Chapter 1
Units and Measurement

1.1 Introduction

Measurement of any physical quantity involves comparison with a certain basic, internationally accepted reference standard called unit.
The result of a measurement of a physical quantity is expressed by a number accompanied by a unit.

Fundamental and Derived Quantities.

The quantities ,which can be measured directly or indirectly are called physical quantities. There are two types of physical quantities-Fundamental quantities(Base quantities) and Derived quantities.

- **The physical quantities, which are independent of each other and cannot be expressed in terms of other physical qualities are called fundamental quantities.**
  Eg: length, mass, time.
- **The physical quantities , which can be expressed in terms of fundamental quantities are called derived quantities.**
  Eg: volume, velocity, force

Fundamental and Derived Units

- The units for the fundamental or base quantities are called fundamental or base units. The units of all other physical quantities can be expressed as combinations of the base units.
- The units of the derived quantities are called derived units.

1.2 The International System of Units

A complete set of both the base and derived units, is known as the system of units. Three such systems, the CGS, the FPS (or British) system and the MKS system were in use extensively till recently.
The base units for length, mass and time in these systems were as follows :

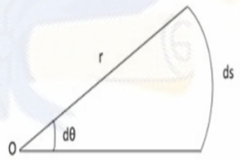
- CGS system - centimetre, gram and second.
- FPS system - foot, pound and second.
- MKS system - metre, kilogram and second.

- In 1971 the General Conference on Weights and Measures developed an internationally accepted system of units for measurement with standard scheme of symbols, units and abbreviations.
- **This is the Système Internationale d' Unites (French for International System of Units), abbreviated as SI system.**
- SI system is now for international usage in scientific, technical, industrial and commercial work.

In SI system there are seven base units and two supplementary units.

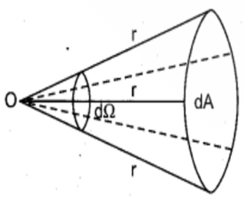
BASE QUANTITY	BASE UNIT	SYMBOL
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Thermodynamic Temperature	kelvin	K
Amount of Substance	mole	mol
Luminous Intensity	candela	cd

SUPPLEMENTARY QUANTITY	SUPPLEMENTARY UNITS	SYMBOL
Plane Angle	radian	rad
Solid Angle	steradian	sr



$$\text{Angle} = \frac{arc}{radius}$$

$$d\theta = \frac{ds}{r}$$



$$\text{Solid angle} = \frac{\text{Intercepted } Area}{Square\ of radius}$$

$$d\Omega = \frac{dA}{r^2}$$

Multiples and Sub multiples of Units

Submultiple	Prefix	Symbol	Multiple	Prefix	Symbol
10^{-1}	deci	d	10	deca	da
10^{-2}	centi	c	10^{2}	hecto	h
10^{-3}	milli	m	10^{3}	kilo	k
10^{-6}	micro	μ	10^{6}	mega	M
10^{-9}	nano	n	10^{9}	giga	G
10^{-12}	pico	p	10^{12}	tera	T
10^{-15}	femto	f	10^{15}	peta	P

1.3 Significant figures

The result of measurement is a number that includes all digits in the number that are non reliable plus the first digit that is uncertain.

The reliable digits plus the first uncertain digit in a measurement are known as significant digits or significant figures.

If the period of oscillation of a symbol pendulum is 1.6 s, the digits 1 and 6 are reliable and certain, while the digit 2 is uncertain

Rules for the determination of number of significant figures

A choice of change of different units does not change the number of significant digits or figures in a measurement.

For example, the length 2.308 cm has four significant figures.

But in different units, the same value can be written
0.02308 m - 4 significant figures.

23.08 mm - 4 significant figures.

23080 μm - 4 significant figures.

Rule1: All the non-zero digits are significant.

38 - 2

123 - 3

23.453 - 5

Rule2: All the zeros between two non-zero digits are significant, no matter where the decimal point is,

1204 - 4

30007 - 5

20.03 - 4

Rule3: If the number is less than one, the zero(s) on the right of decimal point but to the left of the first non-zero digit are not significant. The zero conventionally put on the left of decimal point for a number less than one is not significant.

0.032 - 2

0.004002 - 4

0.0132 - 3

Rule4:For a number without any decimal point the terminal or trailing zero(s) are not significant.

4200 - 2

23040 - 4

38100 - 3

Rule5: For a number with a decimal point, the trailing zero(s) are significant.

20.0 - 3

43.00 - 4

1203.0 - 5

Rule 6: The power of 10, in scientific notation is irrelevant to the determination of significant figures.

