Exercise 9.3

Question 1:

Reduce the following equation into slope-intercept form and find their slopes and the y-intercepts.

(i) $x+7y=0$ (ii) $6x+3y-5=0$ (iii) $y=0$

Solution:

(i) The given equation is $x+7y=0$

It can be written as

$$y=-\frac{1}{7}x+0$$

This equation is of the form $y=mx+c$, where $m=-\frac{1}{7}$ and $c=0$

Therefore, equation $x+7y=0$ is the slope-intercept form, where the slope and the y-intercept are $-\frac{1}{7}$ and 0 respectively.

(ii) The given equation is $6x+3y-5=0$

It can be written as

$$y=\frac{1}{3}(-6x+5)$$

$$y=-2x+\frac{5}{3}$$

This equation is of the form $y=mx+c$, where $m=-2$ and $c=\frac{5}{3}$

Therefore, equation $6x+3y-5=0$ is the slope-intercept form, where the slope and the y-intercept are -2 and $\frac{5}{3}$ respectively.

(iii) The given equation is $y=0$.

It can be written as $y=0.x+0$

This equation is of the form $y=mx+c$, where $m=0$ and $c=0$.

Therefore, equation $y=0$ is in the slope-intercept form, where the slope and the y-intercept are 0 and 0 respectively.

Question 2:

Reduce the following equations into intercept form and find their intercepts on the axis.

(i) $3x+2y-12=0$ (ii) $4x-3y=6$ (iii) $3y+2=0$

Solution:

(i) The given equation is $3x+2y-12=0$

It can be written as

$$3x+2y=12$$

$$\frac{3x}{12}+\frac{2y}{12}=1$$

$$\frac{x}{4}+\frac{y}{6}=1 \qquad \ldots(1)$$

This equation is of the form $\frac{x}{a}+\frac{y}{b}=1$, where $a=4$ and $b=6$.

Therefore, equation (1) is in the intercept form, where the intercepts on the x and y axes are 4 and 6 respectively.

(ii) The given equation is $4x-3y=6$.

It can be written as

$$\frac{4x}{6}-\frac{3y}{6}=1$$

$$\frac{2x}{3}-\frac{y}{2}=1$$

$$\frac{x}{\frac{3}{2}}+\frac{y}{(-2)}=1 \qquad \ldots(2)$$

This equation is of the form $\frac{x}{a}+\frac{y}{b}=1$, where $a=\frac{3}{2}$ and $b=-2$.

Therefore, equation (2) is in the intercept form, where the intercepts on x and y-axes are $\frac{3}{2}$ and -2 respectively.

(iii) The given equation is $3y+2=0$

It can be written as

$$3y=-2$$

$$\frac{y}{\left(-\frac{2}{3}\right)}=1 \qquad \ldots(3)$$

This equation is of the form $\frac{x}{a}+\frac{y}{b}=1$, where $a=0$ and $b=-\frac{2}{3}$.

Therefore, equation (3) is in the intercept form, where the intercepts on the y-axis is $-\frac{2}{3}$ and it has no intercept on the x-axis.

Question 3:

Reduce the following equations into normal form. Find their perpendicular distance from the origin and angle between perpendicular and the positive x-axis.

(i) $x-3\sqrt{y}+8=0$ (ii) $y-2=0$ (iii) $x-y=4$

Solution:

(i) The given equation is $x-3\sqrt{y}+8=0$

It can be written as

$$x-3\sqrt{y}=-8$$
$$-x+3\sqrt{y}=8$$

On dividing both sides by $\sqrt{(-1)^2+\left(\sqrt{3}\right)^2}=\sqrt{4}=2$, we obtain

$$-\frac{x}{2}+\frac{\sqrt{3}}{2}y=\frac{8}{2}$$
$$\left(-\frac{1}{2}\right)x+\left(\frac{\sqrt{3}}{2}\right)y=4$$
$$x\cos 120°+y\sin 120°=4 \qquad \ldots(1)$$

Equation (1) is in the normal form.

