Miscellaneous

Q1. Decide, among the following sets, which sets are subsets of one and another:
A = { x: $x \in$ R and x satisfy $x^2 - 8x + 12 = 0$ },
B = { 2, 4, 6 }, C = { 2, 4, 6, 8, . . . }, D = { 6 }.

A.1. A = {x: $x \in$R and x satisfy $x^2 - 8x + 12 = 0$}

So, $x^2 - 8x + 12 = 0$

$\Rightarrow x^2 - 6x - 2x + 12 = 0$

$\Rightarrow x(x-6) - 2(x-6) = 0$

$\Rightarrow (x-6)(x-2) = 0$

$\Rightarrow x = 6, 2.$

So, A = {2, 6)   B = {2, 4, 6}   C = {2, 4, 6, 8, ….}

D = {6}

$\therefore$ D $\subset$ A $\subset$ B $\subset$ C

i e, D $\subset$ A, D $\subset$ B, D $\subset$ C, A $\subset$ B, A $\subset$ C and B $\subset$ C.

Q2. In each of the following, determine whether the statement is true or false. If it is true, prove it. If it is false, give an example.

(i) If $x \in$ A and A $\in$ B, then $x \in$ B
(ii) If A $\subset$ B and B $\in$ C, then A $\in$ C
(iii) If A $\subset$ B and B $\subset$ C , then A $\subset$ C
(iv) If A $\not\subset$ B and B $\not\subset$ C, then A $\not\subset$ C
(v) If $x \in$ A and A $\not\subset$ B , then $x \in$ B
(vi) If A $\subset$ B and $x \in$ B , then $x \notin$ A

A.2. (i) False Let A = {a}, $a \in$ A then B = {{a}, b} i e, $a \notin$B.

(ii) False. Let A. = {a}, *if* A $\subset$ B, B = {a, b} and B $\in$ C i e, C = {{a, b}, c} i e, A = {a}. $\notin$C.

(iii) True. Let $x \in$A, if A $\subset$ B then $x \in$B and if B $\subset$ C, $x \in$ C i e, elements of A are also elements of C.

$\therefore$ A $\subset$ C.

(iv) False. Let A = {a} and B = {b} then A$\not\subset$B . Let C = {a, c} then B$\not\subset$$\subset$ but a $\in$A and $a \in$ c, i.e., A $\subset$ C.

(v) False. Let A = {a} and B = {b} so, A$\not\subset$B i.e., $a \notin$B .

(vi) True. Let A $\subset$ B such that $y \in$ B i.e., $y \in$ A But $x \notin$B and suppose $x \in$A.

Then by above definition,

A $\subset$ B i.e., $x \in$ B and $x \in$ A which is not the case

Q3. Let A, B, and C be the sets such that A $\cup$ B = A $\cup$ C and A $\cap$ B = A $\cap$ C. Showthat B = C.

A.3. Let $x \in b$

As B $\subset$ A $\cup$ B we can write

Let $x \in$ A $\cup$ B.

as A $\cup$ B = A $\cup$ C.

$x \in$ A $\cup$ C.

i.e., $x \in$ A or $x \in$ C

when $x \in$ A, and $x \in$ B,

$x \in$ A $\cap$ B

But $A \cap B = A \cap C$
So, $x \in A \cap C$
i.e., $x \in A$ and $x \in C$
$x \in C$
when, $x \in C$
as $x \in B$ and $x \in C$
So, $B \subset C$
Similarly, $C \subset B$
So, $B = C$

Q4. Show that the following four conditions are equivalent :
(i) $A \subset B$ (ii) $A - B = \phi$ (iii) $A \cup B = B$ (iv) $A \cap B = A$

A.4. (i) Let $A - B \neq \phi$, our assumption.
i.e., $x \in A$ But $x \neq B$ where x is an element.
But as $A \subset B$, the above condition of assumption is wrong $\therefore$ if $A \subset B$ then $A - B = \phi$

(ii) Let $x \in A$.
As $A - B = \phi$ we can say that $x \in B$ because if $x \notin B$, $A - B \neq \phi$
$\therefore$ *if* $A - B = \phi$ then $A \subset B$.

