Exercise 5.1

Question 1:

Solve $24x < 100$, when

(i) x is a natural number (ii) x is an integer

Solution:

The given inequality is $24x < 100$,

$$24x < 100$$

$$\Rightarrow \frac{24x}{24} < \frac{100}{24} \quad \text{[Dividing both sides by same positive number]}$$

$$\Rightarrow x < \frac{25}{6}$$

(i) It is evident that $1, 2, 3$ and 4 are the only natural numbers less than $\frac{25}{6}$

Thus, when x is a natural number, the solutions of the given inequalities are $1, 2, 3$ and 4

Hence, in this case, the solution set is $\{1, 2, 3, 4\}$.

(ii) The integer less than $\frac{25}{6}$ are $\ldots. -3, -2, -1, 0, 1, 2, 3, 4$.

Thus, when x is an integer, the solutions of the given inequality are $\ldots, -3, -2, -1, 0, 1, 2, 3, 4$

Hence, in this case, the solution set is $\{-3, -2, -1, 0, 1, 2, 3, 4\}$

Question 2:

Solve $-12x > 30$, when

(i) x is a natural number (ii) x is an integer

Solution:

The given inequality is $-12x > 30$

$$\Rightarrow -12x > 30$$

$$\Rightarrow \frac{-12x}{-12} < \frac{30}{-12} \quad \text{[Dividing both sides by same negative number]}$$

$$\Rightarrow x < -\frac{5}{2}$$

(i) There is no natural number less than $\left(-\frac{5}{2}\right)$.

Thus, when x is a natural number, there is no solution of the given inequality.

(ii) The integer less than $\left(-\frac{5}{2}\right)$ are $\ldots,-5,-4,-3$.

Thus, when x is an integer, the solutions of the given inequality are $\ldots,-5,-4,-3$

Hence, in this case, the solution set is $\{\ldots,-5,-4,-3\}$.

Question 3:

Solve $5x-3<7$, when

(i) x is an integer (ii) x is a real number

Solution:

The given inequality is $5x-3<7$.

$$\begin{aligned}&5x-3<7\\ \Rightarrow\ &5x-3+3<7+3\\ \Rightarrow\ &5x<10\\ \Rightarrow\ &\frac{5x}{5}<\frac{10}{5}\\ \Rightarrow\ &x<2\end{aligned}$$

(i) The integers less than 2 are $\ldots,-4,-3,-2,-1,0,1$.

Thus, when x is an integer, the solutions of the given inequality are $\ldots,-4,-3,-2,-1,0,1$

Hence, in this case, the solution set is $\{\ldots,-4,-3,-2,-1,0,1\}$

(ii) When x is a real number, the solutions of the given inequality are given by $x<2$ that is all real numbers x which are less than 2.

Thus, the solution set of the given inequality is $(-\infty,2)$.

Question 4:

Solve $3x+8>2$, when

(i) x is an integer (ii) x is a real number

Solution:

The given inequality is $3x+8>2$

$$
\begin{aligned}
& 3x+8>2 \\
\Rightarrow\ & 3x+8-8>2-8 \\
\Rightarrow\ & 3x>-6 \\
\Rightarrow\ & \frac{3x}{3}>\frac{-6}{3} \\
\Rightarrow\ & x>-2
\end{aligned}
$$

(i) The integers greater than -2 are $-1,0,1,2,\ldots$

Thus, when x is an integer, the solutions of the given inequality are $-1,0,1,2\ldots$

Hence, in this case, the solution set is $\{-1,0,1,2,\ldots\}$.

(ii) When x is a real number, the solutions of the given of the inequality are all the real numbers, which are greater than -2.

Thus, in this case the solution set is $(-2,\infty)$.

Question 5:

Solve the given inequality for real $x: 4x+3<5x+7$

Solution:

$$
\begin{aligned}
& 4x+3<5x+7 \\
\Rightarrow\ & 4x+3-7<5x+7-7 \\
\Rightarrow\ & 4x-4<5x \\
\Rightarrow\ & 4x-4-4x<5x-4x \\
\Rightarrow\ & -4<x \\
\Rightarrow\ & x>-4
\end{aligned}
$$

Thus, all real numbers x, which are greater than -4, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-4,\infty)$.