On comparing equation (1) with the normal form of equation of the line

$x\cos\omega+y\sin\omega=p$, we obtain $\omega=120°$ and $p=4$.

Thus, the perpendicular distance of the line from the origin is 4, while the angle between the perpendicular and the positive x-axis is 120°.

(ii) The given equation is $y-2=0$

It can be represented as $0.x+1.y=2$

On dividing both sides by $\sqrt{0^2+1^2}=1$, we obtain

$$0.x+1.y=2$$
$$\Rightarrow x\cos 90°+y\sin 90°=2 \qquad \ldots(2)$$

Equation (2) is in the normal form.

On comparing equation (2) with the normal form of equation of line

$x\cos\omega + y\sin\omega = p$, we obtain $\omega = 90°$ and $p = 2$.

Thus, the perpendicular distance of the line from the origin is 2, while the angle between the perpendicular and the positive *x*-axis is $90°$.

(iii) The given equation is $x - y = 4$.

It can be reduced as $1.x + (-1)y = 4$

On dividing both sides by $\sqrt{1^2 + (-1)^2} = \sqrt{2}$, we obtain

$$\frac{1}{\sqrt{2}}x + \left(-\frac{1}{\sqrt{2}}\right)y = \frac{4}{\sqrt{2}}$$

$$\Rightarrow x\cos\left(2\pi - \frac{\pi}{4}\right) + y\sin\left(2\pi - \frac{\pi}{4}\right) = 2\sqrt{2}$$

$$\Rightarrow x\cos 315° + y\sin 315° = 2\sqrt{2} \qquad \ldots(3)$$

Equation (3) is in the normal form.

On comparing equation (3) with the normal form of the equation of the line $x\cos\omega + y\sin\omega = p$, we obtain $\omega = 315°$ and $p = 2\sqrt{2}$.

Thus, the perpendicular distance of the line from the origin is $2\sqrt{2}$, while the angle between the perpendicular and the positive *x*-axis is $315°$.

Question 4:

Find the distance of the points $(-1,1)$ from the line $12(x+6) = 5(y-2)$.

Solution:

The given equation of the line is $12(x+6) = 5(y-2)$

$$\Rightarrow 12x + 72 = 5y - 10$$

$$\Rightarrow 12x - 5y + 82 = 0 \qquad \ldots(1)$$

On comparing equation (1) with general equation of line $Ax + By + C = 0$ we obtain $A = 12, B = -5$ and $C = 82$.

It is known that the perpendicular distance (d) of a line $Ax + By + C = 0$ from a point

(x_1, y_1) is given by $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$

The given point is $(x_1, y_1) = (-1, 1)$.

Therefore, the distance of point $(-1,1)$ from the given line is

$$\frac{|12(-1)+(-5)(1)+82|}{\sqrt{12^2+(-5)^2}} = \frac{|-12-5+82|}{\sqrt{169}}$$
$$= \frac{|65|}{13}$$
$$= 5$$

Hence, the distance of point $(-1,1)$ from the given line is 5 units.

Question 5:

Find the points on the x-axis whose distance from the line $\frac{x}{3} + \frac{y}{4} = 1$ are 4 units.

Solution:

The given equation of line is

$$\frac{x}{3} + \frac{y}{4} = 1$$
$$4x + 3y - 12 = 0 \qquad \ldots(1)$$

On comparing equation (1) with general equation of line $Ax + By + C = 0$ we obtain $A = 4$, $B = 3$, and $C = -12$.

Let $(a, 0)$ be the point on the *x*-axis whose distance from the given line is 4 units.

It is known that the perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$

Therefore,

$$\Rightarrow (4a - 12) = 20 \text{ or } -(4a - 12) = 20$$
$$\Rightarrow 4a = 20 + 12 \text{ or } 4a = -20 + 12$$
$$\Rightarrow a = 8 \text{ or } a = -2$$

Thus, the required points on *x*-axis are $(-2,0)$ and $(8,0)$.