Now suppose we change units,

then 4.700 m = 470.0 cm = 4700 mm = 0.004700 km

All these measurements have four significant figures and a mere change of units cannot change the number of significant figures.

To remove such ambiguities in determining the number of significant figures, the best way is to report every measurement in scientific notation (in the power of 10).

$$4.700 \text{ m}$$
$$= 4.700 \times 10^2 \text{ cm}$$
$$= 4.700 \times 10^3 \text{ mm}$$
$$= 4.700 \times 10^{-3} \text{ km}$$

All these numbers have 4 significant namaste figures.

Rounding off the Uncertain Digits

1) If the insignificant digit to be dropped is more than 5, the preceding digit is raised by 1

A number 2.746 rounded off to three significant figures is 2.75

Here the insignificant digit , $6 > 5$ and hence 1 is added to the preceeding digit 4 .(4+1=5)

2) If the insignificant digit to be dropped less than 5, the preceding digit is left unchanged .

A number 2.743 rounded off to three significant figures is be 2.74.

Here the insignificant digit , $3 < 5$ and hence the preceeding number 4 does not change.

3) If the insignificant digit to be dropped is 5,

Case i) If the preceding digit is even, the insignificant digit is simply dropped.

A number 2.745 rounded off to three significant figures is 2.74.

Here the preceding digit 4 is even and hence 5 is simply dropped.

Case ii-) If the preceding digit is odd, the preceding digit is raised by 1.

A number 2.735 rounded off to three significant figures is 2.74

Here the preceding digit 3 ,is odd and hence 1 is added to It. (3+1=4)

Rules for Arithmetic Operations with Significant Figures

(1)In multiplication or division, the final result should retain as many significant figures as are there in the original number with the least significant figures.

Eg:If mass of an object is measured to be, 4.237 g (four significant figures) and its volume is measured to be 2.51cm3(3 significant figures), then find its density in appropriate significant figures.

$$\text{Density} = \frac{mass}{volume} = \frac{4.237\ g}{2.51 cm^3} = 1.688047$$

As per rule the final result should be rounded to 3 significant figures .

So the answer is 1.69 g/ cm^3

(2) In addition or subtraction, the final result should retain as many decimal places as are there in the number with the least decimal places.

Eg:Find the sum of the numbers 436.32 g, 227.2 g and 0.301 g to appropriate significant figures.

436.32 g +	(2 decimal places)
227.2 g +	(1 decimal place)
0.301 g	(3 decimal places)
663.821 g	

As per rule ,the final result should be rounded to 1 decimal place.

So the answer 663.8 g

1.4 Dimensions of Physical Quantities

The nature of a physical quantity is described by its dimensions. All the physical quantities represented by derived units can be expressed in terms of some combination of seven fundamental or base quantities. This base quantities are known as seven dimensions of physical world ,which are denoted with square brackets [ ]. Thus, length has the dimension [L], mass [M], time [T], electric current [A], thermodynamic temperature K], luminous intensity [cd] and amount of substance [mol].

The dimensions of a physical quantity are the powers (or exponents) to which the base quantities are raised to represent that quantity.

Eg: The volume occupied by an object= lengthx breadth x thickness

The dimensions of volume is represented as [V]

$$[V] = [L] \times [L] \times [L] = [L]^3 = [L^3].$$

$$[V] = [L^3]$$

$$[V] = [M^0 L^3 T^0]$$

Thus volume has zero dimension in mass, zero dimension in time and three dimensions in length.

1.5 Dimensional Formulae and Dimensional Equations

Dimensional Formula

The expression which shows how and which of the base quantities represent the dimensions of a physical quantity is called the dimensional formula of the given physical quantity.

The dimensional formula of volume [V] is expressed as $[M^0 L^3 T^0]$

Dimensional Equation

An equation obtained by equating a physical quantity with its dimensional formula is called the dimensional equation of the physical quantity.

$$[V] = [M^0 L^3 T^0]$$

Dimensional Formula of some derived quantities

1. Area = Length x Breadth

$[A] = [L] \times [L]$

$[A] = [L^2]$. Or $[A] = [M^0 L^2 T^0]$

Unit of area = m^2

2. Volume = Length x Breadth x Height

$[V] = [L] \times [L] \times [L]$

$[V] = [L^3]$

Unit of volume = m^3

3. Density = $\frac{Mass}{Volume}$

$[\rho] = \frac{[M]}{[L^3]}$

$[\rho] = [ML^{-3}]$.

