Chapter 4
Laws of Motion

Galileo's Law of Inertia

If the net external force is zero, a body at rest continues to remain at rest and a body in motion continues to move with a uniform velocity.
This property of the body is called inertia.Inertia means 'resistance to change'.

Suppose a person is standing in a stationary bus and the driver starts the bus suddenly. He gets thrown backward with a jerk. This is due to his inertia of rest.

Similarly if a person is standing in a moving bus and if the bus suddenly stops he is thrown forward. This is due to his inertia of motion.

Newton's Laws of Motion

Newton built on Galileo's ideas and laid the foundation of mechanics in terms of three laws of motion. Galileo's law of inertia was his starting point on which he formulated as the First Law of motion.

Newton's First Law of Motion (Law of inertia)

Every body continues to be in its state of rest or of uniform motion in a straight line unless compelled by some external force to change that state.

The state of rest or uniform linear motion both imply zero acceleration. If the net external force on a body is zero, its acceleration is zero. Acceleration can be non zero only if there is a net external force on the body.

Momentum

Momentum, P of a body is defined to be the product of its mass m and velocity v, and is denoted by p.

$$p = m\,v$$

Momentum is a vector quantity.

Unit = kgm/s

$$[p] = ML\,T^{-1}$$

- Suppose a light-weight vehicle car) and a heavy weight vehicle are parked on a horizontal road. A greater force is needed to push the truck than the car to bring them to the sam e speed in same time. Similarly, a greater opposing force is needed to stop a heavy body than a light body in the same time, if they are moving with the same speed.
- Speed is another important parameter to consider. A bullet fired by a gun can easily pierce human tissue before it stops, resulting in casualty. The same bullet fired with moderate speed will not cause much damage.Thus for a given mass, the greater the speed, the greater is the opposing force needed to stop the body in a certain time.

Newton's Second Law f Motion

The rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction in which the force acts.

$$\mathbf{F} \propto \frac{\Delta \mathbf{p}}{\Delta t}$$

$$F = k\frac{\Delta p}{\Delta t}$$

For simplicity we choose k=1

In the limit $\Delta t \to 0$

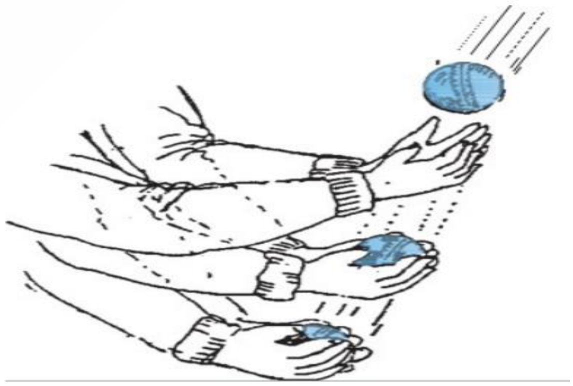
$$\mathbf{F} = \frac{dp}{dt}$$

Why a cricketer draws his hands backwards during a catch?

By Newton's second law of motion ,

$$\mathbf{F} = \frac{\Delta \mathbf{p}}{\Delta t}$$

When he draws his hands backwards, the time interval (Δt) to stop the ball increases . Then force decreases and it does not hurt his hands. Force not only depends on the change in momentum but also on how fast the change is brought about.



Derivation of Equation of force from Newton's second law of motion

By Newton's second law of motion ,

$$F = \frac{dp}{dt}$$

For a body of fixed mass m, p=mv

$$F = \frac{d}{dt}mv$$

$$F = m\frac{dv}{dt}$$

$$F = ma$$

Force is a vector quantity

Unit of force is $kgms^{-2}$ or newton (N)

Definition of newton

$$F = m\,a$$

If $m=1$ kg , $a=1m\,s^{-2}$

$$F=1kg \times 1\ ms^{-2}$$

$$F=1N$$

One newton is that which causes an acceleration of $m\,s^{-2}$ to a mass of 1kg

Important points about second law

1.The second Law is consistent with the first law.

From Newton's second law,

$$F = ma$$

If $F = 0$, $ma = 0$

(since $m \neq 0$)

$$a = 0$$

Zero acceleration implies the state of rest or uniform linear motion . i.e, when there is no external force , the body will remain in its state of rest or of uniform motion in a straight line. This is Newtons first law of motion.

