MISCELLANEOUS EXERCISE

Question 1:

A box contains 10 red marbles, 20 blue marbles and 30 green marbles. 5 marbles are drawn from the box, what is the probability that

(i) all will be blue? (ii) atleast one will be green?

Solution:

Total number of marbles $= 10 + 20 + 30 = 60$

Therefore, number of ways of drawing 5 marbles from 60 marbles $= {}^{60}C_5$

(i) All the drawn marbles will be blue if we draw 5 marbles out of 20 blue marbles.

5 blue marbles can be drawn from 20 blue marbles in ${}^{20}C_5$ ways.

Therefore,

Probability that all marbles will be blue $= \dfrac{{}^{20}C_5}{{}^{60}C_5}$

(ii) Number of ways in which the drawn marbles is not green $= {}^{(20+10)}C_5 = {}^{30}C_5$

Hence, Probability that no marble is green $= \dfrac{{}^{30}C_5}{{}^{60}C_5}$

Therefore,

Probability that at least one marble is green $= 1 - \dfrac{{}^{30}C_5}{{}^{60}C_5}$

Question 2:

4 cards are drawn from a well-shuffled deck of 52 cards. What is the probability of obtaining 3 diamonds and one spade?

Solution:

Number of ways of drawing 4 cards from 52 cards $= {}^{52}C_4$

We know that; in a deck of 52 cards, there are 13 diamonds and 13 spades.

Therefore, number of ways of drawing 3 diamonds and one spade $= {}^{13}C_3 \times {}^{13}C_1$

Thus, the probability of obtaining 3 diamonds and one spade $= \dfrac{{}^{13}C_3 \times {}^{13}C_1}{{}^{52}C_4}$

Question 3:

A die has two faces each with number '1', three faces each with number '2' and one face with number '3'. If die is rolled once, determine

(i) $P(2)$ (ii) $P(1\ or\ 3)$ (iii) $P(not\ 3)$

Solution:

Total number of faces $=6$

(i) Number of faces with number $'2'=3$

Therefore,

$$P(2)=\frac{3}{6}$$
$$=\frac{1}{2}$$

(ii) Since,

$$P(1\ or\ 3)=P(not\ 2)$$
$$=1-P(2)$$
$$=1-\frac{1}{2}$$
$$=\frac{1}{2}$$

(iii) Number of faces with number $'3'=1$

Therefore,

$$P(3)=\frac{1}{6}$$

Thus,

$$P(not\ 3)=1-P(3)$$
$$=1-\frac{1}{6}$$
$$=\frac{5}{6}$$

Question 4:

In a certain lottery $10,000$ tickets are sold, and ten equal prizes are awarded. What is the probability of not getting a prize if you buy

(a) one ticket (b) two tickets? (c) 10 tickets?

Solution:

(a) If we by one ticket, then

Total number of tickets sold $=10000$

Number of prizes awarded $=10$

$$P(getting\ a\ prize)=\frac{10}{10000}$$
$$=\frac{1}{1000}$$

Therefore,

$$P(not\ getting\ a\ prize)=1-\frac{1}{1000}$$
$$=\frac{999}{1000}$$

(b) If we buy two tickets, then

Number of tickets not awarded $=10000-10=9990$

$$P(not\ getting\ a\ prize)=\frac{{}^{9990}C_2}{{}^{10000}C_2}$$

(c) If we buy ten tickets, then

$$P(not\ getting\ a\ prize)=\frac{{}^{9990}C_{10}}{{}^{10000}C_{10}}$$

Question 5:

Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among the 100 students, what is the probability that

(a) you both enter the same section?

(b) you both enter the different sections?

Solution:

My friend and I are among the 100 students.

Total number of ways of selecting 2 students out of 100 students $= {}^{100}C_2$

(a) The two of us will enter the same section if both of us are among 40 students or among 60 students.

Therefore,

Number of ways in which we both enter the same section $= {}^{40}C_2 + {}^{60}C_2$

Hence,

The probability that both of us enter the same section, $P(A)=\frac{{}^{40}C_2+{}^{60}C_2}{{}^{100}C_2}$

$$\frac{{}^{40}C_2 + {}^{60}C_2}{{}^{100}C_2} = \frac{\frac{(40)!}{(2)!(40-2)!} + \frac{(60)!}{(2)!(60-2)!}}{\frac{(100)!}{(2)!(100-2)!}}$$

$$= \frac{\frac{40 \times 39 \times (38)!}{2 \times (38)!} + \frac{60 \times 59 \times (58)!}{2 \times (58)!}}{\frac{100 \times 99 \times (98)!}{2 \times (98)!}}$$

$$= \frac{(20 \times 39) + (30 \times 59)}{50 \times 99}$$

$$= \frac{780 + 1770}{4950}$$

$$= \frac{2550}{4950}$$

$$= \frac{17}{33}$$

(b) Probability that both of us enter the different sections, $P(B) = 1 - P(A)$

$$P(B) = 1 - \frac{17}{33}$$

$$= \frac{16}{33}$$

Question 6:

Three letters are dictated to three persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each employee contains exactly one letter. Find the probability that at least one letter is in its proper envelope.

Solution:

Let L_1, L_2, L_3 be three letters and E_1, E_2, E_3 be their corresponding envelops respectively.
There are 6 ways of inserting 3 letters in envelops as follows:

L_1E_1, L_2E_3, L_3E_2

L_2E_2, L_1E_3, L_3E_1

L_3E_3, L_1E_2, L_2E_1

L_1E_1, L_2E_2, L_3E_3

L_1E_2, L_2E_3, L_3E_1

L_1E_3, L_2E_1, L_3E_2

We can see that there are 4 ways in which at least one letter is inserted in a proper envelope.