Question 6:

Find the distance between parallel lines

(i) $15x+8y-34=0$ and $15x+8y+31=0$

(ii) $l(x+y)+p=0$ and $l(x+y)-r=0$

Solution:

It is known that the distance (d) between parallel lines $Ax+By+C_1=0$ and $Ax+By+C_2=0$ is given by $d=\frac{|C_1-C_2|}{\sqrt{A^2+B^2}}$

(i) The given parallel lines are $15x+8y-34=0$ and $15x+8y+31=0$

Here, $A=15, B=8, C_1=-34$ and $C_2=-31$

Therefore, the distance between the parallel lines is

$$d=\frac{|C_1-C_2|}{\sqrt{A^2+B^2}}$$
$$=\frac{|34-31|}{\sqrt{(15)^2+(8)^2}}\,units$$
$$=\frac{|-65|}{\sqrt{289}}\,units$$
$$=\frac{65}{17}\,units$$

(ii) The given parallel lines are $l(x+y)+p=0$ and $l(x+y)-r=0$

Here, $A=B=l, C_1=p$ and $C_2=-r$

Therefore, the distance between the parallel lines is

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$$
$$= \frac{|p+r|}{\sqrt{l^2 + l^2}} units$$
$$= \frac{|p+r|}{\sqrt{2l^2}} units$$
$$= \frac{|p+r|}{l\sqrt{2}} units$$
$$= \frac{1}{\sqrt{2}} \frac{|p+r|}{l} units$$

Question7:

Find equation of the line parallel to the line $3x - 4y + 2 = 0$ and passing through the point $(-2,3)$.

Solution:

The equation of the given line is

$$3x - 4y + 2 = 0$$
$$y = \frac{3x}{4} + \frac{2}{4}$$

$y = \frac{3}{4}x + \frac{2}{4}$, which is of the form $y = mx + c$

Therefore, slope of the given line is $\frac{3}{4}$

It is known that parallel lines have the same slope.

Slope of the other line is $m = \frac{3}{4}$

Now, the equation of the line that has a slope of $\frac{3}{4}$ and passes through the points $(-2,3)$ is

$$(y-3) = \frac{3}{4}\{x - (-2)\}$$
$$4y - 12 = 3x + 6$$
$$3x - 4y + 18 = 0$$

Question 8:

Find the equation of the line perpendicular to the line $x - 7y + 5 = 0$ and having x intercept 3.

Solution:

The given equation of the line is $x-7y+5=0$

Or $y=\frac{1}{7}x+\frac{5}{7}$, which is of the form $y=mx+c$

Therefore, slope of the given line is $\frac{1}{7}$

The slope of the line perpendicular to the line having a slope is $m=-\frac{1}{\left(\frac{1}{7}\right)}=-7$

The equation of the line with slope -7 and *x*-intercept 3 is given by

$$y=m(x-d)$$
$$y=-7(x-3)$$
$$y=-7x+21$$
$$7x+y-21=0$$

Hence, the required equation of the line is $7x+y-21=0$

Question 9:

Find the angles between the lines $\sqrt{3}x+y=1$ and $x+\sqrt{3}y=1$

Solution:

The given lines are $\sqrt{3}x+y=1$ and $x+\sqrt{3}y=1$

$y=-\sqrt{3}x+1 \quad \ldots(1)$ and $y=-\frac{1}{\sqrt{3}}x+\frac{1}{\sqrt{3}} \quad \ldots(2)$

The slope of line (1) is $m_1=-3$, while the slope of the line (2) is $m_2=-\frac{1}{\sqrt{3}}$

The actual angle i.e., θ between the two lines is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{-\sqrt{3} + \frac{1}{\sqrt{3}}}{1 + \left(-\sqrt{3}\left(-\frac{1}{\sqrt{3}}\right)\right)}\right|$$

$$= \left|\frac{\frac{-3+1}{\sqrt{3}}}{1+1}\right| = \left|\frac{-2}{2 \times \sqrt{3}}\right|$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = 30^\circ$$

Thus, the angle between the given lines is either 30° or $180^\circ - 30^\circ = 150^\circ$.