(iii) We know that,
$B \subset A \cup B$ always true
Let $x \in A \cup B$ i.e., $x \in A$ or $x \in B$.
As $A \subset B$,
If $x \in A$ then $x \in B$, all elements of A are among the elements of B
So, $(A \cup B) = B$

(iv) We know that,
$(A \cap B) \subset A$ as $A \subset B$.
Let $x \in A$ then $x \in B$
So, $x \in (A \cap B)$
i.e., $A \subset (A \cap B)$
So, $A = (A \cap B)$
Hence, $A \subset B$
$\Rightarrow A - B = \phi$.
$\Rightarrow A \cup B = B$
$\Rightarrow A \cap B = A$.
i.e., the 4 conditions are equivalent.

Q5. Show that if $A \subset B$, then $C - B \subset C - A$.

A.5. Given, $A \subset B$.
Let $x \in C - B$ then $x \in C$ but $x \notin B$.
However, $A \subset B$, elements of B should have elements of A

i.e., $x \notin B \Rightarrow x \notin A$

So, $x \in C - A$ i.e., $x \in C$ but $x \notin A$

$\therefore C - B \subset C - A$

Q6. Assume that P (A) = P (B). Show that A = B.

A.6. Given, P(A) = P(B) where P is power set

Let $x \in A$.

Then, $\{x\} \subset P(A) \subset P(B)$

i.e., $x \in B$

$\therefore A \subset B$

Similarly, $B \subset A$

$\therefore A = B$

Q7. Is it true that for any sets A and B, P (A) $\cup$ P(B) = P (A $\cup$ B) ? Justify your answer.

A.7. Let $A = \{a\}$, $B = \{b\}$.

So, $P(A) = \{\phi, \{a\}\}$ and $B(A) = \{\phi, (b\}\}$.

So, $P(A) \cup P\{B\} = \{\phi, \{a\}, \{b\}\}$ ______(1)

Now, $A \cup B = \{a, b\}$.

$P(A \cup B) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$ ____(2)

So. From (1) and (2) we see that,

$P(A) \cup P(B) \neq P(A \cup B)$

Q8. Show that for any sets A and B,
A = (A $\cap$ B) $\cup$ (A – B) and A $\cup$ (B – A) = (A $\cup$ B)

A.8. Here,

$(A \cap B) \cup (A - B) = (A \cap B) \cup (A \cap B')$ as $(A - B) = A \cap B'$

$= A \cap (B \cup B')$ [$\therefore$ by converse of distributive law]

$= A \cap U$ [$\therefore$ $B \cup B' = U$, sample space set or universal set]

$= A$

And $(A \cup (B - A)) = A \cup (B \cap A')$ [as $B - A = B \cap A'$]

$= (A \cup B) \cap (A \cup A')$

$= (A \cup B) \cap U$ [$\therefore$ $A \cup A' = U$, universal set]

$= A \cup B$.

Q9. Using properties of sets, show that
(i) A $\cup$ (A $\cap$ B) = A (ii) A $\cap$ (A $\cup$ B) = A

A.9.

(i) We know that,

$A \subset A$

$(A \cap B) \subset A$

$[A \cup (A \cap B)] \subset (A \cup A)$

$[A \cup (A \cap B)] \subset A$

and also

$A \subset [A \cup (A \cap B)]$

So, $A \cup (A \cap B) = A$.

(ii) $A \cap (A \cup B) = (A \cap A) \cup (A \cap B)$ [By distributive law]

$= A \cup (A \cap B)$

$= A$ as $(A \cap B) \subset A$

Q10. Show that $A \cap B = A \cap C$ need not imply $B = C$.