2. The second law of motion is a vector law.

$$F_x = \frac{dP_x}{dt} = m\frac{dv_x}{dt} = ma_x$$

$$F_y = \frac{dP_y}{dt} = m\frac{dv_y}{dt} = ma_y$$

$$F_z = \frac{dP_z}{dt} = m\frac{dv_z}{dt} = ma_z$$

3. If a force makes some angle with velocity, the force changes only the component of velocity along the direction of force.

4. For a system of particles we can write

$$F_{net} = ma$$

F_{net} refers to the total external force on the system and a refers to the acceleration of the system as a whole.

5. In equation F=ma

Acceleration at any instant is determined by the force at that instant , not by any history of the motion of the particle.

Example

A bullet of mass 0.04 kg moving with a speed of 90 m/s enters a heavy wooden block and is stopped after a distance of 60 cm. What is the average resistive force exerted by the block on the bullet?

m=0.04 kg

u=90 m/s

v=0

s=60cm=0.6 m

The retardation 'a' of the bullet is assumed to be constant.

$$v^2 - u^2 = 2as$$

$$0 - 90^2 = 2 \text{x a x} 0.6$$

$$a = \frac{-90^2}{2 \text{ x } 0.6}$$

$$a = -6750 \text{ m}/s^2$$

The retarding force, F= ma

$$F = 0.04 \text{ x} - 6750$$

$$F = -270 \text{ N}$$

The negative sign shows that the force is resistive or retarding.

Example

The motion of a particle of mass m is described by $y = ut + \frac{1}{2}gt^2$. Find the force acting on the particle.

$$y = ut + \frac{1}{2}gt^2$$

Velocity, $v = \frac{dy}{dt}$

$$v = \frac{d}{dt}(ut + \frac{1}{2}gt^2)$$

$$v = u\frac{dt}{dt} + \frac{1}{2}g\frac{d}{dt}t^2)$$

$$v = u + \frac{1}{2}g \times 2t$$

$\frac{d}{dt}t^n = nt^{n-1}$

$\frac{d}{dt}t^2 = 2t^{2-1} = 2t$

$$v = u + gt$$

Acceleration, $a = \frac{dv}{dt}$

$$a = \frac{d}{dt}(u + gt)$$

$$a = \frac{du}{dt} + g\frac{dt}{dt})$$

$$a = g$$

Force, $F = ma$

$$F = mg$$

Impulse (I)

There are some situations where a large force acts for a very short duration producing a finite change in momentum of the body. For example, when a ball hits a wall and bounces back, the force on the ball by the wall acts for a very short time when the two are in contact, yet the force is large enough to reverse the momentum of the ball.

Impulse is the the product of force and time duration, which is the change in momentum of the body.

Impulse = Force × time duration

$$I = F \times t$$

Unit = kg m s^{-1}

$[I] = M L T^{-1}$

Impulsive force.

A large force acting for a short time to produce a finite change in momentum is called an impulsive force.

Impulse momentum Principle

Impulse is equal to the change in momentum of the body.

By Newton's second law of motion,

$$F=\frac{dp}{dt}$$

$$F \times dt = dp$$

$$I = dp$$

$$\text{Impulse} = \text{change in momentum}$$

Example

A batsman hits back a ball straight in the direction of the bowler without changing its initial speed of 12 m s^{-1} . If the mass of the ball is 0.15 kg, determine the impulse imparted to the ball.

Impulse = change of momentum

Change in momentum = final momentum – initial momentum

Change in momentum = $0.15 \times 12-(0.15\times -12)$

Impulse = 3.6 N s

Newton's Third Law of Motion

To every action, there is always an equal and opposite reaction.

Important points about Third law

1. Forces always occur in pairs. Force on a body A by B is equal and opposite to the force on the body B by A.

2. There is no cause- effect relation implied in the third law. The force on A by B and the force on B by A act at the same instant. By the same reasoning, any one of them may be called action and the other reaction.

3. Action and reaction forces act on different bodies, not on the same body. So they do not cancel each other , eventhough they are equal and opposite.

According to the third law,

$$F_{AB} = -F_{BA} \quad \text{(force on A by B)} = -\text{(force on B by A)}$$

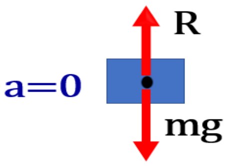
4. However, if you are considering the system of two bodies as a whole, F_{AB} and F_{BA} are internal forces of the system (A + B). They add up to give a null force.

