Express each of the complex number given in the Exercises 1 to 10 in the form *a* + *ib*.

Q1. $(5i)\left(\frac{-3}{5}i\right)$

A.1. $(5i)\left(\frac{-3}{5}i\right)$

$= 5 \times \left(-\frac{3}{5}\right) \times i^2$ [since $i^2 = -1$]

$= -3 \times (-1)$

$= 3$

So, $(5i)\left(-\frac{3}{5}i\right) = 3 + i0$

Q2. $i^9 + i^{19}$

A.2. $i^9 + i^{19}$

$= i^{4x2+1} + i^{4x4+3}$ [Since, $i^{4k+1} = i$ and $i^{4k+3} = -i$]

$= i - i$

$= 0$

So, $i^9 + i^{19} = 0 + i0$

Q3. i^{-39}

A.3.

i^{-39}

$= \frac{1}{i^{39}}$

$= \frac{1}{i^{4x9+3}}$

$= \frac{1}{-i}$ [Since, $i^{4k+3} = -i$]

$= \frac{1}{-i} \times \frac{i}{i}$ [multiplying numerator and denominator by i]

$= \frac{i}{-i^2}$

$= \frac{i}{- -1}$ [since, $i^2 = -1$]

$= i$

So, $i^{-39} = 0 + i$

Q4. $3(7 + i7) + i(7 + i7)$

A.4. $3(7 + i7) + i(7 + i7)$

$= 21 + 21i + 7i + 7i^2$

$= 21 + 28i + 7(-1)$ [since $i^2 = -1$]

$= 21 + 28i - 7$

$= 14 + 28i$

Q5. $\quad$ **$(1 - i) - (-1 + i6)$**

Answer

$(1 - i) - (-1 + i6)$

$= 1 - i + 1 - i6$

$= 2 - 7i$

Q6. $\quad \left(\frac{1}{5} + i\frac{2}{5}\right) - \left(4 + i\frac{5}{2}\right)$

A.6. $\quad \left(\frac{1}{5} + i\frac{2}{5}\right) - \left(4 + i\frac{5}{2}\right)$

$$= \frac{1}{5} + i\frac{2}{5} - 4 - i\frac{5}{2}$$

$$= \frac{1}{5} - 4 + i\left(\frac{2}{5} - \frac{5}{2}\right)$$

$$= \frac{1-20}{5} + i\left(\frac{4-25}{10}\right)$$

$$= \frac{-19}{5} + i\left(\frac{-21}{10}\right)$$

$$= \frac{-19}{5} - i\frac{21}{10}$$

Q7. $\quad \left[\left(\frac{1}{3} + i\frac{7}{3}\right) + \left(4 + i\frac{1}{3}\right)\right] - \left(-\frac{4}{3} + i\right)$

A.7. $\quad \left[\left(\frac{1}{3} + i\frac{7}{3}\right) + \left(4 + i\frac{1}{3}\right)\right] - \left(-\frac{4}{3} + i\right)$

$$= \frac{1}{3} + i\frac{7}{3} + 4 + i\frac{1}{3} + \frac{4}{3} - i$$

$$= \left(\frac{1}{3} + 4 + \frac{4}{3}\right) + i\left(\frac{7}{3} + \frac{1}{3} - 1\right)$$

$$= \frac{1+12+4}{3} + i\frac{7+1-3}{3}$$

$$= \frac{17}{3} + i\frac{5}{3}$$

Q8. $\quad$ **$(1 - i)^4$**

A.8. $\quad (1 - i)^4$

$= [(1 - i)^2]^2$

$= [1 + i^2 - 2.i]^2$ $\quad$ [since, $(a - b)^2 = a^2 + b^2 - 2ab$]

$= [1 - 1 - 2i]^2$ [since, $i^2 = -1$]

$= (-2i)^2$

$= 4i^2$

$= -4$

$= -4 + 0i$

Q9. $\left(\frac{1}{3}+3i\right)^3$

A.9. $\left(\frac{1}{3}+3i\right)^3$

$= \left(\frac{1}{3}\right)^3 + (3i)^3 + 3\left(\frac{1}{3}\right)(3i)\left(\frac{1}{3}+3i\right)$ [since, $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$]