Thus, the required probability $=\frac{4}{6}=\frac{2}{3}$

Question 7:

A and B are two events such that $P(A)=0.54,\ P(B)=0.69$ and $P(A\cap B)=0.35$. Find

(i) $P(A\cup B)$ (ii) $P(A'\cap B')$ (iii) $P(A\cap B')$ (iv) $P(B\cap A')$

Solution:

It is given that $P(A)=0.54,\ P(B)=0.69$ and $P(A\cap B)=0.35$

(i) We know that

$$\begin{aligned}P(A\cup B)&=P(A)+P(B)-P(A\cap B)\\&=0.54+0.69-0.35\\&=0.88\end{aligned}$$

(ii) By De Morgan's law

$$(A'\cap B')=(A\cup B)'$$

Therefore,

$$\begin{aligned}P(A'\cap B')&=P(A\cup B)'\\&=1-P(A\cup B)\\&=1-0.88\\&=0.12\end{aligned}$$

(iii) We know that

$$\begin{aligned}P(B\cap A')&=P(A)-P(A\cap B)\\&=0.54-0.35\\&=0.19\end{aligned}$$

(iv) We know that

$$n(B\cap A')=n(B)-n(A\cap B)$$

Therefore,

$$\begin{aligned}P(B\cap A')&=P(B)-P(A\cap B)\\&=0.69-0.35\\&=0.34\end{aligned}$$

Question 8:

From the employees of a company, 5 persons are selected to represent them in the managing committee of the company. Particulars of five persons are as follows:

S. No.	Name	Sex	Age in years
1.	Harish	M	30
2.	Rohan	M	33
3.	Sheetal	F	46
4.	Alis	F	28
5.	Salim	M	41

A person is selected at random from this group to act as a spokesperson. What is the probability that the spokesperson will be either male or over 35 years?

Solution:

Let A be the event in which the spokesperson will be a male

Then, $P(A)=\frac{3}{5}$ and $P(F)=\frac{2}{5}$

B be the event in which the spokesperson will be over 35 years of age

Then, $P(B)=\frac{2}{5}$

Since there is only one male who is over 35 years of age,

Therefore, $P(A\cap B)=\frac{1}{5}$

We know that

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$
$$=\frac{3}{5}+\frac{2}{5}-\frac{1}{5}$$
$$=\frac{4}{5}$$

Thus, the probability that the spokesperson will either be a male or over 35 years of age is $\frac{4}{5}$.

Question 9:

If 4-digits numbers greater than 5000 are randomly formed from the digits 0, 1, 3, 5 and 7, what is the probability of forming a number divisible by 5 when,

(i) the digits are repeated? (ii) the repetition of digits is not allowed?

Solution:

(i) When the digits are repeated

Since 4-digit numbers greater than 5000 are randomly formed from the digits 0, 1, 3, 5 and 7, the leftmost digit is either 7 or 5.

The remaining three places can be filled by any of the digits 0, 1, 3, 5 and 7, as repetition of digits is allowed.

Therefore,

Total number of such 4-digit numbers greater than 5000 will be equal to

$$2\times5\times5\times5-1=250-1=249$$

[Since, 5000 cannot be counted; so, 1 is subtracted]

A number is divisible by 5, if the digit at its units place is either 0 or 5.

Total number of 4-digit numbers greater than 5000 that are divisible by 5 are

$$2\times5\times5\times2-1=100-1=99$$

Thus, the probability of forming a number divisible by 5 when the digits are repeated is

$$\frac{99}{249}=\frac{33}{83}$$

(ii) When repetition of digits is not allowed

The thousands place can be filled with either of the two digits 5 or 7.

The remaining three places can be filled with any of the remaining four digits.

Hence, Total number of 4-digit numbers greater than 5000 will be

$$2\times4\times3\times2=48$$

When the digit at the thousands place is 5, the units place can be filled only with 0 and the tens and hundreds places can be filled with any two of the remaining three digits.

Therefore, number of 4-digit numbers starting with 5 and divisible by 5 are

$$3\times2=6$$

When the digit at the thousands place is 7, the units place can be filled in two ways, 0 or 5 and the tens and hundreds places can be filled with any two of the remaining three digits.

Therefore, number of 4-digit numbers starting with 7 and divisible by 5 are

$$1\times2\times3\times2=12$$

Hence,

Total number of 4-digit numbers greater than 50000 that are divisible by 5 are

$$6+12=18$$

Thus, the probability of forming a number divisible by 5 when the repetition of digits is not allowed is $\frac{18}{48}=\frac{3}{8}$.

Question 10:

The number lock of a suitcase has 4 wheels, each labelled with ten digits i.e., from 0 to 9. The lock opens with a sequence of four digits with no repeats. What is the probability of a person getting the right sequence to open the suitcase?

Solution:

The number lock has 4 wheels, each labelled with digits from 0 to 9.

Number of ways of selecting 4 different digits out of 10 digits $= {}^{10}C_4$

Now, each combination of 4 different digits can be arranged in $(4)!$ ways.

Hence the number sequence of four digits with no repetitions $= {}^{10}C_4 \times (4)!$

$$\begin{aligned} {}^{10}C_4 \times (4)! &= \frac{(10)!}{(4)!(10-4)!} \times (4)! \\ &= \frac{10 \times 9 \times 8 \times 7 \times (6)!}{(6)!} \\ &= 5040 \end{aligned}$$

There is only one number that can be open the suitcase.

Thus, the required probability is $\frac{1}{5040}$.