Exercise 3.1

Question 1:

Find the radian measures corresponding to the following degree measures:
(i) 25° (ii) – 47°30′ (iii) 240° (iv) 520°

Solution:

We know that $180° = \pi$ radian
Therefore,

(i) $25° = \frac{\pi}{180} \times 25$ radian

$= \frac{5\pi}{36}$ radian

(ii) $-47°30' = -\left(47\frac{1}{2}\right)° = -\left(\frac{95}{2}\right)°$

$-\left(\frac{95}{2}\right)° = \frac{\pi}{180} \times -\left(\frac{95}{2}\right)$ radian

$= -\frac{19\pi}{72}$ radian

(iii) $245° = \frac{\pi}{180} \times 240$ radian

$= \frac{3\pi}{4}$ radian

(iv) $520° = \frac{\pi}{180} \times 520$ radian

$= \frac{26\pi}{9}$ radian

Question 2:

Find the degree measures corresponding to the following radian measures $\left(\text{Use } \pi = \frac{22}{7}\right)$.

(i) $\frac{11}{16}$ (ii) – 4 (iii) $\frac{5\pi}{3}$ (iv) $\frac{7\pi}{6}$

Solution:

We know that $180° = \pi$ radian

Therefore,

(i) $\frac{11}{16}$ radian $= \frac{180°}{\pi} \times \frac{11}{16}$

$= 180 \times \frac{7}{22} \times \frac{11}{16} \text{deg}$

$= \frac{315}{8} \text{deg}$

$= 39\frac{3}{8} \text{deg}$

$= 39 \text{deg} + \frac{3}{8} \times 60 \text{min}$ $[\because 1° = 60']$

$= 39 \text{deg} + 22\frac{1}{2} \text{min}$

$= 39 \text{deg} + 22 \text{min} + \frac{60}{2} \text{sec}$ $[\because 1' = 60'']$

$= 39°22'30''$

(ii) -4 radian $= \frac{180°}{\pi} \times (-4)$

$= 180 \times \frac{7}{22} \times (-4) \text{deg}$

$= -\frac{2520}{11} \text{deg}$

$= -229\frac{1}{11} \text{deg}$

$= -229 \text{deg} + \frac{60}{11} \text{min}$ $[\because 1° = 60']$

$= -22 \text{deg} + 5\frac{5}{11} \text{min}$

$= -22 \text{deg} + 5 \text{min} + \frac{5}{11} \times 60 \text{sec}$ $[\because 1' = 60'']$

$= -22°5'27''$

(iii) $\frac{5\pi}{3}$ radian $= \frac{180°}{\pi} \times \frac{5\pi}{3}$

$= 300°$

(iv) $\frac{7\pi}{6}$ radian $= \frac{180°}{\pi} \times \frac{7\pi}{6}$

$= 210°$

Question 3:
A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

Solution:
A wheel makes 360 revolutions in 1 minute (60 seconds)

Therefore,

Number of revolutions made by the wheel in 1 second $= \frac{360}{60} = 6$
In one complete revolution, the wheel turns an angle of 2π radians
Hence, in 6 complete revolutions, it will turn an angle of $6 \times 2\pi = 12\pi$ radians

Thus, in one second, the wheel turns an angle of 12π radians.

Question 4:
Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm. $\left(\text{Use } \pi = \frac{22}{7}\right)$

Solution:
As we know that if in a circle of radius r, an arc of length l subtends an angle of θ radians,
Then $l = r\theta$

Therefore, $\theta = \frac{l}{r}$ radian

$$\theta = \frac{22cm}{100cm} \text{ radian}$$

$$\theta = \frac{11}{50} \times \frac{180}{\pi} \text{deg}$$

$$= \frac{11}{50} \times 180 \times \frac{7}{22} \text{deg}$$

$$= \frac{63}{5} \text{deg}$$

$$= 12\frac{3}{5} \text{deg}$$

$$= 12\,\text{deg} + \frac{3}{5} \times 60\,\text{min} \qquad [\because 1° = 60']$$

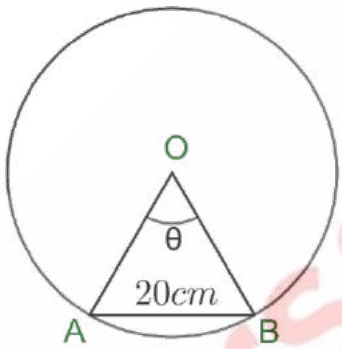
$$= 12°36'$$

Thus, the required angle is $12°36'$

Question 5:
In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.

Solution:
Diameter of the circle = 40 cm

Therefore, Radius of the circle $r = \frac{40cm}{2} = 20cm$
Let AB be a chord of length 20 cm of the circle.



In ΔAOB
$AB = 20cm$
$OA = OB = r = 20cm$

Hence, ΔAOB is an equilateral triangle

Thus, $\theta = 60°$ or $\theta = \frac{\pi}{3}$ radian

As we know that if in a circle of radius r, an arc of length l subtends an angle of θ radians,
Then $l = r\theta$
Therefore,

$$l = r\theta$$
$$AB = 20cm \times \frac{\pi}{3}$$
$$= \frac{20\pi}{3} cm$$

Hence, the length of the minor arc of the chord is $\frac{20\pi}{3} cm$.

Question 6:
If in two circles, arcs of the same length subtend angles 60° and 75° at the centre, find the ratio of their radii.

Solution:

Let the radii of the two circles be r and R.

Let an arc of length l subtend an angle of 60° at the centre of the circle of radius r, and 75° at the centre of the circle of radius R.

Now,

$$60° = \frac{\pi}{3} \text{ radian and } 75° = \frac{5\pi}{12} \text{ radian}$$

As we know that if in a circle of radius r, an arc of length l subtends an angle of θ radians,

Then $l = r\theta$

Therefore,

$$l = r \times \frac{\pi}{3} \qquad l = R \times \frac{5\pi}{12}$$

$$= \frac{\pi r}{3} \text{ and } = \frac{5\pi R}{12}$$

Thus,

$$\frac{\pi r}{3} = \frac{5\pi R}{12}$$

$$\frac{r}{R} = \frac{5}{4}$$

$$r : R = 5 : 4$$

Hence, the ratio of their radii is $5:4$.

Question 7:

Find the angle in radian through which a pendulum swings if its length is 75 cm and the tip describes an arc of length

(i) 10 cm (ii) 15 cm (iii) 21 cm

Solution:

As we know that if in a circle of radius r, an arc of length l subtends an angle of θ radians,

Then $l = r\theta$

(i) Radius, $r = 75cm$ and length of the arc, $l = 10cm$

$$\theta = \frac{l}{r}$$

$$= \frac{10cm}{75cm}$$

$$= \frac{2}{15}$$

Thus, $\theta = \frac{2}{15}$ radian

(ii) Radius, $r = 75cm$ and length of the arc, $l = 15cm$

$$\theta = \frac{l}{r}$$
$$= \frac{15cm}{75cm}$$
$$= \frac{1}{5}$$

Thus, $\theta = \frac{1}{5}$ radian

(iii) Radius, $r = 75cm$ and length of the arc, $l = 21cm$

$$\theta = \frac{l}{r}$$
$$= \frac{21cm}{75cm}$$
$$= \frac{7}{25}$$

Thus, $\theta = \frac{7}{25}$ radian