Question 10:

The line through the points $(h, 3)$ and $(4, 1)$ intersects the line $7x - 9y - 19 = 0$. At right angle. Find the value of *h*.

Solution:

The slope of the line passing through points $(h, 3)$ and $(4, 1)$ is

$$m_1 = \frac{1-3}{4-h} = \frac{-2}{4-h}$$

The slope of the line $7x - 9y - 19 = 0$ or $y = \frac{7}{9}x - \frac{19}{9}$ is $m_2 = \frac{7}{9}$

It is given that the two lines are perpendicular

Therefore,

$$\Rightarrow m_1 \times m_2 = -1$$

$$\Rightarrow \frac{-14}{36 - 9h} = -1$$

$$\Rightarrow 14 = 36 - 9h$$

$$\Rightarrow 9h = 36 - 14$$

$$\Rightarrow h = \frac{22}{9}$$

Thus, the value of $h = \frac{22}{9}$.

Question 11:

Prove that the line through the point (x_1, y_1) and parallel to the line $Ax + By + C = 0$ is $A(x - x_1) + B(y - y_1) = 0$

Solution:

The slope of line $Ax + By + C = 0$ or $y = \left(-\frac{A}{B}\right)x + \left(-\frac{C}{B}\right)$ is $m = -\frac{A}{B}$

It is known that parallel lines have the same slope.

Therefore, slope of the other line $= m = -\frac{A}{B}$

The equation of the line passing through point (x_1, y_1) and having slope $m = -\frac{A}{B}$ is

$$y - y_1 = m(x - x_1)$$

$$y - y_1 = -\frac{A}{B}(x - x_1)$$

$$B(y - y_1) = -A(x - x_1)$$

$$A(x - x_1) + B(y - y_1) = 0$$

Hence, the line through point (x_1, y_1) and parallel to line $Ax + By + C = 0$ is $A(x - x_1) + B(y - y_1) = 0$

Question 12:

Two lines passing through the points $(2,3)$ intersects each other at an angle of $60°$. If slope of one line is 2, find equation of the other line.

Solution:

It is given that the slope of the first line, $m_1 = 2$.

Let the slope of the other line be m_2.

The angle between the two lines is $60°$.

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan 60° = \left|\frac{2 - m_2}{1 + 2m_2}\right|$$

$$\sqrt{3} = \pm\left(\frac{2 - m_2}{1 + 2m_2}\right)$$

$$\sqrt{3}=\left(\frac{2-m_2}{1+2m_2}\right) \qquad \sqrt{3}=-\left(\frac{2-m_2}{1+2m_2}\right)$$

$$\sqrt{3}(1+2m_2)=2-m_2 \qquad \sqrt{3}(1+2m_2)=-(2-m_2)$$

$$\sqrt{3}+2\sqrt{3}m_2+m_2=2 \qquad \sqrt{3}+2\sqrt{3}m_2-m_2=-2$$

$$\sqrt{3}+\left(2\sqrt{3}+1\right)m_2=2 \qquad \sqrt{3}+\left(2\sqrt{3}-1\right)m_2=-2$$

$$m_2=\frac{2-\sqrt{3}}{\left(2\sqrt{3}+1\right)} \quad \text{or} \quad m_2=\frac{-\left(2+\sqrt{3}\right)}{\left(2\sqrt{3}-1\right)}$$

