EXERCISE 3.3

Prove that:

Question 1:

$$\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}=-\frac{1}{2}$$

Solution:

$$LHS=\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}$$

$$=\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2-(1)^2$$

$$=\frac{1}{4}+\frac{1}{4}-1$$

$$=\frac{1+1-4}{4}$$

$$=\frac{-2}{4}$$

$$=-\frac{1}{2}=RHS$$

Question 2:

$$2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}=\frac{3}{2}$$

Solution:

$$LHS=2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$$

$$=2\left(\frac{1}{2}\right)^2+\operatorname{cosec}^2\left(\pi+\frac{\pi}{6}\right)\times\left(\frac{1}{2}\right)^2$$

$$=2\times\frac{1}{4}+\left(-\operatorname{cosec}\frac{\pi}{6}\right)^2\times\frac{1}{4}$$

$$=\frac{1}{2}+(-2)^2\times\frac{1}{4}$$

$$=\frac{1}{2}+1$$

$$=\frac{1+2}{2}$$

$$=\frac{3}{2}=RHS$$

Question 3:

$$\cot^2\frac{\pi}{6}+\text{cosec}^2\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}=6$$

Solution:

$$\begin{aligned}LHS &= \cot^2\frac{\pi}{6}+\text{cosec}\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}\\ &=\left(\sqrt{3}\right)^2+\text{cosec}\left(\pi-\frac{\pi}{6}\right)+3\left(\frac{1}{\sqrt{3}}\right)^2\\ &=3+\text{cosec}\frac{\pi}{6}+3\times\frac{1}{3}\\ &=3+2+1\\ &=6=RHS\end{aligned}$$

Question 4:

$$2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}=10$$

Solution:

$$\begin{aligned}LHS &= 2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}\\ &=2\sin^2\left(\pi-\frac{\pi}{4}\right)+2\times\left(\frac{1}{\sqrt{2}}\right)^2+2\times(2)^2\\ &=2\sin^2\frac{\pi}{4}+2\times\frac{1}{2}+2\times4\\ &=2\times\left(\frac{1}{\sqrt{2}}\right)^2+1+8\\ &=1+9\\ &=10=RHS\end{aligned}$$

Question 5:

Find the value of:

(i) $\sin 75°$

(ii) $\tan 15°$

Solution:

(i) $\sin 75°$

$$\sin 75° = \sin(45° + 30°)$$
$$= \sin 45° \cos 30° + \cos 45° \sin 30° \qquad [\because \sin(x+y) = \sin x \cos y + \cos x \sin y]$$
$$= \left(\frac{1}{\sqrt{2}}\right) \times \left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right) \times \left(\frac{1}{2}\right)$$
$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$
$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

(ii) $\tan 15°$

$$\tan 15° = \tan(45° - 30°)$$
$$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°} \qquad \left[\because \tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}\right]$$
$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \times \frac{1}{\sqrt{3}}}$$
$$= \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$$
$$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$$
$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} \qquad [\text{By rationalizing}]$$
$$= \frac{(\sqrt{3}-1)^2}{3-1}$$
$$= \frac{3+1-2\sqrt{3}}{2}$$
$$= \frac{2(2-\sqrt{3})}{2}$$
$$= 2 - \sqrt{3}$$

Prove the following:

Question 6:

$$\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)=\sin(x+y)$$

Solution:

$$LHS=\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)$$

$$=\frac{1}{2}\left[2\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)\right]+\frac{1}{2}\left[-2\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)\right]$$

$$=\left(\begin{array}{l}\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}+\cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\}\right]\\ \qquad+\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}-\cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\}\right]\end{array}\right)$$

$$\left[\begin{array}{l}\because 2\cos A\cos B=\cos(A+B)+\cos(A-B)\\ -2\sin A\sin B=\cos(A+B)-\cos(A-B)\end{array}\right]$$

$$=\sin(x+y)=RHS$$

$$=\left(\begin{array}{l}\frac{1}{2}\left[\cos\left\{\frac{\pi}{4}-x+\frac{\pi}{4}-y\right\}+\cos\left\{\frac{\pi}{4}-x-\frac{\pi}{4}+y\right\}\right]\\ \qquad+\frac{1}{2}\left[\cos\left\{\frac{\pi}{4}-x+\frac{\pi}{4}-y\right\}-\cos\left\{\frac{\pi}{4}-x-\frac{\pi}{4}+y\right\}\right]\end{array}\right)$$