Unit of density = kg m^{-3}

4. Frequency = $\frac{1}{Time\ period}$

$[f] = \frac{1}{[T]}$

$[f] = [T^{-1}]$

Unit of frequency = s^{-1}

5. Speed = $\frac{\text{Distance}}{\text{time}}$

$$[s] = \frac{[L]}{[T]}$$

$$[s] = [LT^{-1}]$$

Unit of speed = m s^{-1}

6. Velocity = $\frac{\text{Displacement}}{\text{time}}$

$$[v] = \frac{[L]}{[T]}$$

$$[v] = [LT^{-1}]$$

Unit of velocity = m s^{-1}

Speed and Velocity have same dimensional formula

7. Momentum = Mass x Velocity

$$[p] = [M] \times [LT^{-1}]$$

$$[p] = [MLT^{-1}]$$

Unit of momentum = kg m s^{-1}

8. Angular Momentum = momentum x Distance

$$[L] = [MLT^{-1}] \times [L]$$

$$[L] = [M L^2 T^{-1}]$$

Unit of angular momentum = kg m^2 s^{-1}

9. Acceleration = $\frac{\text{Change in velocity}}{\text{time}}$

$$[a] = \frac{[LT^{-1}]}{[T]}$$

$$[a] = [LT^{-1}] \times [T^{-1}]$$

$$[a] = [LT^{-2}]$$

Unit of acceleration = m s^{-2}

10. Force = Mass x Acceleration

$$[F] = [M] \times [LT^{-2}]$$

$$[F] = [MLT^{-2}]$$

Unit of force = kg m s^{-2} or newton(N)

$1 kgms^{-2} = 1N$

11. Impulse = Force x Time

$$[I] = [MLT^{-2}] \times [T]$$

$$[I] = [MLT^{-1}]$$

Unit of Impulse = kg m s^{-1}

12. Work = Force x Displacement

$[W] = [MLT^{-2}]$ x $[L]$

$[W] = [M L^2 T^{-2}]$

Unit of work $= kg\ m^2\ s^{-2}$ or joule (J)

$1 kgm^2 s^{-2} = 1J$

13. Energy = Workdone

$[E] = [M L^2 T^{-2}]$

Unit of energy $= kg\ m^2\ s^{-2}$ or joule (J)

14. Torque = Force x perpendicular Distance

$[\tau] = [MLT^{-2}]$ x $[L]$

$[\tau] = [M L^2 T^{-2}]$

Unit of torque $= kg\ m^2 s^{-2}$

Work, Energy and Torque have same dimensional formula

15. Pressure $= \frac{\text{Force}}{\text{Area}}$

$[P] = \frac{[MLT^{-2}]}{[L^2]}$

$[P] = [MLT^{-2}]$ x $[L^{-2}]$

$[P] = [ML^{-1}T^{-2}]$

Unit of pressure $= kg\ m^{-1} s^{-2}$ or pascal(Pa)

16. Stress $= \frac{\text{Force}}{\text{Area}}$

$[stress] = \frac{[MLT^{-2}]}{[L^2]}$

$[stress] = [ML^{-1}T^{-2}]$

Unit of stress $= kg\ m^{-1}\ s^{-2}$

Pressure and Stress have same dimensional formula

17.Power $= \frac{\text{Work}}{\text{Time}}$

$[P] = \frac{[M L^2 T^{-2}]}{[T]}$

$[P] = [M L^2 T^{-2}]$x $[T^{-1}]$

$[P] = [M L^2 T^{-3}]$

Unit of power$= kg\ m^2\ s^{-3}$ or watt(W)

Physical quantities having no dimension and no unit

$$\text{Strain} = \frac{\textit{Change in dimension}}{\textit{Original dimension}} = \frac{[L]}{[L]} = [L^0]$$

$$\text{Relative Density} = \frac{\textit{Density of substance}}{\textit{Density of water}} = \frac{[ML^{-3}]}{[ML^{-3}]} = [L^0]$$

Physical quantities having units, but no dimension

Plane angle
Solid Angle
Angular Displacement

Find the dimensional formula of Gravitational constant using equation $F = \frac{Gm_1m_2}{r^2}$

$$G = \frac{Fr^2}{m_1m_2}$$

$$[G] = \frac{[F][r^2]}{[m_1]\,[m_2]}$$

$$[G] = \frac{[MLT^{-2}][L^2]}{[M][M]}$$

$$= [MLT^{-2}]\,[L^2][M^{-1}][M^{-1}]$$

$$[G] = [M^{-1}L^3T^{-2}]$$

Find the dimensional formula of Planck's constant using equation $\lambda = \frac{h}{mv}$