Question 6:

Solve the given inequality for real $x: 3x-7>5x-1$

Solution:

$$
\begin{aligned}
& 3x-7>5x-1 \\
\Rightarrow\ & 3x-7+7>5x-1+7 \\
\Rightarrow\ & 3x>5x+6 \\
\Rightarrow\ & 3x-5x>5x+6-5x \\
\Rightarrow\ & -2x>6 \\
\Rightarrow\ & -x>3 \\
\Rightarrow\ & x<-3
\end{aligned}
$$

Thus, all real numbers x, which are less than -3, are the solutions of the given inequality. Hence, the solution set of the given inequality is $(-\infty, -3)$

Question 7:

Solve the given inequality for real $x : 3(x-1) \le 2(x-3)$

Solution:

$$\begin{aligned}
&3(x-1) \le 2(x-3) \\
&\Rightarrow 3x - 3 \le 2x - 6 \\
&\Rightarrow 3x - 3 + 3 \le 2x - 6 + 3 \\
&\Rightarrow 3x \le 2x - 3 \\
&\Rightarrow 3x - 2x \le 2x - 3 - 2x \\
&\Rightarrow x \le -3
\end{aligned}$$

Thus, all real numbers x, which are less than or equal to -3, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-\infty, -3]$

Question 8:

Solve the given inequality for real $x : 3(2-x) \ge 2(1-x)$

Solution:

$$\begin{aligned}
&3(2-x) \ge 2(1-x) \\
&\Rightarrow 6 - 3x \ge 2 - 2x \\
&\Rightarrow 6 - 3x + 2x \ge 2 - 2x + 2x \\
&\Rightarrow 6 - x \ge 2 \\
&\Rightarrow 6 - x - 6 \ge 2 - 6 \\
&\Rightarrow -x \ge -4 \\
&\Rightarrow x \le 4
\end{aligned}$$

Thus, all real numbers x, which are less than or equal to 4, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-\infty, 4]$

Question 9:

Solve the given inequality for real $x : x + \frac{x}{2} + \frac{x}{3} < 11$

Solution:

$$x+\frac{x}{2}+\frac{x}{3}<11$$

$$\Rightarrow x\left(1+\frac{1}{2}+\frac{1}{3}\right)<11$$

$$\Rightarrow x\left(\frac{6+3+2}{6}\right)<11$$

$$\Rightarrow \frac{11}{6}x<11$$

$$\Rightarrow \frac{11x}{6\times 11}<\frac{11}{11}$$

$$\Rightarrow \frac{x}{6}<1$$

$$\Rightarrow x<6$$

Thus, all real numbers x, which are less than 6, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-\infty,6)$.

Question 10:

Solve the given inequality for real $x: \frac{x}{3} > \frac{x}{2} + 1.$

Solution:

$$\begin{aligned}
&\frac{x}{3} > \frac{x}{2} + 1 \\
&\Rightarrow \frac{x}{3} - \frac{x}{2} > 1 \\
&\Rightarrow \frac{2x - 3x}{6} > 1 \\
&\Rightarrow -\frac{x}{6} > 1 \\
&\Rightarrow -x > 6 \\
&\Rightarrow x < -6
\end{aligned}$$

Thus, all real numbers *x*, which are less than -6, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-\infty, -6)$

Question 11:

Solve the given inequality for real $x: \frac{3(x-2)}{5} \le \frac{5(2-x)}{3}$

Solution:

$$\begin{aligned}
&\frac{3(x-2)}{5} \le \frac{5(2-x)}{3} \\
&\Rightarrow 9(x-2) \le 25(2-x) \\
&\Rightarrow 9x - 18 \le 50 - 25x \\
&\Rightarrow 9x + 25x \le 50 + 18 \\
&\Rightarrow 34x \le 68 \\
&\Rightarrow x \le \frac{68}{34} \\
&\Rightarrow x \le 2
\end{aligned}$$

