

## MISCELLANEOUS EXERCISE

### Question 1:

Three vertices of a parallelogram ABCD are  $A(3, -1, 2)$ ,  $B(1, 2, -4)$  and  $C(-1, 1, 2)$ . Find the coordinates of the fourth vertex.

### Solution:

The three vertices of a parallelogram ABCD are given as  $A(3, -1, 2)$ ,  $B(1, 2, -4)$  and  $C(-1, 1, 2)$ .  
Let the coordinates of the fourth vertex be  $D(x, y, z)$ .

We know that the diagonals of a parallelogram bisect each other.  
Therefore, in parallelogram ABCD, diagonals AC and BD bisect each other.

i.e., Mid-point of AC = Mid-point of BD

$$\Rightarrow \left( \frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2} \right) = \left( \frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2} \right)$$
$$\Rightarrow (1, 0, 2) = \left( \frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2} \right)$$

Hence,

$$\frac{x+1}{2} = 1, \frac{y+2}{2} = 0 \text{ and } \frac{z-4}{2} = 2$$

$$\Rightarrow x = 1, y = -2 \text{ and } z = 8$$

Thus, the coordinates of the fourth vertex D are  $(1, -2, 8)$ .

### Question 2:

Find the lengths of the medians of the triangle with vertices  $A(0, 0, 6)$ ,  $B(0, 4, 0)$  and  $C(6, 0, 0)$ .

### Solution:

Let AD, BE and CF be the medians of the given triangle.

Since, AD is the median, D is the mid-point of BC

Coordinates of point  $D = \left( \frac{0+6}{2}, \frac{4+0}{2}, \frac{0+0}{2} \right) = (3, 2, 0)$

$$\begin{aligned}
 AD &= \sqrt{(0-3)^2 + (0-2)^2 + (6-0)^2} \\
 &= \sqrt{9+4+36} \\
 &= \sqrt{49} \\
 &= 7
 \end{aligned}$$

Since, BE is the median, E is the mid-point of AC

Coordinates of point  $E = \left( \frac{0+6}{2}, \frac{0+0}{2}, \frac{0+6}{2} \right) = (3, 0, 3)$

$$\begin{aligned}
 BE &= \sqrt{(3-0)^2 + (0-4)^2 + (3-0)^2} \\
 &= \sqrt{9+16+9} \\
 &= \sqrt{34}
 \end{aligned}$$

Since CF is the median, F is the mid-point of AB

Coordinates of point  $F = \left( \frac{0+0}{2}, \frac{0+4}{2}, \frac{0+6}{2} \right) = (0, 2, 3)$

$$\begin{aligned}
 CF &= \sqrt{(6-0)^2 + (0-2)^2 + (0-3)^2} \\
 &= \sqrt{36+4+9} \\
 &= \sqrt{49} \\
 &= 7
 \end{aligned}$$

Thus, the lengths of the medians of triangle ABC are  $7, \sqrt{34}$  and  $7$ .

### Question 3:

If the origin is the centroid of the triangle PQR with vertices  $P(2a, 2, 6)$ ,  $Q(-4, 3b, -10)$  and  $R(8, 14, 2c)$ , then find the values of  $a, b$  and  $c$ .

### Solution:

It is known that the coordinates of the centroid of the triangle, whose vertices are

$(x_1, y_1, z_1), (x_2, y_2, z_2)$  and  $(x_3, y_3, z_3)$  are  $\left( \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3} \right)$

Therefore, coordinates of the centroid of

$$\begin{aligned}
 \Delta PQR &= \left( \frac{2a-4+8}{3}, \frac{2+3b+14}{3}, \frac{6-10+2c}{3} \right) \\
 &= \left( \frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3} \right)
 \end{aligned}$$

It is given that origin  $(0,0,0)$  is the centroid of the  $\Delta PQR$   
Hence,

$$\frac{2a+4}{3}=0, \frac{3b+16}{3} \text{ and } \frac{2c-4}{3}=0$$

Thus, the values of  $a=-2, b=-\frac{16}{3}$  and  $c=2$ .