Weight of a body in a lift

Weight of a body is the normal reaction exerted by the surface in contact with that body.

Case:1

When lift is at rest(or moving up or down with uniform velocity)

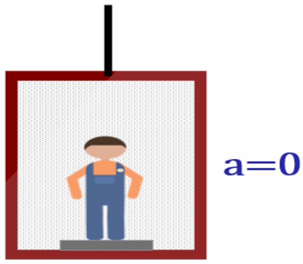


$$F_{net} = ma$$

$$R - mg = m \times 0$$

$$R - mg = 0$$

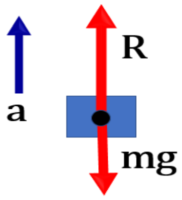
$$R = mg$$



No change in weight of the body.

Case 2

When lift is moving up with an acceleration ‘a ‘

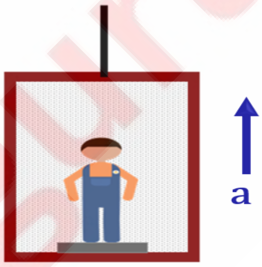


$$F_{net} = ma$$

$$R - mg = ma$$

$$R = mg + ma$$

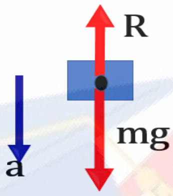
$$R = m(g + a)$$



The weight of the body increases

Case 3

When lift is moving down with an acceleration ‘a ‘

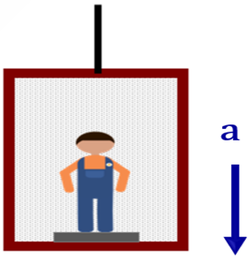


$$F_{net} = ma$$

$$mg - R = ma$$

$$R = mg - ma$$

$$R = m(g - a)$$



The weight of the body decreases

Case 4

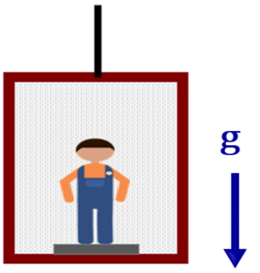
When the lift mechanism failed and it moves down freely under gravity.

$$R = mg - ma$$

$$a = g$$

$$R = m(g - g)$$

$$R = 0$$



The body experiences weightlessness.

Example

A man of mass 70 kg stands on a weighing scale in a lift which is moving,

(a) upwards with a uniform speed of 10 m s^{-1}

(b) downwards with a uniform acceleration of 5 m s^{-2}

(c) upwards with a uniform acceleration of 5 m s^{-2}

What would be the readings on the scale in each case?

(d) What would be the reading if the lift mechanism failed and it falls down freely under gravity? Take g=10 **m s^{-2}**

(a)When lift moves with uniform speed , a=0

R= mg = 70 x 10=700 N

Reading= 700/ 10= 70 kg

(b)Acceleration a = 5m s^{-2} downwards

R= m(g-a) =70 (10-5)= 70x5= 350N

Reading = 350/ 10 = 35 kg

(c) Acceleration a = 5m s^{-2} upwards

R= m(g+a) = 70(10+5) = 70x 15= 1050N

Reading = 1050/ 10 = 105 kg

(d) when lift falls freely a=g

R= m(g-g) = 0

Reading = 0

Law of Conservation of Momentum

The total momentum of an isolated system of interacting particles is conserved.

Or

When there is no external force acting on a system of particles ,their total momentum remains constant.

Proof of law of conservation of momentum

By Newton's second law of motion , $F=\frac{dp}{dt}$

When F = 0

$$\frac{dp}{dt} = 0$$

dp = 0 ,

p=constant

Thus when there is no external force acting on a system of particles, their total momentum remains constant.

Applications of law of conservation of linear momentum

1.Recoil of gun

When a bullet is fired from a gun , the backward movement of gun is called recoil of the gun.

By the law of conservation of momentum, as the system is isolated,

$$\mathbf{P = constant}$$

Initial momentum = Final momentum

Initial momentum of gun+ bullet system = 0

Final momentum of gun+ bullet system = 0

If p_b and p_g are the momenta of the bullet and gun after firing

$$p_b + p_g = 0$$

$$p_b = -p_g$$

The negative sign shows that the gun recoils to conserve momentum.