Thus, all real numbers *x*, which are less than or equal to 2, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-\infty, 2]$

Question 12:

Solve the given inequality for real $x : \frac{1}{3}\left(\frac{3x}{5}+4\right) \geq \frac{1}{3}(x-6)$

Solution:

$$\frac{1}{2}\left(\frac{3x}{5}+4\right) \geq \frac{1}{3}(x-6)$$
$$\Rightarrow 3\left(\frac{3x}{5}+4\right) \geq 2(x-6)$$
$$\Rightarrow \frac{9x}{5}+12 \geq 2x-12$$
$$\Rightarrow 12+12 \geq 2x-\frac{9x}{5}$$
$$\Rightarrow 24 \geq \frac{10x-9x}{5}$$
$$\Rightarrow 24 \geq \frac{x}{5}$$
$$\Rightarrow x \leq 120$$

Thus, all real numbers x, which are less than or equal to 120, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-\infty, 120]$

Question 13:

Solve the given inequality for real $x : 2(2x+3)-10 < 6(x-2)$

Solution:

$$2(2x+3)-10 < 6(x-2)$$
$$\Rightarrow 4x+6-10 < 6x-12$$
$$\Rightarrow 4x-4 < 6x-12$$
$$\Rightarrow 12-4 < 6x-4x$$
$$\Rightarrow 8 < 2x$$
$$\Rightarrow x > 4$$

Thus, all real numbers x, which are greater than 4, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(4, \infty)$.

Question 14:

Solve the given inequality for real $x : 37 - (3x + 5) \geq 9x - 8(x - 3)$

Solution:

$$37 - (3x + 5) \geq 9x - 8(x - 3)$$
$$\Rightarrow 37 - 3x - 5 \geq 9x - 8x + 24$$
$$\Rightarrow 32 - 3x \geq x + 24$$
$$\Rightarrow 32 - 24 \geq x + 3x$$
$$\Rightarrow 8 \geq 4x$$
$$\Rightarrow x \leq 2$$

Thus, all real numbers x, which are less than or equal to 2, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-\infty, 2]$

Question 15:

Solve the given inequality for real $x : \frac{x}{4} < \frac{(5x - 2)}{3} + \frac{(7x - 3)}{5}$

Solution:

$$\frac{x}{4} < \frac{(5x - 2)}{3} + \frac{(7x - 3)}{5}$$
$$\Rightarrow \frac{x}{4} < \frac{5(5x - 2) - 3(7x - 3)}{15}$$
$$\Rightarrow \frac{x}{4} < \frac{25x - 10 - 21x + 9}{3}$$
$$\Rightarrow \frac{x}{4} < \frac{4x - 1}{15}$$
$$\Rightarrow 15x < 4(4x - 1)$$
$$\Rightarrow 15x < 16x - 4$$
$$\Rightarrow 4 < 16x - 15x$$
$$\Rightarrow x > 4$$

Thus, all real numbers x, which are greater than 4, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(4, \infty)$

Question 16:

Solve the given inequality for real $x: \frac{(2x-1)}{3} \geq \frac{(3x-2)}{4} - \frac{(2-x)}{5}$

Solution:

$$\frac{(2x-1)}{3} \geq \frac{(3x-2)}{4} - \frac{(2-x)}{5}$$
$$\Rightarrow \frac{(2x-1)}{3} \geq \frac{5(3x-2)-4(2-x)}{20}$$
$$\Rightarrow \frac{(2x-1)}{3} \geq \frac{15x-10-8+4x}{20}$$
$$\Rightarrow \frac{(2x-1)}{3} \geq \frac{19x-18}{20}$$
$$\Rightarrow 20(2x-1) \geq 3(19x-18)$$
$$\Rightarrow 40x-20 \geq 57x-54$$
$$\Rightarrow 40x-57x \geq 20-54$$
$$\Rightarrow -17x \geq -34$$
$$\Rightarrow x \leq 2$$

Thus, all real numbers x, which are less than or equal to 2, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-\infty, 2]$