A.10. Let $A = \{a\}$, $B = \{a, b\}$, $C = \{a, c\}$

So, $A \cap B = \{a\} \cap \{a, b\} = \{a\}$

$A \cap C = \{a\} \cap \{a, c\} = \{a\}$

i.e., $A \cap B = A \cap C = \{a\}$

But $B \neq C$. as $b \in B$ but $b \notin C$ vice-versa

Q11. Let A and B be sets. If $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X, show that A = B.

(Hints $A = A \cap (A \cup X)$, $B = B \cap (B \cup X)$ and use Distributive law)

A.11. Let A, B and x be sets such that,

$A \cap x = B \cap x = \phi$ and $A \cup x = B \cup x$.

We know that,

$A = A \cap (A \cup x)$

$= A \cap (B \cup x)$ [$\because A \cup x = B \cup x$]

$= (A \cap B) \cup (A \cap x)$ [by distributive law]

$= (A \cap B) \cup \Phi$ [$\because A \cap x = \Phi$]

$\Rightarrow A = A \cap B$ [$\because A \cup \Phi = A$]

And $B = \cap (B \cup x)$

$= B \cap (A \cup x)$ [$\because B \cup x = A \cup x$]

$= (B \cap A) \cup (B \cap x)$ [By distributive law]

$= (B \cap A) \cup \phi$ [$\because B \cap x = \Phi$]

$B = B \cap A$ [$\because A \cup \phi = A$]

So, $A = B = A \cap B$.

Q12. Find sets A, B and C such that $A \cap B$, $B \cap C$ and $A \cap C$ are non-empty sets and $A \cap B \cap C = \phi$

A.12. Let $A = \{x, y\}$

$B = \{y, z\}$

$C = \{x, z\}$

So, $A \cap B = \{x, y\} \cap \{y, z\} = \{y\} \neq \phi$

$B \cap C = \{y, z\} \cap \{x, z\} = \{z\} \neq \phi$

$A \cap C = \{x, y\} \cap \{x, z\} = \{x\} \neq \phi$

But $A \cap B \cap C = (A \cap B) \cap C$

$= \{y\} \cap (x, z\}$

$= \phi$

Q13. In a survey of 600 students in a school, 150 students were found to be taking teaand 225 taking coffee, 100 were taking both tea and coffee. Find how manystudents were taking neither tea nor coffee?

A.13. Let T and C be sets of students taking tea and coffee.

Then, $n(T) = 150$, number of students taking tea

$n(C) = 225$, number of students taking coffee

$n(T \cap C) = 100$, number of students taking both tea and coffee.

So, Number of students taking either tea or coffee is.

$n(T \cup C) = n(T) + n(C) - n(T \cap C)$

$= 150 + 225 - 100$

$= 275$

$\therefore$ Number of students taking neither tea coffee

= Total number of students – No of students taking either tea or coffee

$= 600 - 275$

$= 325.$

Q14. In a group of students, 100 students know Hindi, 50 know English and 25 knowboth. Each of the students knows either Hindi or English. How many studentsare there in the group?

A.14. Let H and E be set of students who known Hindi and English respectively.

Then, number of students who know Hindi = $n(H) = 100$

Number of students who know English = $n(E) = 50$

Number of students who know both English & Hindi = $25 = n(H \cap E)$

As each of students knows either Hindi or English,

Total number of students in the group,

$n(H \cup E) = n(H) + n(E) - n(H \cap E)$

$= 100 + 25 - 25$

$= 125,$

Q15. In a survey of 60 people, it was found that 25 people read newspaper H, 26 readnewspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T,8 read both T and I, 3 read all three newspapers. Find:

(i) the number of people who read at least one of the newspapers.

(ii) the number of people who read exactly one newspaper.

A.15. Let H,T and I be of people who reads newspaper H,T and I respectively.