Expression for Recoil velocity and muzzle velocity

Momentum of bullet after firing , $p_b = mv$

Recoil momentum of the gun after firing , $p_g = MV$

$$p_b = -p_g$$

$$mv = -MV$$

Recoil velocity of gun , $V = \frac{-mv}{M}$

Muzzle velocity of bullet , $v = \frac{-MV}{m}$

M= mass of gun, V= recoil velocity of bullet
m= mass of bullet, v=muzzle velocity of bullet

2. The collision of two bodies

Before collision After collision

$$\text{By Newton's second law},\ F = \frac{\Delta P}{\Delta t}$$

$$F\Delta t = \Delta P$$

F_{AB} changes the momentum of body A

$$F_{AB}\,\Delta t = p'_A - p_A \text{-----------------(1)}$$

F_{BA} changes the momentum of body B

$$F_{BA}\,\Delta t = p'_B - p_B \text{ ------------(2)}$$

By Newton's third law

$$F_{AB} = -F_{BA} \qquad \text{-----------(3)}$$

$$p'_A - p_A = -(p'_B - p_B)$$

$$p'_A + p'_B = p_A + p_B$$

Total Final momentum = Total initial momentum
i.e. , the total final momentum of the isolated system equals its total initial momentum.

Equilibrium of a particle

Equilibrium of a particle in mechanics refers to the situation when the net external force on the particle is zero.
According to the first law, this means that, the particle is either at rest or in uniform motion.

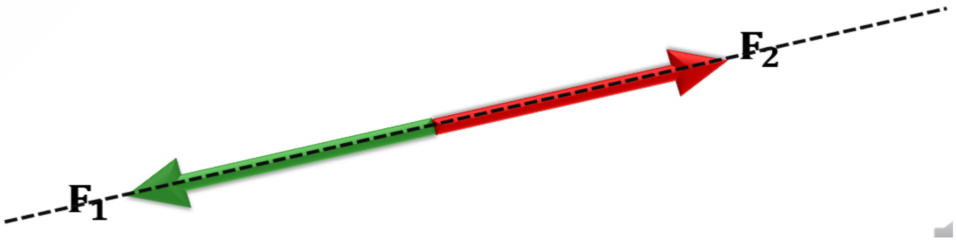
If two forces F_1 and F_2, act on a particle

The equilibrium requires,

$$F_1 + F_2 = 0$$

$$F_1 = -F_2$$

i.e. the two forces on the particle must be equal and opposite.



If three forces F_1, F_2 and F_3, act on a particle

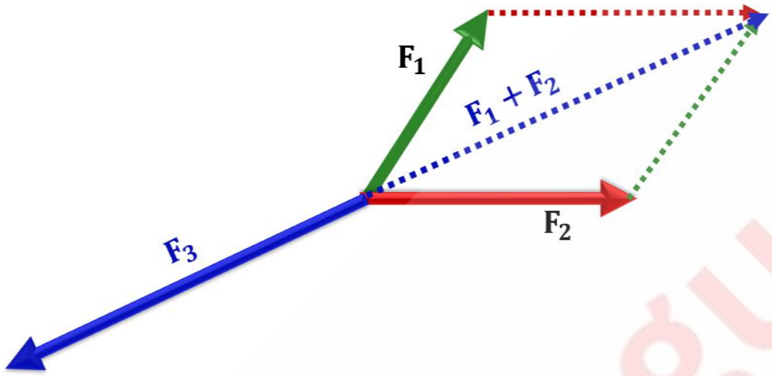
The equilibrium requires,

$$\mathbf{F_1} + \mathbf{F_2} + \boldsymbol{F_3} = 0$$

$$\mathbf{F_1} + \mathbf{F_2} = -\mathbf{F_3}$$

The resultant of any two forces say F_1 and F_2 , obtained by the parallelogram law of forces must be equal and opposite to the third force, F_3 .