Question 17:

Solve the given inequality and show the graph of the solution on number line:

$3x-2 < 2x+1$

Solution:

$$3x-2 < 2x+1$$
$$\Rightarrow 3x-2x < 1+2$$
$$\Rightarrow x < 3$$

Hence, the solution set of the given inequality is $(-\infty, 3)$

The graphical representation of the solutions of the given inequality is as follows:

Question 18:

Solve the given inequality and show the graph of the solution on number line:

$5x-3\geq 3x-5$

Solution:

$$\begin{aligned}&5x-3\geq 3x-5\\ \Rightarrow\ &5x-3x\geq -5+3\\ \Rightarrow\ &2x\geq -2\\ \Rightarrow\ &x\geq -1\end{aligned}$$

Hence, the solution set of the given inequality is $[-1,\infty)$

The graphical representation of the solutions of the given inequality is as follows:

Question 19:

Solve the given inequality and show the graph of the solution on number line:

$3(1-x)<2(x+4)$

Solution:

$$\begin{aligned}&3(1-x)<2(x+4)\\ \Rightarrow\ &3-3x<2x+8\\ \Rightarrow\ &3-8<2x+3x\\ \Rightarrow\ &-5<5x\\ \Rightarrow\ &x>-1\end{aligned}$$

Hence, the solution set of the given inequality is $(-1,\infty)$

The graphical representation of the solution of the inequality is as follows:

Question 20:

Solve the given inequality and show the graph of the solution on number line:

$$\frac{x}{2} \geq \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$$

Solution:

$$\frac{x}{2} \geq \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$$

$$\Rightarrow \frac{x}{2} \geq \frac{25x-10-21x+9}{15}$$

$$\Rightarrow \frac{x}{2} \geq \frac{4x-1}{15}$$

$$\Rightarrow 15x \geq 8x - 2$$

$$\Rightarrow 15x - 8x \geq -2$$

$$\Rightarrow 7x \geq -2$$

$$\Rightarrow x \geq -\frac{2}{7}$$

Hence, the solution set of the given inequality is $\left[-\frac{2}{7}, \infty\right)$

The graphical representation of the solution of the given inequality is as follows:

Question 21:

Ravi obtained 70 and 75 marks in first two-unit test. Find the minimum marks he should get in the third test to have an average of at least 60 marks.

Solution:

Let x be the marks obtained by Ravi in the third unit test.

Since the students should have an average of at least 60 marks,

$$\frac{70+75+x}{3} \geq 60$$

$$\Rightarrow 145 + x \geq 180$$

$$\Rightarrow x \geq 180 - 145$$

$$\Rightarrow x \geq 35$$

Thus, the student must obtain a minimum of 35 marks to have an average of at least 60 marks.

Question 22:

To receive Grade 'A' in a course, one must obtain an average of 90 marks or more in five examinations (each of 100 marks). If Sunita's marks in first four examinations are 87, 92, 94 and 95, find minimum marks that Sunita must obtain in fifth examination to get grade 'A' in the course.

Solution:

Let x be the marks obtained by Sunita in the fifth examination.
In order to receive grade 'A' in the course, she must obtain an average of 90 marks or more in five examinations.
Therefore,

$$\frac{87+92+94+95+x}{5} \geq 90$$
$$\Rightarrow \frac{368+x}{5} \geq 90$$
$$\Rightarrow 368+x \geq 450$$
$$\Rightarrow x \geq 450-368$$
$$\Rightarrow x \geq 82$$

Thus, Sunita must obtain a minimum of 82 marks in the fifth examination

Question 23:

Find all pairs of consecutive odd positive integers both of which are smaller than 10 such that their sum is more than 11.