$$=\frac{1}{2}\left[\cos\left\{\frac{\pi}{2}-(x+y)\right\}+\cos\{-(x-y)\}+\cos\left\{\frac{\pi}{2}-(x+y)\right\}-\cos\{-(x-y)\}\right]$$

$$=\frac{1}{2}\left[2\cos\left\{\frac{\pi}{2}-(x+y)\right\}\right]$$

$$=\sin(x+y)\qquad\left[\because\cos\left(\frac{\pi}{2}-A\right)=\sin A\right]$$

$$=RHS$$

Question 7:

$$\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\left(\frac{1+\tan x}{1-\tan x}\right)^2$$

Solution:

$$LHS=\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}$$

$$=\frac{\left(\frac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}\right)}{\left(\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right)}$$

$$\left[\because \tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}\right.$$
$$\left.\& \tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}\right]$$

$$=\frac{\left(\frac{1+\tan x}{1-\tan x}\right)}{\left(\frac{1-\tan x}{1+\tan x}\right)}$$

$$=\left(\frac{1+\tan x}{1-\tan x}\right)\times\left(\frac{1+\tan x}{1-\tan x}\right)$$

$$=\left(\frac{1+\tan x}{1-\tan x}\right)^2$$

$$=RHS$$

Question 8:

$$\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}=\cot^2 x$$

Solution:

$$LHS=\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}$$

$$=\frac{(-\cos x)\times(\cos x)}{(\sin x)\times(-\sin x)} \qquad \left[\begin{array}{l}\because \cos(\pi+x)=-\cos x\\ \Rightarrow \cos(-x)=\cos x\\ \Rightarrow \cos\left(\frac{\pi}{2}+x\right)=-\sin x\\ \Rightarrow \sin(\pi-x)=\sin x\end{array}\right]$$

$$=\frac{-\cos^2 x}{-\sin^2 x}$$

$$=\left(\frac{\cos x}{\sin x}\right)^2$$

$$=\cot^2 x \qquad \left[\because \cot x=\frac{\cos x}{\sin x}\right]$$

$$=RHS$$

Question 9:

$$\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]=1$$

Solution:

$$LHS = \cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$$

$$= \cos\left\{\pi+\left(\frac{\pi}{2}+x\right)\right\}\cos x\left[\cot\left\{\pi+\left(\frac{\pi}{2}-x\right)\right\}+\cot x\right] \qquad \left[\begin{array}{l}\because \cos(2n\pi+\theta)=\cos\theta \\ \Rightarrow \cot(2n\pi+\theta)=\cot\theta\end{array}\right]$$

$$= -\cos\left(\frac{\pi}{2}+x\right)\cos x\left[\cot\left(\frac{\pi}{2}-x\right)+\cot x\right] \qquad \left[\begin{array}{l}\because \cos(\pi+\theta)=-\cos\theta \\ \Rightarrow \cot(\pi+\theta)=\cot\theta\end{array}\right]$$

$$= -(-\sin x)\cos x[\tan x+\cot x] \qquad \left[\begin{array}{l}\because \cos\left(\frac{\pi}{2}+\theta\right)=-\sin\theta \\ \Rightarrow \cot(2n\pi+\theta)=\cos\theta\end{array}\right]$$

$$= \sin x\cos x\left[\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right]$$

$$= \sin^2 x+\cos^2 x$$

$$= 1$$

$$= RHS$$

Question 10:

$$\sin(n+1)x\sin(n+2)x+\cos(n+1)x\cos(n+2)x=\cos x$$

Solution:

$$LHS = \sin(n+1)x\sin(n+2)x+\cos(n+1)x\cos(n+2)x$$

$$= \cos(n+2)x.\cos(n+1)x+\sin(n+2)x.\sin(n+1)x$$

$$= \cos\{(n+2)x-(n+1)x\} \qquad [\because \cos(A-B)=\cos A\cos B+\sin A\sin B]$$

$$= \cos\{n+2-n-1\}x$$

$$= \cos x$$

$$= RHS$$

Question 11:

$$\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)=-\sqrt{2}\sin x$$

Solution:

$$LHS = \cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)$$

$$= -2\sin\left(\frac{\left(\frac{3\pi}{4}+x\right)+\left(\frac{3\pi}{4}-x\right)}{2}\right)\sin\left(\frac{\left(\frac{3\pi}{4}+x\right)-\left(\frac{3\pi}{4}-x\right)}{2}\right)$$