$$\mathbf{F_1} + \mathbf{F_2} = -\mathbf{F_3}$$



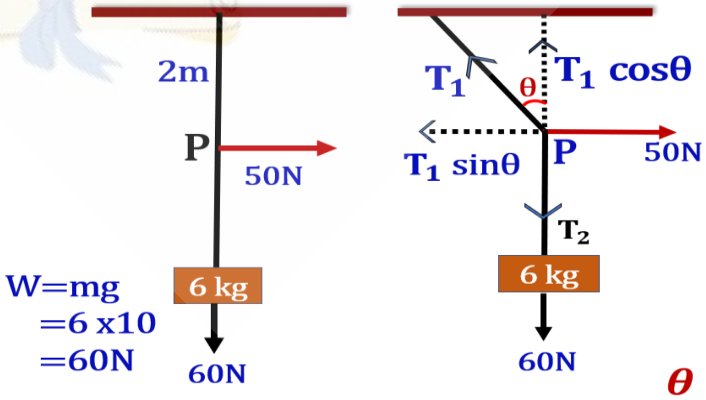
If a number of forces F_1, F_2,........F_n, act on a particle

$$\mathbf{F_1} + \mathbf{F_2} + \ldots\ldots\ldots + \mathbf{F_n} = 0$$

Example

A mass of 6 kg is suspended by a rope of length 2 m from the ceiling. A force of 50 N in the horizontal direction is applied at the midpoint P of the rope, as shown. What is the angle the rope makes with the vertical in equilibrium ? (Take $g = 10\ \mathrm{m\ s^{-2}}$).

Neglect the mass of the rope.



$$T_1 \cos\theta = 60$$

$$T_1 \sin\theta = 50$$

$$\tan\theta = \frac{50}{60} = \frac{5}{6}$$

$$\theta = \tan^{-1}\left(\frac{5}{6}\right) = 40^0$$

Common Forces in Mechanics

There are two types of forces in mechanics- Contact forces and Non contact forces.

Contact forces

A contact force on an object arises due to contact with some other object: solid or fluid.

Eg: Frictional force, viscous force, air resistance

Non contact forces

A non contact force can act at a distance without the need of any intervening medium.

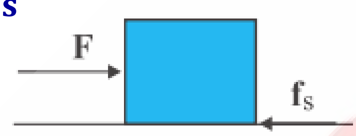
Eg: Gravitational force.

Friction

The force that opposes (impending or actual) relative motion between two surfaces in contact is called frictional force.

There are two types of friction-Static and Kinetic friction

Static friction f_s



- Static friction is the frictional force that acts between two surfaces in contact before the actual relative motion starts. Or Static friction f_s opposes impending relative motion.
  The maximum value of static friction is $(f_s)_{max}$
- The limiting value of static friction $(f_s)_{max}$, is independent of the area of contact.
- The limiting value of static friction $(f_s)_{max}$, varies with the normal force(N)

$$(f_s)_{max} \alpha N$$

$$(f_s)_{max} = \mu_s N$$

Where the **constant μ_s is called the coefficient of static friction** and depends only on the nature of the surfaces in contact.

The Law of Static Friction

The law of static friction may thus be written as , $f_s \leq \mu_s N$

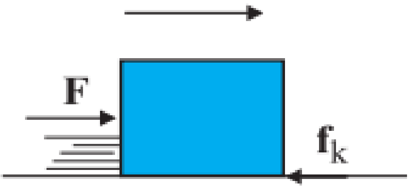
Or

$$(f_s)_{max} = \mu_s N$$

Note:

If the applied force F exceeds $(f_s)_{max}$, the body begins to slide on the surface. When relative motion has started, the frictional force decreases from the static maximum value $(f_s)_{max}$

Kinetic friction f_k



Frictional force that opposes (actual) relative motion between surfaces in contact is called kinetic or sliding friction and is denoted by f_k. .

- Kinetic friction is independent of the area of contact.
- Kinetic friction is nearly independent of the velocity.
- Kinetic friction , f_k varies with the normal force(N)

$$f_k \propto N$$

$$f_k = \mu_k N$$

where μ_k the **coefficient of kinetic friction**, depends only on the surfaces in contact.

μ_k is less than μ_s

The Law of Kinetic Friction

The law of kinetic friction can be written as , $f_k = \mu_k N$

where μ_k the **coefficient of kinetic friction,**

Example

▶ ***Example 5.7*** Determine the maximum acceleration of the train in which a box lying on its floor will remain stationary, given that the co-efficient of static friction between the box and the train's floor is 0.15.