Then,

number of people who reads newspaper H, $n(H) = 25.$

number of people who T, $n(T) = 26.$

number of people who I , $n(I) = 26$

number of people who both H and T, $n(H \cap I) = 9$

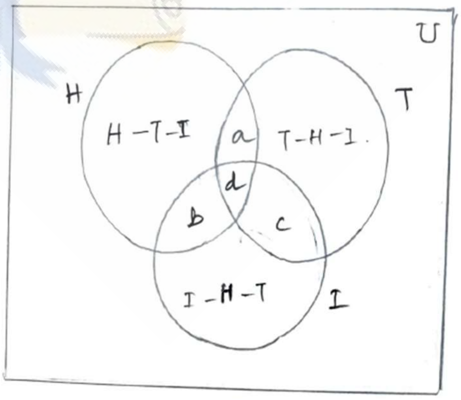
number of people who both H and T, $n(H \cap T) = 11$

number of people who both T and I, $n(T \cap I) = 8$

number of people who reads all newspaper, $n(H \cap T \cap I) = 3.$

Total no. of people surveyed = 60

The given sets can be represented by venn diagram



(i) The number of people who reads at least one of the newspaper.

in $(H\cup T\cup I) = n(H) + n(T) + n(I) - n(H\cap T) - n(H\cap I) - n(T\cap I) + n(H\cap T\cap I)$

$= 25 + 26 + 26 - 11 - 9 - 8 + 3$

$= 80 - 28$

$= 52$

(ii) From venn diagram,

a = number of people who reads newspapers H and T only.

b = number of people who reads newspapers H and I only.

c = number of people who reads newspapers T and I only.

d = number of people who reads all newspaper.

So, $n(H\cap T) = a + d$.

$n(H\cap I) = b + d$

$n(T\cap I) = c + d$

So, $a + d + c + d + b + d = n(H\cap T) + n(H\cap I) + n(T\cap I)$

$\Rightarrow a + d + c + b + 2d = 11 + 9 + 8$

$\Rightarrow a + b + c + d = 28 - 2d$

$= 28 - 2 \times 3 \qquad [\therefore d = n(H\cap T\cap I) = 3]$

$= 28 - 6$

$= 22$

$\therefore$ Number of people who reads exactly one newspaper

= Total no. of people – No. of people who reads more than one newspaper

$= 52 - (a + b + c + d)$

$= 52 - 22$

$= 30$

Q16. In a survey it was found that 21 people liked product A, 26 liked product B and29 liked product C. If 14 people liked products A and B, 12 people liked productsC and A, 14 people liked products B and C and 8 liked all the three products.Find how many liked product C only.

A.16. Let A, B and C be the set of people who like product A, B and C respectively.

Then,

Number of people who like product A, $n(A) = 21$

Number of people who like product B, $n(B) = 26$

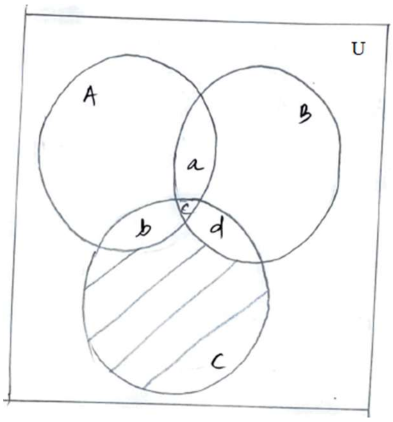
Number of people who like product C, $n(C) = 29$.

Number of people likes both product A and B, $n(A\cap B) = 14$

Number of people likes both product A and C, $n(A\cap C) = 12$

Number of people likes both product B and C, $n(B\cap C) = 14$.

No. of people who likes all product, $n(A\cap B\cap C) = 8$



$a \rightarrow n(A \cap B)$

$b \rightarrow n(A \cap C)$

$d \rightarrow n(B \cap C)$

$c \rightarrow n(A \cap B \cap C)$

From the above venn diagram we can see that,

Number of people who likes product C only

$= n(C) - b - d + c$

$= n(C) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

$= 29 - 12 - 14 + 8$

$= 11$