EXERCISE 3.2

Find the values of other five trigonometric functions in Exercises 1 to 5.

Question 1:

$\cos x = -\frac{1}{2}$, x lies in third quadrant.

Solution:

As we know that

$$\sec x = \frac{1}{\cos x}$$
$$= \frac{1}{\left(-\frac{1}{2}\right)}$$
$$= -2$$

Now, $\sin^2 x + \cos^2 x = 1$

$$\sin^2 x = 1 - \cos^2 x$$
$$\sin x = \pm\sqrt{1 - \cos^2 x}$$
$$= \pm\sqrt{1 - \left(-\frac{1}{2}\right)^2}$$
$$= \pm\sqrt{1 - \frac{1}{4}}$$
$$= \pm\sqrt{\frac{3}{4}}$$
$$= \pm\frac{\sqrt{3}}{2}$$

Since, x lies in third quadrant, the value of $\sin x$ will be negative.
Therefore,

$$\sin x = -\frac{\sqrt{3}}{2}$$

Now,

$$\operatorname{cosec} x = \frac{1}{\sin x}$$
$$= \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)}$$
$$= -\frac{2}{\sqrt{3}}$$

Now,

$$\tan x = \frac{\sin x}{\cos x}$$
$$= \frac{\left(-\frac{1}{2}\right)}{\left(-\frac{\sqrt{3}}{2}\right)}$$
$$= \sqrt{3}$$

Now,

$$\cot x = \frac{1}{\tan x}$$
$$= \frac{1}{\sqrt{3}}$$

Hence, $\sin x = -\frac{\sqrt{3}}{2}$, $\operatorname{cosec} x = -\frac{2}{\sqrt{3}}$, $\sec x = -2$, $\tan x = \sqrt{3}$, and $\cot x = \frac{1}{\sqrt{3}}$

Question 2:

$\sin x = \frac{3}{5}$, x lies in second quadrant.

Solution:

As we know that

$$\operatorname{cosec} x = \frac{1}{\sin x}$$
$$= \frac{1}{\left(\frac{3}{5}\right)}$$
$$= \frac{5}{3}$$

Now, $\sin^2 x + \cos^2 x = 1$

$$\cos^2 x = 1 - \sin^2 x$$

$$\cos x = \pm\sqrt{1 - \sin^2 x}$$

$$= \pm\sqrt{1 - \left(\frac{3}{5}\right)^2}$$

$$= \pm\sqrt{1 - \frac{9}{25}}$$

$$= \pm\sqrt{\frac{16}{25}}$$

$$= \pm\frac{4}{5}$$

Since, x lies in second quadrant, the value of $\cos x$ will be negative.
Therefore,

$$\cos x = -\frac{4}{5}$$

Now,

$$\sec x = \frac{1}{\cos x}$$

$$= \frac{1}{\left(-\frac{4}{5}\right)}$$

$$= -\frac{5}{4}$$

Now,

$$\tan x = \frac{\sin x}{\cos x}$$

$$= \frac{\left(\frac{3}{5}\right)}{\left(-\frac{4}{5}\right)}$$

$$= -\frac{3}{4}$$

Now,

$$\cot x = \frac{1}{\tan x}$$

$$= \frac{1}{\left(-\frac{3}{4}\right)}$$

$$= -\frac{4}{3}$$

Hence, $\operatorname{cosec} x = \frac{5}{3}$, $\cos x = -\frac{4}{5}$, $\sec x = -\frac{5}{4}$, $\tan x = -\frac{3}{4}$, and $\cot x = -\frac{4}{3}$

Question 3:

$\cot x = \frac{3}{4}$, x lies in third quadrant.

Solution:

As we know that

$$\tan x = \frac{1}{\cot x}$$
$$= \frac{1}{\left(\frac{3}{4}\right)}$$
$$= \frac{4}{3}$$

Now, $1 + \tan^2 x = \sec^2 x$

$$\sec x = \pm\sqrt{1 + \tan^2 x}$$
$$= \pm\sqrt{1 + \left(\frac{4}{3}\right)^2}$$
$$= \pm\sqrt{1 + \frac{16}{9}}$$
$$= \pm\sqrt{\frac{25}{9}}$$
$$= \pm\frac{5}{3}$$

Since, x lies in third quadrant, the value of $\sec x$ will be negative.
Therefore,

$$\sec x = -\frac{5}{3}$$

Now,

$$\cos x = \frac{1}{\sec x}$$
$$= \frac{1}{\left(-\frac{5}{3}\right)}$$
$$= -\frac{3}{5}$$

Now,

$$\tan x = \frac{\sin x}{\cos x}$$

$$\sin x = \tan x \cos x$$

$$= \left(\frac{4}{3}\right) \times \left(-\frac{3}{5}\right)$$

$$= -\frac{4}{5}$$

Now,

$$\operatorname{cosec} x = \frac{1}{\sin x}$$

$$= \frac{1}{\left(-\frac{4}{5}\right)}$$

$$= -\frac{5}{4}$$

Hence, $\sin x = -\frac{4}{5}$, $\operatorname{cosec} x = -\frac{5}{4}$, $\cos x = -\frac{3}{5}$, $\sec x = -\frac{5}{3}$, and $\tan x = \frac{4}{3}$

Question 4:

$\sec x = \frac{13}{5}$, x lies in fourth quadrant.