Answer Since the acceleration of the box is due to the static friction,

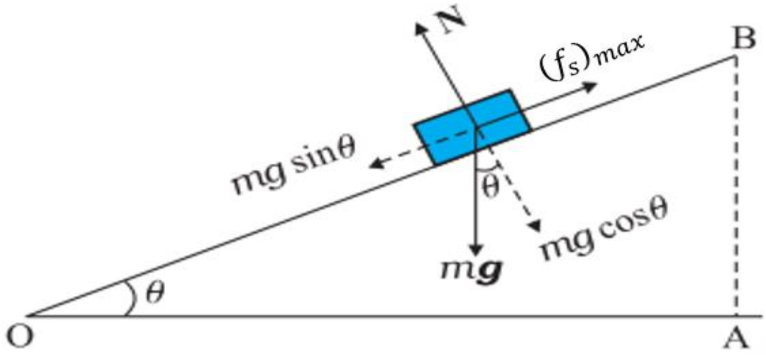
$$ma = f_s \le \mu_s N = \mu_s mg$$

i.e. $a \le \mu_s g$

$$\therefore a_{max} = \mu_s g = 0.15 \times 10 \text{ m s}^{-2}$$
$$= 1.5 \text{ m s}^{-2}$$

◀

Body on an inclined surface



The forces acting on a block of mass m When it just begins to slide are

(i) the weight mg

(ii) the normal force N

(iii) the maximum static frictional force $(f_s)_{max}$

In equilibrium, the resultant of these forces must be zero.

$$m\,g\sin\theta = (f_s)_{max}$$

$$\text{But } (f_s)_{max} = \mu_s N$$

$$mg\sin\theta = \mu_s N \text{------------}(1)$$

$$m\,g\cos\theta = N\text{------------}(2)$$

$$\text{Eqn}\frac{(1)}{(2)} \text{ --------- } \frac{mg\sin\theta}{m\,g\cos\theta} = \frac{\mu_s N}{N}$$

$$\boldsymbol{\mu_s = \tan\theta}$$

$$\boldsymbol{\theta = \tan^{-1}\mu_s}$$

This angle whose tangent gives the coefficient of friction is called angle of friction.

> **Example 5.8** See Fig. 5.11. A mass of 4 kg rests on a horizontal plane. The plane is gradually inclined until at an angle θ = 15° with the horizontal, the mass just begins to slide. What is the coefficient of static friction between the block and the surface ?

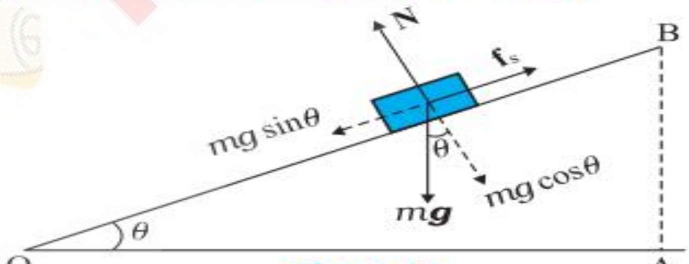


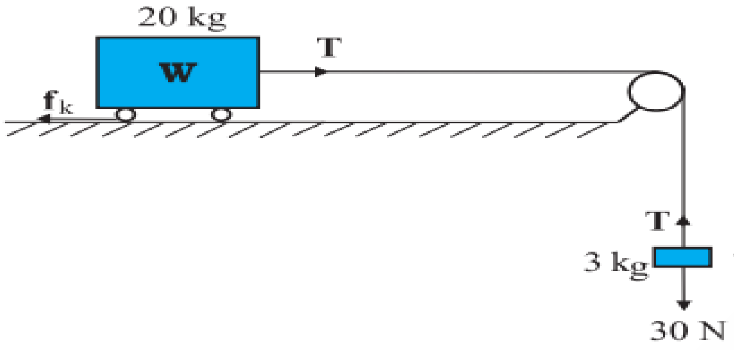
Fig. 5.11

$$\mu_s = \tan\theta$$

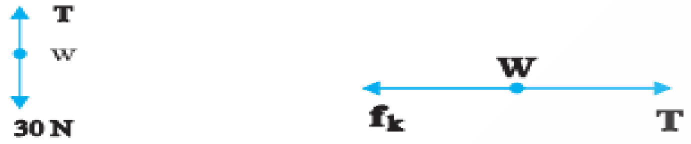
$$\theta = 15°,$$

$$\mu_s = \tan 15° = 0.27$$

Example 5.9 What is the acceleration of the block and trolley system shown in a Fig. 5.12(a), if the coefficient of kinetic friction between the trolley and the surface is 0.04? What is the tension in the string? (Take $g = 10\ m\ s^{-2}$). Neglect the mass of the string.