#### Question 4:

Find the coordinates of a point on  $y$ -axis which are at distance of  $5\sqrt{2}$  from the point  $P(3,-2,5)$ .

#### Solution:

If a point is on the  $y$ -axis, then  $x$ -coordinate and the  $z$ -coordinate of the point are zero.

Let  $A(0,b,0)$  be the point on the  $y$ -axis at a distance of  $5\sqrt{2}$  from point  $P(3,-2,5)$ .

Accordingly,  $AP=5\sqrt{2}$

On squaring both sides, we obtain  $AP^2=50$

Therefore,

$$(3-0)^2 + (-2-b)^2 + (5-0)^2 = 50$$

$$9 + 4 + b^2 + 4b + 25 = 50$$

$$b^2 + 4b - 12 = 0$$

$$b^2 + 6b - 2b - 12 = 0$$

$$(b+6)(b-2) = 0$$

$$\Rightarrow b = -6 \text{ and } b = 2$$

Thus, the coordinates of the required point are  $(0,2,0)$  and  $(0,-6,0)$ .

#### Question 5:

A point R with  $x$ -coordinate 4 lies on the line segment joining the points  $P(2,-3,4)$  and  $Q(8,0,10)$ . Find the coordinates of the point R.

[Hint Suppose R divides PQ in the ratio  $k:1$ . The coordinates of the point R are given

$$\text{by } \left( \frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1} \right)]$$

**Solution:**

The coordinates of points P and Q are given as  $P(2, -3, 4)$  and  $Q(8, 0, 10)$ .

Let R divide the line segment PQ in the ratio  $k:1$ .

Hence, by section formula, the coordinates of point R are given by

$$\left( \frac{k(8)+2}{k+1}, \frac{k(0)-3}{k+1}, \frac{k(10)+4}{k+1} \right) \\ \Rightarrow \left( \frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1} \right)$$

It is given that the  $x$ -coordinate of point R is 4.

Hence,

$$\frac{8k+2}{k+1} = 4 \\ 8k+2 = 4k+4 \\ 4k = 2 \\ k = \frac{1}{2}$$

Therefore, the coordinates of point R are

$$\left( 4, \frac{-3}{\frac{1}{2}+1}, \frac{10\left(\frac{1}{2}\right)+4}{\frac{1}{2}+1} \right) \\ \Rightarrow (4, -2, 6)$$

Thus, the coordinates of point R are  $(4, -2, 6)$ .

**Question 6:**

If A and B be the points  $(3, 4, 5)$  and  $(-1, 3, -7)$  respectively, find the equation of the set of points P such that  $PA^2 + PB^2 = k^2$ , where  $k$  is a constant.

**Solution:**

The coordinates of points A and B are given as  $A(3, 4, 5)$  and  $B(-1, 3, -7)$  respectively.

Let the coordinates of point P be  $(x, y, z)$ .

On using distance formula, we obtain

$$\begin{aligned}
 PA^2 &= (x-3)^2 + (y-4)^2 + (z-5)^2 \\
 &= x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 - 10z \\
 &= x^2 - 6x + y^2 - 8y + z^2 - 10z + 50
 \end{aligned}$$

$$\begin{aligned}
 PB^2 &= (x+1)^2 + (y-3)^2 + (z+7)^2 \\
 &= x^2 + 1 + 2x + y^2 + 9 - 6y + z^2 + 49 + 14z \\
 &= x^2 + 2x + y^2 - 6y + z^2 + 14z + 59
 \end{aligned}$$

Now, if  $PA^2 + PB^2 = k^2$ , then,

$$\begin{aligned}
 &\Rightarrow (x^2 - 6x + y^2 - 8y + z^2 - 10z + 50) + (x^2 + 2x + y^2 - 6y + z^2 + 14z + 59) = k^2 \\
 &\Rightarrow 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = k^2 \\
 &\Rightarrow 2(x^2 + y^2 + z^2 - 2x - 7y + 2z) = k^2 - 109 \\
 &\Rightarrow x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2 - 109}{2}
 \end{aligned}$$

Thus, the required equation is  $x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2 - 109}{2}$ .





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