$$h = \lambda mv$$

$$[h] = [\lambda][m][v]$$

$$[h] = [L][M]\,[LT^{-1}]$$

$$[h] = [ML^2T^{-1}]$$

Find the dimensional formula of Coefficient of Viscosity using equation $F = \eta A\left(\frac{dv}{dx}\right)$

$$\eta = \frac{F}{A\left(\frac{dv}{dx}\right)}$$

$$\eta = \frac{F dx}{A\ dv}$$

$$[\eta] = \frac{[MLT^{-2}]\ x\ [L]}{[L^2]\ [L\ T^{-1}]}$$

$$[\eta] = [MLT^{-2}] \times [L] \times [L^{-2}] \times [L^{-1}] \times [T]$$

$$[\eta] = [ML^{-1}T^{-1}]$$

1.6 Dimensional Analysis and its Applications

1. Checking the dimensional consistency (correctness) of equatons.
2. Deducing relation among the physical quantities.

1) Checking the Dimensional Consistency(correctness) of Equations

The principle called the principle of homogeneity of dimensions is used to check the dimensional correctness of an equation.

The principle of homogeneity states that, for an equation to be correct, the dimesions of each terms on both sides of the equation must be the same.

The magnitudes of physical quantities may be added or subtracted only if they have the same dimensions.

If $X + Y = Z$

$[X] = [Y] = [Z]$

We cannot add 5m and 10kg

Velocity cannot be added to force.

eg1) Check the dimensional consistency(correctness) or homogeneity of the equation

$s = ut + \frac{1}{2}at$

s= displacement
u =initial velocity
a=acceleration
t=time

$[s] = L$

$[ut] = LT^{-1} \times T$

$= L$

$[\frac{1}{2}at] = LT^{-2} \times T$

$= LT^{-1}$

Since the dimensions of all terms of the equation are not same, this equation is wrong.

2) Check the dimensional consistency(correctness) or homogeneity of the equation

$s = ut + \frac{1}{2}at^2$

s= displacement
u =initial velocity
a=acceleration
t=time

$[s] = L$

$[ut] = LT^{-1} \times T$

$= L$

$[\frac{1}{2}at^2] = LT^{-2} \times T^2$

$= L$

Since each term on both sides of equation has the same dimension, this equation is dimensionally correct

If an equation passes this consistency test it is not proved right.

3) Check the dimensional consistency(correctness) or homogeneity of the equation

$s = ut + \frac{2}{3}at^2$

s= displacement
u =initial velocity
a=acceleration
t=time

$[s] = L$

$[ut] = LT^{-1} \times T$

$= L$

$[\frac{2}{3}at^2] = LT^{-2} \times T^2$

$= L$

Since each term on both sides of equation has the same dimension, this equation is dimensionally correct.

Eventhough this equation is is dimensionally correct,it is not an exact equation.

A dimensionally correct equation need not be an exact (correct) equation, but a dimensionally wrong (incorrect) must be wrong.

4) Check the dimensional consistency(correctness) or homogeneity of the equation

$$\frac{1}{2}mv^2 = mgh$$

m=mas of th body
v= velocity of body
g=acceleration due to gravity
h=height

$$[\tfrac{1}{2}mv^2] = M\,[LT^{-1}]^2$$
$$= ML^2T^{-2}$$

$$[mgh] = M\ LT^{-2}\ L$$
$$= ML^2T^{-2}$$

The dimensions of LHS and RHS are the same and hence the equation is dimensionally correct.

5) Check the dimensional correctness of the equation $E = m\,c^2$

E=energy
m=mass
c=velocity of light

$$[E] = ML^2T^{-2}$$

$$[m\,c^2] = M\,[LT^{-1}]^2$$
$$= ML^2T^{-2}$$

The dimensions of LHS and RHS are the same and hence the equation is dimensionally correct.

6) In the given equation v=x + at , find the dimensions of x.

(where v = velocity, a= acceleration, t = time)

$v = x + at$

$[v] = [x] = [at]$

$[x] = [v]$

$[x] = LT^{-1}$

7) In the given equation $x = a + bt + ct^2$, find the dimensions of a,b and c.