(a)



Applying second law to motion of the block

$$30 - T = 3a \text{ ---------------(1)}$$

Apply the second law to motion of the trolley

$$T - f_k = 20\ a \text{ ------------(2)}$$

Now $f_k = \mu_k\ N$,

Here $\mu_k = 0.04$, $N = 20 \times 10 = 200\ N$.

Substituting in eq(2)

$$T - 0.04 \times 200 = 20\ a$$

$$T - 8 = 20a \text{-------------(3)}$$

Solving eqns (1) and (3)

$$a = \frac{22}{23} m\ s^{-2} = 0.96\ m\ s^{-2}$$

$$T = 27.1\ N$$

Rolling Friction

It is the frictional force that acts between the surfaces in contact when one body rolls over the other.

Rolling friction is much smaller than static or sliding friction

Disadvantages of friction

friction is undesirable in many situations, like in a machine with different moving parts, friction opposes relative motion and thereby dissipates power in the form of heat, etc.

Advantages of friction

Friction is a necessary evil .In many practical situations friction is critically needed. Kinetic friction is made use of by brakes in machines and automobiles. We are able to walk because of static friction. It is impossible for a car to move on a very slippery road. On an ordinary road, the friction between the tyres and the road provides the necessary external force to accelerate the car.

Methods to reduce friction

(1)Lubricants are a way of reducing kinetic friction in a machine.
(2)Another way is to use ball bearings between two moving parts of a machine.
(3) A thin cushion of air maintained between solid surfaces in relative motion is another effective way of reducing friction.

Circular Motion

The acceleration of a body moving in a circular path is directed towards the centre and is called centripetal acceleration.

$$a = \frac{v^2}{R}$$

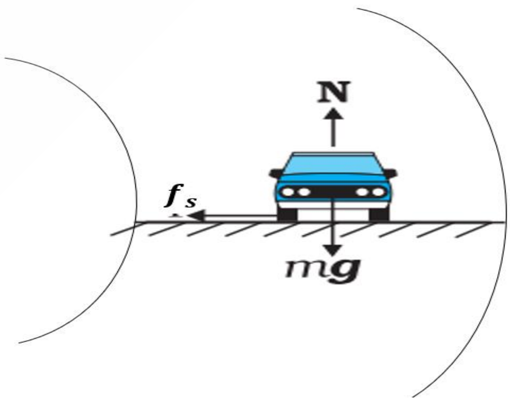
The force f providing centripetal acceleration is called the centripetal force and is directed towards the centre of the circle.

$$f_s = \frac{mv^2}{R}$$

where m is the mass of the body, R is the radius of circle.
For a stone rotated in a circle by a string, the centripetal force is provided by the tension in the string.

The centripetal force for motion of a planet around the sun is the gravitational force on the planet due to the sun.

Motion of a car on a curved level road



Three forces act on the car.

(i) The weight of the car, mg

(ii) Normal reaction, N

(iii) Frictional force, f_s

As there is no acceleration in the vertical direction

$$N = mg$$

The static friction provides the centripetal acceleration

$$f_s = \frac{mv^2}{R}$$

$$\text{But}, f_s \leq \mu_s N$$

$$\frac{mv^2}{R} \leq \mu_s mg \quad (N=mg)$$

$$\mathbf{v^2 \leq \mu_s Rg}$$

$$\mathbf{v_{max} = \sqrt{\mu_s Rg}}$$

This is the maximum safe speed of the car on a circular level road.

Example

A cyclist speeding at 18 km/h on a level road takes a sharp circular turn of radius 3 m without reducing the speed. The co-efficient of static friction between the tyres and the road is 0.1. Will the cyclist slip while taking the turn ?