$$\left[\because \cos A-\cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$$= -2\sin\left(\frac{\frac{3\pi}{4}+x+\frac{3\pi}{4}-x}{2}\right)\sin\left(\frac{\frac{3\pi}{4}+x-\frac{3\pi}{4}+x}{2}\right)$$

$$= -2\sin\left(\frac{3\pi}{4}\right)\sin\left(\frac{2x}{2}\right)$$

$$= -2\sin\left(\pi-\frac{\pi}{4}\right)\sin x$$

$$= -2\sin\frac{\pi}{4}\sin x \qquad \left[\because \sin(\pi-\theta)=\sin\theta\right]$$

$$= -2\times\frac{1}{\sqrt{2}}\times\sin x$$

$$= -\sqrt{2}\sin x$$

$$= RHS$$

Question 12:

$\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

Solution:

$$
\begin{aligned}
LHS &= \sin^2 6x - \sin^2 4x \\
&= (\sin 6x + \sin 4x)(\sin 6x - \sin 4x) \\
&= \left[2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)\right] \times \left[2\cos\left(\frac{6x+4x}{2}\right)\sin\left(\frac{6x-4x}{2}\right)\right]
\end{aligned}
$$

$$
\left[\begin{aligned}
&\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \\
&\& \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)
\end{aligned}\right]
$$

$$
\begin{aligned}
&= [2\sin 5x \cos x] \times [2\cos 5x \sin x] \\
&= [2\sin 5x \cos 5x] \times [2\sin x \cos x] \\
&= [\sin(5x+5x) + \sin(5x-5x)] \times [\sin(x+x) + \sin(x-x)]
\end{aligned}
$$

$$
[\because \ 2\sin A \cos B = \sin(A+B) + \sin(A-B)]
$$

$$
\begin{aligned}
&= [\sin 10x + \sin 0] \times [\sin 2x + \sin 0] \\
&= [\sin 10x + 0] \times [\sin 2x + 0] \\
&= \sin 2x \sin 10x \\
&= RHS
\end{aligned}
$$

Question 13:

$\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

Solution:

$$LHS = \cos^2 2x - \cos^2 6x$$

$$= (\cos 2x + \cos 6x)(\cos 2x - \cos 6x) \qquad [\because a^2 - b^2 = (a+b)(a-b)]$$

$$= \left[2\cos\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right] \times \left[-2\sin\left(\frac{2x+6x}{2}\right)\sin\left(\frac{2x-6x}{2}\right)\right]$$

$$\left[\begin{array}{l}\because \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \\ \& \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\end{array}\right]$$

$$= [2\cos 4x \cos(-2x)] \times [-2\sin 4x \sin(-2x)]$$

$$= [2\cos 4x \cos 2x] \times [-2\sin 4x(-\sin 2x)] \qquad \left[\begin{array}{l}\because \cos(-\theta) = \cos\theta \\ \& \sin(-\theta) = -\sin\theta\end{array}\right]$$

$$= [2\cos 4x \cos 2x] \times [2\sin 4x \sin 2x]$$

$$= [2\cos 4x \sin 4x] \times [2\cos 2x \sin 2x]$$

$$= [\sin(4x+4x) - \sin(4x-4x)] \times [\sin(2x+2x) - \sin(2x-2x)]$$

$$[\because 2\cos A \sin B = \sin(A+B) - \sin(A-B)]$$

$$= [\sin 8x - \sin 0] \times [\sin 4x - \sin 0]$$

$$= [\sin 8x - 0] \times [\sin 4x - 0]$$

$$= \sin 4x \sin 8x$$

$$= RHS$$

Question 14:

$\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$

Solution:

$$
\begin{aligned}
LHS &= \sin 2x + 2\sin 4x + \sin 6x \\
&= [\sin 2x + \sin 6x] + 2\sin 4x \\
&= \left[2\sin\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right] + 2\sin 4x && \left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right] \\
&= [2\sin 4x\cos(-2x)] + 2\sin 4x \\
&= 2\sin 4x\cos 2x + 2\sin 4x \\
&= 2\sin 4x(\cos 2x + 1) \\
&= 2\sin 4x\left(2\cos^2 x - 1 + 1\right) && \left[\because \cos 2x = 2\cos^2 x - 1\right] \\
&= 2\sin 4x\left(2\cos^2 x\right) \\
&= 4\cos^2 x\sin 4x \\
&= RHS
\end{aligned}
$$