$= \frac{1}{27} + 27i^3 + 3i\left(\frac{1}{3}+3i\right)$ [since, $i^3 = i^2.i = -i$ and $i^2 = -1$]

$= \frac{1}{27} + 27(-i) + \frac{3i}{3} + (3i \times 3i)$

$= \frac{1}{27} - 27i + i - 9$

$= \left(\frac{1}{27} - 9\right) + i(-27 + 1)$

$= \frac{-242}{27} - i26$

Q10. $\left(-2-\frac{1}{3}i\right)^3$

A.10. $\left(-2-\frac{1}{3}i\right)^3$

$= \left[-\left(2+\frac{1}{3i}\right)\right]^3$

$= (-1)^3\left[\left(2+\frac{1}{3}i\right)^3\right]$

$= (-8)\left[8+\frac{i^3}{27}+3.2.\frac{1}{3}i\left(2+\frac{i}{3}\right)\right]$ [since, $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$]

$= (-1)\left[8-\frac{i}{27}+(2i\times 2)+\left(2i\times\frac{i}{3}\right)\right]$ [since, $i^3 = i^2.i = -i$ and $i^2 = -1$]

$= (-1)\left[8-\frac{i}{27}+4i+\frac{2i^2}{3}\right]$

$= (-1)\left[8-\frac{2}{3}+i\left(4-\frac{1}{27}\right)\right]$

$= -\frac{22}{3} - \frac{i107}{27}$

Find the multiplicative inverse of each of the complex numbers given in the Exercises 11 to 13.

Q11. **$4 - 3i$**

A.11. Let $Z = 4 - 3i$

Then $\bar{z} = 4 + 3i$

And, $|z|^2 = 4^2 + (-3)^2 = 16 + 9 = 25$

Hence, $z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{4+3i}{25} = \frac{4}{25} + i\frac{3}{25}$

Q12. **$\sqrt{5} + 3i$**

A.12. Let $z = \sqrt{5} + 3i$

Then, $\bar{z} = \sqrt{5} - 3i$

And, $|z|^2 = (\sqrt{5})^2 + (3)^2 = 5 + 9 = 14$

So, multiplicative inverse of $\sqrt{5} + 3i$ is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{\sqrt{5}-3i}{14} = \frac{\sqrt{5}}{14} - i\frac{3}{14}$$

Q13. **$-i$**

A.13. Let $z = -i$

Then $\bar{z} = i$

And $|z|^2 = (-1)^2 = 1$

So, multiplicative inverse of $-i$ is given by

$$z^{-1} = \frac{\bar{z}}{|z|} = \frac{i}{1} = i = 0 + i1$$

Q14. **Express the following expression in the form of $a + ib$:**

$$\frac{3+i\sqrt{5}}{\sqrt{3}+\sqrt{2}i \;-\; \sqrt{3}-i\sqrt{2}}$$

A.14. $$\frac{3+i\sqrt{5}}{\sqrt{3}+\sqrt{2}i \;-\; \sqrt{3}-i\sqrt{2}}$$

$$= \frac{3^2-(i\sqrt{5})^2}{\sqrt{3}+\sqrt{2}i-\sqrt{3}+\sqrt{2}i}$$ [since, $a^2 - b^2 = (a + b)(a - b)$]

$$= \frac{3^2 - i^2.5}{2\sqrt{2}i}$$

$$= \frac{[9- \;-1\;5}{2\sqrt{2}i} \times \frac{i\sqrt{2}}{i\sqrt{2}}$$ [since, $i^2 = -1$] [multiply numerator and denominator by $i\sqrt{2}$]

$$= \frac{9+5\;i\sqrt{2}}{2(\sqrt{2})^2 i^2}$$

$$= \frac{14i\sqrt{2}}{2\times 2 \; -1} \qquad \text{[since, } i^2 = -1\text{]}$$

$$= -\frac{7\sqrt{2}i}{2}$$

$$= 0 - i\frac{7\sqrt{2}}{2}$$