Solution:

Let x be the smaller of the two consecutive odd positive integers. Then, the other integer is $(x+2)$
Since both the integers are smaller than 10,

$$x+2<10$$
$$\Rightarrow x<10-2$$
$$\Rightarrow x<8 \qquad \ldots(1)$$

Also, the sum of the two integers is more than 11.
Therefore,

$$x+(x+2)>11$$
$$\Rightarrow 2x+x>11$$
$$\Rightarrow 2x>11-2$$
$$\Rightarrow 2x>9$$
$$\Rightarrow x>\frac{9}{2}$$
$$\Rightarrow x>4.5 \qquad \ldots(2)$$

From (1) and (2), we obtain

$$4.5<x<8$$

Since x is an odd positive integer, then values of x are 5 and 7.

When $x=5$, the pair is $(5,7)$ and when $x=7$, the pair is $(7,9)$

Thus, the required possible pairs are $(5,7)$ and $(7,9)$.

Question 24:

Find all pairs of consecutive even positive integers, both of which are larger than 5 such that their sum is less than 23.

Solution:

Let x be the smaller of the two consecutive even positive integers, then the other integer is $(x+2)$

Since both the integers are larger than 5,

$$x>5 \qquad \ldots(1)$$

Also, the sum of the two integers is less than 23

$$x+(x+2)<23$$
$$\Rightarrow 2x+2<23$$
$$\Rightarrow 2x<21$$
$$\Rightarrow x<\frac{21}{2}$$
$$\Rightarrow x<10.5. \qquad \ldots(2)$$

From (1) and (2), we obtain

$$5<x<10.5.$$

Since x is an even positive integer, then values of x are 6, 8 and 10.

When $x = 6$, the pair is $(6, 8)$

When $x = 8$, the pair is $(8, 10)$

When $x = 10$, the pair is $(10, 12)$

Thus, the required possible pairs are $(6, 8)$, $(8, 10)$ and $(10, 12)$.

Question 25:

The longest side of a triangle is 3 times the shortest side and the third side is 2 cm shorter than the longest side. If the perimeter of the triangle is at least 61 cm, find the minimum length of the shortest side.

Solution:

Let the length of the shortest side of the triangle in cm be x.

Then, length of the longest side in cm $= 3x$

Length of the third side in cm $= (3x - 2)$

Since the perimeter of the triangle is at least 61 cm,

Therefore,

$$
\begin{aligned}
& x + 3x + (3x - 2) \geq 61 \\
& \Rightarrow 7x - 2 \geq 61 \\
& \Rightarrow 7x \geq 61 + 2 \\
& \Rightarrow 7x \geq 63 \\
& \Rightarrow x \geq \frac{63}{7} \\
& \Rightarrow x \geq 9
\end{aligned}
$$

Thus, the minimum length of the shortest side is 9 cm.

Question 26:

A man wants to cut three lengths from a single piece of board of length 91 cm. The second length is to be 3 cm longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths of the shortest board if the third piece is to be at least 5 cm longer than the second?

[Hint: If x is the length of the shortest board, then $(x + 3)$ and $2x$ are the length of the second and third piece, respectively. Thus $x = (x + 3) + 2x \leq 91$ and $2x \geq (x + 3) + 5$]

Solution:

Let the length of the shortest piece in cm be x.

Then, the length of second and third piece in cm are $(x+3)$ and $2x$ respectively.

Since the three lengths are to be cut from a single piece of board of length 91 cm.

$$\begin{aligned} & x+(x+3)+2x \leq 91 \\ \Rightarrow\ & 4x+3 \leq 91 \\ \Rightarrow\ & 4x \leq 91-3 \\ \Rightarrow\ & 4x \leq 88 \\ \Rightarrow\ & x \leq \frac{88}{4} \\ \Rightarrow\ & x \leq 22 \qquad \ldots(1) \end{aligned}$$

Also, the third piece is at least 5 cm longer than the second piece.

$$\begin{aligned} & 2x \geq (x+3)+5 \\ \Rightarrow\ & 2x \geq x+8 \\ \Rightarrow\ & 2x-x \geq 8 \\ \Rightarrow\ & x \geq 8 \qquad \ldots(2) \end{aligned}$$

From (1) and (2), we obtain

$$8 \leq x \leq 22$$

Thus, the possible length of the shortest board is greater than or equal to 8 cm but less than or equal to 22 cm.