) > \frac{45}{100} \times 1125$$

$$\Rightarrow 30(1125 + x) > 45 \times 1125$$

$$\Rightarrow 30 \times 1125 + 30x > 45 \times 1125$$

$$\Rightarrow 30x > 45 \times 1125 - 30 \times 1125$$

$$\Rightarrow 30x > (45 - 30) \times 1125$$

$$\Rightarrow x > \frac{15 \times 1125}{30}$$

$$\Rightarrow x > 562.5 \qquad \ldots(1)$$

$$25\% \text{ of } (1125+x) < 45\% \text{ of } 1125$$

$$\Rightarrow \frac{25}{100}(1125+x) < \frac{45}{100}\times 1125$$

$$\Rightarrow 25(1125+x) < 45\times 1125$$

$$\Rightarrow 25\times 1125 + 25x < 45\times 1125$$

$$\Rightarrow 25x < 45\times 1125 - 25\times 1125$$

$$\Rightarrow 25x < (45-25)\times 1125$$

$$\Rightarrow x < \frac{20\times 1125}{25}$$

$$\Rightarrow x < 900 \qquad \ldots(2)$$

From (1) and (2), we get, $562.5 < x < 900$

Thus, the required number of litres of water that is to be added will have to be more than 562.5 litres but less than 900 litres.

Question 14:

IQ of a person is given by the formula

$$IQ = \frac{MA}{CA}\times 100$$

where MA is mental age and CA is chronological age. If $80 \le IQ \le 140$ for a group of 12 years old children, find the range of their mental age.

Solution:

It is given that for a group of 12 years old children, $80 \le IQ \le 140 \qquad \ldots(1)$

For a group of 12 years old children, $CA = 12$ years

Therefore, $IQ = \frac{MA}{12} \times 100$

Putting this value of IQ in (1), we obtain

$$80 \le \frac{MA}{12} \times 100 \le 140$$
$$\Rightarrow 80 \times \frac{12}{100} \le MA \le 140 \times \frac{12}{100}$$
$$\Rightarrow 9.6 \le MA \le 16.8$$

Thus, the range of mental age of the group of 12 years old children is $9.6 \le MA \le 16.8$.

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