Question 15:

$\cot 4x(\sin 5x + \sin 3x) = \cot x(\sin 5x - \sin 3x)$

Solution:

$$
\begin{aligned}
LHS &= \cot 4x(\sin 5x + \sin 3x) \\
&= \cot 4x\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right] && \left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right] \\
&= \frac{\cos 4x}{\sin 4x}[2\sin 4x\cos x] && \left[\because \cos 2x = 2\cos^2 x - 1\right] \\
&= 2\cos 4x\cos x \\
&= 2\cos 4x\cos x \times \frac{\sin x}{\sin x} \\
&= \frac{\cos x}{\sin x} \times [2\cos 4x\sin x] \\
&= \cot x[\sin(4x+x) - \sin(4x-x)] && [\because 2\cos A\sin B = \sin(A+B) - \sin(A-B)] \\
&= \cot x(\sin 5x - \sin 3x) \\
&= RHS
\end{aligned}
$$

Question 16:

$$\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\frac{\sin 2x}{\cos 10x}$$

Solution:

$$LHS=\frac{\cos 9x-\cos 5x}{\sin 17x-\sin 3x}$$

$$=\frac{\left[-2\sin\left(\frac{9x+5x}{2}\right)\sin\left(\frac{9x-5x}{2}\right)\right]}{\left[2\cos\left(\frac{17x+3x}{2}\right)\sin\left(\frac{17x-3x}{2}\right)\right]}\qquad\left[\because \cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\ \&\ \sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$$=\frac{[-2\sin 7x\sin 2x]}{[2\cos 10x\sin 7x]}$$

$$=-\frac{\sin 2x}{\cos 10x}$$

$$=RHS$$

Question 17:

$$\frac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}=\tan 4x$$

Solution:

$$LHS=\frac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}$$

$$=\frac{\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]}{\left[2\cos\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]}\qquad\left[\because \sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\ \&\ \cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$=\frac{\sin 4x}{\cos 4x}$$

$$=\tan 4x$$

$$=RHS$$

Question 18:

$$\frac{\sin x-\sin y}{\cos x+\cos y}=\tan\frac{x-y}{2}$$

Solution:

$$LHS = \frac{\sin x - \sin y}{\cos x + \cos y}$$

$$= \frac{\left[2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)\right]}{\left[2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)\right]} \quad \left[\because \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) \\ \& \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$= \frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)}$$

$$= \tan\frac{x-y}{2}$$

$$= RHS$$

Question 19:

$$\frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$$

Solution:

$$LHS = \frac{\sin x + \sin 3x}{\cos x + \cos 3x}$$

$$= \frac{\left[2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)\right]}{\left[2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)\right]} \quad \left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \\ \& \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$= \frac{\sin 2x}{\cos 2x}$$

$$= \tan 2x$$

$$= RHS$$

Question 20:

$$\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2\sin x$$

Solution:

$$LHS=\frac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}$$

$$=\frac{\left[2\cos\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)\right]}{-\left[\cos^2 x-\sin^2 x\right]}$$

$$\left[\because \sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) \\ \& \cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$=\frac{2\cos 2x\sin(-x)}{-\cos 2x}$$

$$=2\sin x$$

$$=RHS$$

Question 21:

$$\frac{\cos 4x+\cos 3x+\cos 2x}{\sin 4x+\sin 3x+\sin 2x}=\cot 3x$$

Solution:

$$LHS=\frac{\cos 4x+\cos 3x+\cos 2x}{\sin 4x+\sin 3x+\sin 2x}$$

$$=\frac{[\cos 4x+\cos 2x]+\cos 3x}{[\sin 4x+\sin 2x]+\sin 3x}$$

$$=\frac{\left[2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)\right]+\cos 3x}{\left[2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)\right]+\sin 3x}$$

$$\left[\because \cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \\ \& \sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$=\frac{2\cos 3x\cos x+\cos 3x}{2\sin 3x\cos x+\sin 3x}$$

$$=\frac{\cos 3x(2\cos x+1)}{\sin 3x(2\cos x+1)}$$

$$=\frac{\cos 3x}{\sin 3x}$$

$$=\cot 3x$$

$$=RHS$$

Question 22:

$\cot x\cot 2x-\cot 2x\cot 3x-\cot 3x\cot x=1$

Solution:

$$
\begin{aligned}
LHS &= \cot x\cot 2x-\cot 2x\cot 3x-\cot 3x\cot x\\
&= \cot x\cot 2x-\cot 3x\left(\cot 2x+\cot x\right)\\
&= \cot x\cot 2x-\cot\left(2x+x\right)\left(\cot 2x+\cot x\right)\\
&= \cot x\cot 2x-\left[\frac{\cot 2x\cot x-1}{\cot 2x+\cot x}\right]\left(\cot 2x+\cot x\right) \qquad \left[\because \cot\left(A+B\right)=\frac{\cot A\cot B-1}{\cot A+\cot B}\right]\\
&= \cot x\cot 2x-\left[\cot 2x\cot x-1\right]\\
&= \cot x\cot 2x-\cot x\cot 2x+1\\
&= 1\\
&= RHS
\end{aligned}
$$

Question 23:

$\tan 4x=\dfrac{4\tan x\left(1-\tan^2 x\right)}{1-6\tan^2 x+\tan^4 x}$

Solution:

$$LHS = \tan 4x$$

$$= \tan 2(2x)$$

$$= \frac{2\tan 2x}{1-\tan^2 2x} \qquad \left[\because \tan 2A = \frac{2\tan A}{1-\tan^2 A}\right]$$

$$= \frac{2\left(\frac{2\tan x}{1-\tan^2 x}\right)}{1-\left(\frac{2\tan x}{1-\tan^2 x}\right)^2} \qquad \left[\because \tan 2A = \frac{2\tan A}{1-\tan^2 A}\right]$$

$$= \frac{\left(\frac{4\tan x}{1-\tan^2 x}\right)}{1-\left(\frac{4\tan^2 x}{1+\tan^4 x-2\tan^2 x}\right)} \qquad \left[\because (a-b)^2 = a^2+b^2-2ab\right]$$

$$= \frac{\left(\frac{4\tan x}{1-\tan^2 x}\right)}{\left(\frac{1+\tan^4 x-2\tan^2 x-4\tan^2 x}{1+\tan^4 x-2\tan^2 x}\right)}$$

$$= \left(\frac{4\tan x}{1-\tan^2 x}\right)\times\left(\frac{1+\tan^4 x-2\tan^2 x}{1+\tan^4 x-6\tan^2 x}\right)$$

$$= \frac{4\tan x\left(1-\tan^2 x\right)^2}{\left(1-\tan^2 x\right)\left(1+\tan^4 x-6\tan^2 x\right)} \qquad \left[\because a^2+b^2-2ab = (a-b)^2\right]$$

$$= \frac{4\tan x\left(1-\tan^2 x\right)}{1-6\tan^2 x+\tan^4 x}$$

$$= RHS$$

Question 24:

$\cos 4x = 1-8\sin^2 x\cos^2 x$

Solution:

$$LHS = \cos 4x$$

$$= \cos 2(2x)$$

$$= 1-2\sin^2 2x \qquad \left[\because \cos 2A = 1-2\sin^2 x\right]$$

$$= 1-2(2\sin x\cos x)^2 \qquad \left[\because \sin 2A = 2\sin x\cos x\right]$$

$$= 1-2\left(4\sin^2 x\cos^2 x\right)$$

$$= 1-8\sin^2 x\cos^2 x$$

$$= RHS$$

Question 25:

$\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

Solution:

$$
\begin{aligned}
LHS &= \cos 6x \\
&= \cos 3(2x) \\
&= 4\cos^3 2x - 3\cos 2x && \left[\because \cos 3A = 4\cos^3 x - 3\cos x\right] \\
&= 4\left(2\cos^2 x - 1\right)^3 - 3\left(2\cos^2 x - 1\right) && \left[\because \cos 2A = 2\cos^2 x - 1\right] \\
&= 4\left(8\cos^6 x - 1 - 12\cos^4 x + 6\cos^2 x\right) - 6\cos^2 x + 3 && \left[\because (a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2\right] \\
&= 32\cos^6 x - 4 - 48\cos^4 x + 24\cos^2 x - 6\cos^2 x + 3 \\
&= 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1 \\
&= RHS
\end{aligned}
$$