(where x is in metres and t in seconds)

$x = a + bt + ct^2$

$[x] = [a] = [bt] = [ct^2]$

$[a] = [x]$

$[a] = L$

$[bt] = [x]$

$[b] \times T = L$

$[b] = \frac{L}{T}$

$[b] = LT^{-1}$

$[ct^2] = [x]$

$[c] \times T^2 = L$

$[c] = \frac{L}{T^2}$

$[c] = LT^{-2}$

8) The SI unit of energy is $J = kg\, m^2\, s^{-2}$; that of speed v is ms^{-1} and of acceleration a is ms^{-2}
Using dimensional arguments, find which of the formulae can be considered for kinetic energy (K)

(a) $K = m^2 v^3$

(b) $K = \frac{1}{2}mv^2$

(c) $K = ma$

(d) $K = \frac{3}{16}mv^2$

(e) $K = \frac{1}{2}mv^2 + ma$

(a) $K = m^2 v^3$

$$ML^2T^{-2} = [M]^2 [LT^{-1}]^3$$
$$\neq M^2 L^3 T^{-3}$$

(b) $K = \frac{1}{2}mv^2$

$$ML^2T^{-2} = [M] [LT^{-1}]^2$$
$$= ML^2T^{-2}$$

(c) $K = ma$

$$ML^2T^{-2} = M [LT^{-2}]$$
$$\neq MLT^{-2}$$

(d) $K = \frac{3}{16}mv^2$

$$ML^2T^{-2} = [M] [LT^{-1}]^2$$
$$= ML^2T^{-2}$$

(e) $K = \frac{1}{2}mv^2 + ma$

$$[ML^2T^{-2}] = [M][LT^{-1}]^2 + [M][LT^{-2}]$$
$$\neq ML^2T^{-2} + MLT^{-2}$$

Since dimensions of all terms are the same for Equations (b) and (d) , these equations can be considered as the equation for kinetic energy.

9.The Van der waals equation of 'n' moles of a real gas is $(P+\frac{a}{V^2})(V-b)=nRT$. Where P is the pressure, V is the volume, T is absolute temperature, R is molar gas constant and a, b, c are Van der waal constants. Find the dimensional formula for a and b.

$$(P+\frac{a}{V^2})(V-b)=nRT.$$

By principle of homegeneity, the quantities with same dimensions can be added or subtracted.

$$[P] = [\frac{a}{V^2}]$$
$$[a] = [PV^2]$$
$$= ML^{-1}T^{-2} \times L^6$$
$$[a] = ML^5T^{-2}$$

$$[b] = [V]$$
$$[b] = L^3$$

2) Deducing Relation among the Physical Quantities

We can deduce relation of a physical quantity which depends upto three physical quantities.

1) Derive the equation for kinetic energy(E) of a body of mass m moving with velocity v.

$$E \alpha \ m^x \ v^y$$

$$E = k\, m^x v^y \longrightarrow (1)$$

Writing the dimensions on both sides,

$$M L^2 T^{-2} = M^x \left(LT^{-1}\right)^y$$

$$M^1 L^2 \ T^{-2} = M^x \ L^y \ T^{-y}$$

equating the dimensions on both sides,

$$x = 1$$
$$y = 2$$

Substituting in eq (1)

$$E = k\, m^1 v^2$$

$$\boxed{E = k\, mv^2}$$

2) Suppose that the period of oscillation of the simple pendulum depends on its mass of the bob (m), length (l) and acceleration due to gravity (g). Derive the expression for its time period using method of dimensions.

$$T \propto m^x \ l^y \ g^z$$

$$T = k\, m^x l^y g^z \longrightarrow (1)$$

Writing the dimensions on both sides,

$$M^0 L^0 T^1 = M^x \ L^y \ \left(LT^{-2}\right)^z$$

$$M^0 L^0 T^1 = M^x \ L^y \ L^z \ T^{-2z}$$

$$M^0 L^0 T^1 = M^x \ L^{y+z} \ T^{-2z}$$

equating the dimensions on both sides,

$$x = 0$$

$$y + z = 0$$

$$-2z = 1 \qquad z = \frac{-1}{2}$$

$$y + \frac{-1}{2} = 0 \qquad y = \frac{1}{2}$$

$$\boxed{T = k m^0 \, l^{1/2} g^{-1/2}}$$

$$T = k \frac{l^{1/2}}{g^{1/2}}$$

$$T = k \frac{\sqrt{l}}{\sqrt{g}}$$

$$\boxed{T = k \sqrt{\frac{l}{g}}}$$

Limitations of Dimensional Analysis

1) Dimensional analysis check only the dimensional correctness of an equation, but not the exact correctness.
2) The dimensionless constants cannot be obtained by this method.
3) We cannot deduce a relation, if a physical quantity depends on more than three physical quantities.
4) The method cannot be considered to derive equations involving more than one term
5) A formula containing trigonometric , exponential and logarithmic function can not be derived from it.
6) It does not distinguish between the physical quantities having same dimensions.