$$v^2 \leq \mu_s R g$$

$$R = 3\text{ m}, \quad g = 9.8\text{ m s}^{-2}, \mu_s = 0.1.$$

$$\mu_s R g = 2.94\text{ m}^2\ \text{s}^{-2}$$

$$v^2 \leq 2.94\text{ m}^2\ \text{s}^{-2}$$

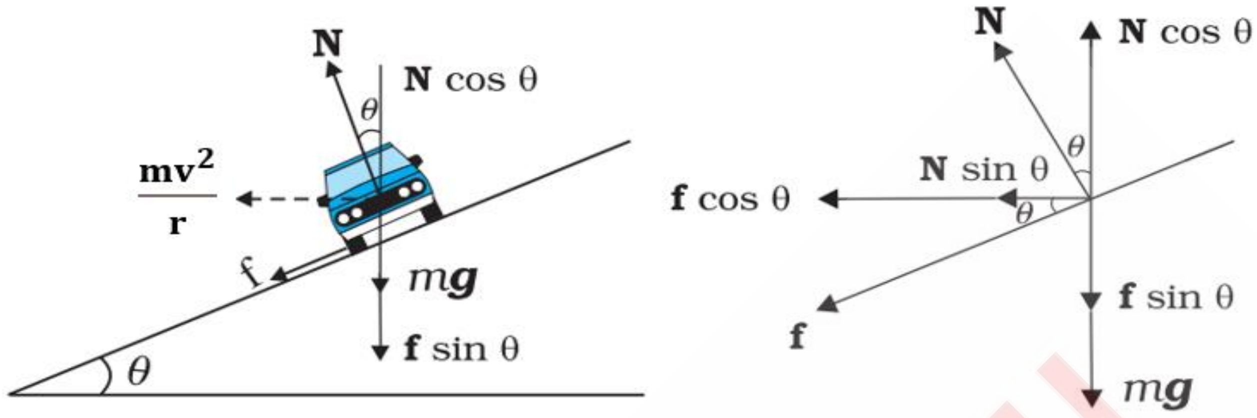
$$\text{Here } v = 18\text{ km/h} = 5\text{ m s}^{-1}$$

$$\text{i.e., } v^2 = 25\text{ m}^2\ \text{s}^{-2}$$

The condition is not obeyed. The cyclist will slip while taking the circular turn.

Motion of a car on a banked road

Raising the outer edge of a curved road above the inner edge is called banking of curved roads.



Since there is no acceleration along the vertical direction, the net force along this direction must be zero.

$$N\cos\theta = mg + f\sin\theta$$

$$N\cos\theta - f\sin\theta = mg \text{ --------------(1)}$$

The centripetal force is provided by the horizontal components of N and f_s.

$$N\sin\theta + f\cos\theta = \frac{mv^2}{R} \text{ -------------(2)}$$

$$\frac{Eqn(1)}{Eqn(2)} \text{ ----- } \frac{N\cos\theta - f\sin\theta}{N\sin\theta + f\cos\theta} = \frac{mg}{\frac{mv^2}{R}}$$

Dividing throughout by N cos θ

$$\frac{1 - \frac{f}{N}\tan\theta}{\tan\theta + \frac{f}{N}} = \frac{Rg}{v^2}$$

But , $\frac{f}{N} = \mu_s$ for maximum speed

$$\frac{1 - \mu_s\tan\theta}{\tan\theta + \mu_s} = \frac{Rg}{v^2}$$

$$v^2 = \frac{Rg(\mu_s + \tan\theta)}{1 - \mu_s\tan\theta}$$

$$v_{max} = \sqrt{\frac{Rg(\mu_s + \tan\theta)}{1 - \mu_s\tan\theta}}$$

This is the maximum safe speed of a vehicle on a banked Curved road.

If friction is absent, $\mu_s = 0$

Then Optimum speed, $v_{optimum} = \sqrt{Rg\tan\theta}$

Example

A circular racetrack of ra dius 300 m is banked at an angle of 15°. If the coefficient of friction between the wheels of a race-car and the road is 0.2, what is the

(a) optimum speed of the racecar to avoid wear and tear on its tyres, and

(b) maximum permissible speed to avoid slipping ?

$r = 300$ m ; $\theta = 15^o$; $\mu_s = 0.2$

So the optimum speed becomes,

$$v_o = \sqrt{rg \tan\theta}$$

$$\therefore v_o = \sqrt{300 \times 9.8 \times \tan 15}$$

$$\therefore v_o = \sqrt{300 \times 9.8 \times 0.2679} = 28.1\, m/s$$

Maximum permissible speed on banked road is,

$$v_{max} = \sqrt{\frac{rg(\tan\theta + \mu_s)}{1 - \mu_s \tan\theta}}$$

$$\therefore v_{max} = \sqrt{\frac{300 \times 9.8 \times (\tan 15 + 0.2)}{(1 - 0.2 \times \tan 15)}}$$

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$$v_{max} = \sqrt{\frac{300 \times 9.8 \times (0.2679 + 0.2)}{(1 - 0.2 \times 0.2679)}} = 38.1\, m/s$$