<u>Case 1</u>:

$$m_2=\frac{2-\sqrt{3}}{\left(2\sqrt{3}+1\right)}$$

The equation of the line passing through the point $(2,3)$ and having a slope of $\frac{2-\sqrt{3}}{\left(2\sqrt{3}+1\right)}$ is

$$(y-3)=\frac{2-\sqrt{3}}{(2\sqrt{3}+1)}(x-2)$$

$$\left(2\sqrt{3}+1\right)y-3\left(2\sqrt{3}+1\right)=\left(2-\sqrt{3}\right)x-2\left(2-\sqrt{3}\right)$$

$$\left(\sqrt{3}-2\right)x+\left(2\sqrt{3}+1\right)y=-1+8\sqrt{3}$$

In this case, the equation of the other line is $\left(\sqrt{3}-2\right)x+\left(2\sqrt{3}+1\right)y=-1+8\sqrt{3}$

<u>Case 2</u>:

$$m_2=\frac{-\left(2+\sqrt{3}\right)}{\left(2\sqrt{3}-1\right)}$$

The equation of the line passing through the point $(2,3)$ and having a slope of $\frac{-\left(2+\sqrt{3}\right)}{\left(2\sqrt{3}-1\right)}$ is

$$(y-3)=\frac{-\left(2+\sqrt{3}\right)}{\left(2\sqrt{3}-1\right)}(x-2)$$

$$\left(2\sqrt{3}-1\right)y-3\left(2\sqrt{3}-1\right)=-\left(2-\sqrt{3}\right)x+2\left(2-\sqrt{3}\right)$$

$$\left(2-\sqrt{3}\right)x+\left(2\sqrt{3}-1\right)y=4-2\sqrt{3}+6\sqrt{3}-3$$

$$\left(2-\sqrt{3}\right)x+\left(2\sqrt{3}-1\right)y=1+8\sqrt{3}$$

If the case of the equation of the other line is $\left(2-\sqrt{3}\right)x+\left(2\sqrt{3}-1\right)y=1+8\sqrt{3}$

Thus, the required equation of the other line is $\left(\sqrt{3}-2\right)x+\left(2\sqrt{3}+1\right)y=-1+8\sqrt{3}$ or $\left(2-\sqrt{3}\right)x+\left(2\sqrt{3}-1\right)y=1+8\sqrt{3}$.

Question 13:

Find the equation of the right bisector of the line segment joining the points $(3,4)$ and $(-1,2)$.

Solution:

The right bisector of a line segment bisects the line segment at 90°.

The end points of the line segment are given as $A(3,4)$ and $B(-1,2)$.

Accordingly, mid-point of $AB=\left(\frac{3-1}{2},\frac{4+2}{0}\right)=(1,3)$

Slope of $AB=\frac{2-4}{-1-3}=\frac{-2}{-4}=\frac{1}{2}$

Slope of the line perpendicular to $AB=-\frac{1}{\left(\frac{1}{2}\right)}=-2$

The equation of the line passing through $(1,3)$ and having a slope of -2 is

$$\begin{aligned}(y-3)&=-2(x-1)\\ y-3&=-2x+2\\ 2x+y&=5\end{aligned}$$

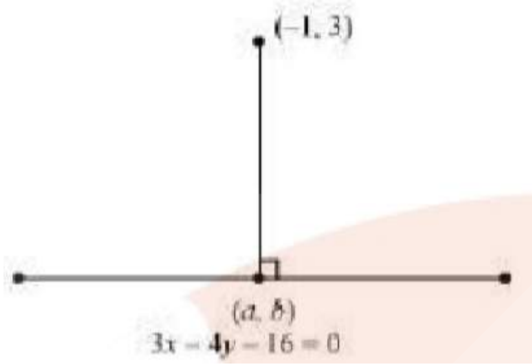
Thus, the required equation of the line is $2x+y=5$.

Question 14:

Find the coordinates of the foot of perpendicular from the points $(-1,3)$ to the line $3x-4y-16=0$.

Solution:

Let (a,b) be the coordinates of the foot of the perpendicular from the points $(-1,3)$ to the line $3x-4y-16=0$.