Solution:

As we know that

$$\cos x = \frac{1}{\sec x}$$

$$= \frac{1}{\left(\frac{13}{5}\right)}$$

$$= \frac{5}{13}$$

Now, $\sin^2 x + \cos^2 x = 1$

$$\sin^2 x = 1 - \cos^2 x$$
$$\sin x = \pm\sqrt{1-\cos^2 x}$$
$$= \pm\sqrt{1-\left(\frac{5}{13}\right)^2}$$
$$= \pm\sqrt{1-\frac{25}{169}}$$
$$= \pm\sqrt{\frac{144}{169}}$$
$$= \pm\frac{12}{13}$$

Since, x lies in fourth quadrant, the value of $\sin x$ will be negative.

Therefore,

$$\sin x = -\frac{12}{13}$$

Now,

$$\text{cosec } x = \frac{1}{\sin x}$$
$$= \frac{1}{\left(-\frac{12}{13}\right)}$$
$$= -\frac{13}{12}$$

Now,

$$\tan x = \frac{\sin x}{\cos x}$$
$$= \frac{\left(-\frac{12}{13}\right)}{\left(\frac{5}{13}\right)}$$
$$= -\frac{12}{5}$$

Now,

$$\cot x = \frac{1}{\tan x}$$
$$= \frac{1}{\left(-\frac{12}{5}\right)}$$
$$= -\frac{5}{12}$$

Hence, $\sin x = -\frac{12}{13}$, $\operatorname{cosec} x = -\frac{13}{12}$, $\cos x = \frac{5}{13}$, $\tan x = -\frac{12}{5}$, and $\cot x = -\frac{5}{12}$

Question 5:

$\tan x = -\frac{5}{12}$, x lies in second quadrant.

Solution:

As we know that

$$\cot x = \frac{1}{\tan x}$$
$$= \frac{1}{\left(-\frac{5}{12}\right)}$$
$$= -\frac{12}{5}$$

Now, $1 + \tan^2 x = \sec^2 x$

$$\sec x = \pm\sqrt{1 + \tan^2 x}$$
$$= \pm\sqrt{1 + \left(-\frac{5}{12}\right)^2}$$
$$= \pm\sqrt{1 + \frac{25}{144}}$$
$$= \pm\sqrt{\frac{169}{144}}$$
$$= \pm\frac{13}{12}$$

Since, x lies in second quadrant, the value of $\sec x$ will be negative.
Therefore,

$$\sec x = -\frac{13}{12}$$

Now,

$$\cos x = \frac{1}{\sec x}$$
$$= \frac{1}{\left(-\frac{13}{12}\right)}$$
$$= -\frac{12}{13}$$

Now,

$$\tan x = \frac{\sin x}{\cos x}$$
$$\sin x = \tan x \cos x$$
$$= \left(-\frac{5}{12}\right) \times \left(-\frac{12}{13}\right)$$
$$= \frac{5}{13}$$

Now,

$$\operatorname{cosec} x = \frac{1}{\sin x}$$
$$= \frac{1}{\left(\frac{5}{13}\right)}$$
$$= \frac{13}{5}$$

Hence, $\sin x = \frac{5}{13}$, $\operatorname{cosec} x = \frac{13}{5}$, $\cos x = -\frac{12}{13}$, $\sec x = -\frac{13}{12}$, and $\cot x = -\frac{12}{5}$

Find the values of the trigonometric functions in Exercises 6 to 10.

Question 6:

$\sin 765°$

Solution:

It is known that the value of $\sin x$ repeat after an interval of 2n or 360°.

Therefore,

$$\begin{aligned}\sin 765^\circ &= \sin\left(2\times 360^\circ + 45^\circ\right)\\ &= \sin 45^\circ\\ &= \frac{1}{\sqrt{2}}\end{aligned}$$

Question 7:

$\operatorname{cosec}\left(-1410^\circ\right)$

Solution:

It is known that the value of $\sin x$ repeat after an interval of 2n or 360°.
Therefore,

$$\begin{aligned}\operatorname{cosec}\left(-1410^\circ\right) &= \operatorname{cosec}\left(4\times 360^\circ - 1410^\circ\right)\\ &= \operatorname{cosec}\left(1440^\circ - 1410^\circ\right)\\ &= \operatorname{cosec} 30^\circ\\ &= 2\end{aligned}$$

Question 8:

$\tan\frac{19\pi}{3}$

Solution:

It is known that the value of $\tan x$ repeat after an interval of n or 180°.
Therefore,

$$\begin{aligned}\tan\frac{19\pi}{3} &= \tan 6\frac{1}{3}\pi\\ &= \tan\left(6\pi + \frac{1}{3}\pi\right)\\ &= \tan\frac{\pi}{3}\\ &= \tan 60^\circ\\ &= \sqrt{3}\end{aligned}$$

Question 9:

$\sin\left(-\frac{11\pi}{3}\right)$

Solution:

It is known that the value of $\sin x$ repeat after an interval of 2n or 360°.

Therefore,

$$\sin\left(-\frac{11\pi}{3}\right) = \sin\left(2\times 2\pi - \frac{11\pi}{3}\right)$$
$$= \sin\frac{\pi}{3}$$
$$= \sin 60°$$
$$= \frac{\sqrt{3}}{2}$$

Question 10:

$\cot\left(-\frac{15\pi}{4}\right)$

Solution:

It is known that the value of $\cos x$ repeat after an interval of n or 180°.

Therefore,

$$\cot\left(-\frac{15\pi}{4}\right) = \cot\left(4\pi - \frac{15\pi}{4}\right)$$
$$= \cot\frac{\pi}{4}$$
$$= \cot 45°$$
$$= 1$$