Slope of the line joining $(-1,3)$ and (a,b), $m_1 = \dfrac{b-3}{a+1}$

Slope of the line $3x-4y-16=0$ or $y=\dfrac{3}{4}x-4,\ m_2=\dfrac{3}{4}$

Since these two lines are perpendicular, $m_1 \times m_2 = -1$

Therefore,

$$\Rightarrow \left(\frac{b-3}{a+1}\right)\times\left(\frac{3}{4}\right)=-1$$

$$\Rightarrow \frac{3b-9}{4a+4}=-1$$

$$\Rightarrow 3b-9=-4a-4$$

$$\Rightarrow 4a+3b=5 \qquad \ldots(1)$$

Point (a,b) lies on the line $3x-4y-16=0$

Therefore,

$$\Rightarrow 3a-4b=16 \qquad \ldots(2)$$

On solving equations (1) and (2), we obtain

$a=\dfrac{68}{25}$ and $b=-\dfrac{49}{25}$

Thus, the required coordinates of the foot of the perpendicular are $\left(\dfrac{68}{25},\dfrac{49}{25}\right)$

Question 15:

The perpendicular from the origin to the line $y=mx+c$ meets it at the a point $(-1,2)$. Find the values of *m* and *c*.

Solution:

The given equation of line is $y=mx+c$

It is given that the perpendicular from the origin meets the given line at $(-1,2)$.

Therefore, the line joining the points $(0,0)$ and $(-1,2)$ is perpendicular to the given line

Slope of the line joining $(0,0)$ and $(-1,2)$ is $\frac{2}{-1}=-2$

The slope of the given line is m
Therefore,

$$m\times(-2)=-1 \quad \text{[The two lines are perpendicular]}$$
$$\Rightarrow m=\frac{1}{2}$$

Since points $(-1,2)$ lies on the given line, it satisfies the equation $y=mx+c$
Therefore,

$$\Rightarrow 2=m(-1)+c$$
$$\Rightarrow 2=2+\frac{1}{2}(-1)+c$$
$$\Rightarrow c=2+\frac{1}{2}=\frac{5}{2}$$

Thus, the respective values of m and c are $\frac{1}{2}$ and $\frac{5}{2}$

Question 16:

If p and q are the lengths of perpendicular from the origin to the lines $x\cos\theta-y\sin\theta=k\cos 2\theta$ and $x\sec\theta+y\operatorname{cosec}\theta=k$, respectively, prove that $p^2+4q^2=k^2$

Solution:

The equations of given lines are

$$x\cos\theta-y\sin\theta=k\cos 2\theta \quad \ldots(1)$$
$$x\sec\theta+y\cos ec\theta=k \quad \ldots(2)$$

The perpendicular distance(d) of a line $Ax+By+C=0$ from a point (x_1,x_2) is given by

$$d=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$$

On comparing equation(1) to the general equation of a line i.e., $Ax+By+C=0$, we obtain $A=\cos\theta, B=-\sin\theta$ and $C=-k\cos 2\theta$

It is given that p is the length of the perpendicular from $(0,0)$ to line (1).

Therefore, $$p=\frac{|A(0)+B(0)+C|}{\sqrt{A^2+B^2}}=\frac{|C|}{\sqrt{A^2+B^2}}=\frac{|-k\cos 2\theta|}{\sqrt{\cos^2\theta+\sin^2\theta}}=|-k\cos 2\theta| \quad \ldots(3)$$

On comparing equation (2) to the general equation of line i.e., $Ax+By+C=0$, we obtain $A=\sec\theta, B=\text{cosec}\,\theta$ and $C=-k$

It is given that q is the length of the perpendicular from $(0,0)$ to line (2)

Therefore, $p=\frac{|A(0)+B(0)+C|}{\sqrt{A^2+B^2}}=\frac{|C|}{\sqrt{A^2+B^2}}=\frac{|-k|}{\sqrt{\sec^2\theta+\cos ec^2\theta}}$ ...(4)

From (3) and (4), we have

$$p^2+4q^2=(|-k\cos 2\theta|)^2+4\left(\frac{|-k|}{\sqrt{\sec^2\theta+\cos ec^2\theta}}\right)^2$$
$$=k^2\cos^2 2\theta+\frac{4k^2}{(\sec^2\theta+\cos ec^2\theta)}$$
$$=k^2\cos^2 2\theta+\frac{4k^2}{\left(\frac{1}{\cos^2\theta}+\frac{1}{\sin^2\theta}\right)}$$
$$=k^2\cos^2 2\theta+\left(\frac{4k^2}{\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta\cos^2\theta}}\right)$$
$$=k^2\cos^2 2\theta+\left(\frac{4k^2}{\frac{1}{\sin^2\theta\cos^2\theta}}\right)$$
$$=k^2\cos^2 2\theta+4k^2\sin^2\theta\cos^2\theta$$
$$=k^2\cos^2 2\theta+k^2(2\sin\theta\cos\theta)^2$$
$$=k^2\cos^2 2\theta+k^2\sin^2 2\theta$$
$$=k^2(\cos^2 2\theta+\sin^2 2\theta)$$
$$=k^2$$

Hence, we proved that $p^2+4q^2=k^2$

Question 17:

In the triangle ABC with vertices $A(2,3)$, $B(4,-1)$ and $C(1,2)$, find the equation and length of altitude from the vertex A.

Solution:

Let AD be the altitude of triangle ABC from vertex A.
Accordingly, $AD\perp BC$

The equation of the line passing through point $(2,3)$ and having a slope of 1 is

$$\Rightarrow (y-3) = 1(x-2)$$
$$\Rightarrow x - y + 1 = 0$$
$$\Rightarrow y - x = 1$$

Therefore, equation of the altitude from vertex $A = y - x = 1$

Length of AD = Length of the perpendicular from $A(2,3)$ to BC
The equation of BC is

$$\Rightarrow (y+1) = \frac{2+1}{1-4}(x-4)$$
$$\Rightarrow (y+1) = -1(x-4)$$
$$\Rightarrow y + 1 = -x + 4$$
$$\Rightarrow x + y - 3 = 0 \qquad \ldots(1)$$

The perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by
$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$

On comparing equation (1) to the general equation of line $Ax + By + C = 0$, we obtain $A = 1, B = 1$ and $C = 3$.

Length of $AD = \frac{|1 \times 2 + 1 \times 3 - 3|}{\sqrt{1^2 + 1^2}} = \frac{2}{\sqrt{2}} = \sqrt{2} units$

Thus, the equation and length of the altitude from vertex A are $y - x = 1$ and $\sqrt{2}$ units.

Question 18:

If p is the length of perpendicular from the origin to the line whose intercepts on the *x*-axis are a and b, then show that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Solution:

It is known that the equation of a line whose intercepts on the axis *a* and *b* is

$\frac{x}{a} + \frac{y}{b} = 1$

$bx + ay = ab$

$bx + ay - ab = 0 \qquad \ldots(1)$

The perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by

$$d=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$$

On comparing equation (1) to the general equation of line $Ax+By+C=0$, we obtain $A=b,\ B=a$ and $C=-ab$.

Therefore, if p is the length of the perpendicular from point $(x_1,y_1)=(0,0)$ to line (1),
We obtain

$$p=\frac{|A(0)+B(0)-ab|}{\sqrt{a^2+b^2}}$$

$$=\frac{|-ab|}{\sqrt{a^2+b^2}}$$

On squaring both sides, we obtain

$$\Rightarrow p^2=\frac{(-ab)^2}{a^2+b^2}$$

$$\Rightarrow p^2\left(a^2+b^2\right)=a^2b^2$$

$$\Rightarrow \frac{a^2+b^2}{a^2b^2}=\frac{1}{p^2}$$

$$\Rightarrow \frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$$

Hence, we showed $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$