EXERCISE 2.3

Question 1:

Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.

(i) $\{(2,1),(5,1),(8,1),(11,1),(14,1),(17,1)\}$

(ii) $\{(2,1),(4,2),(6,3),(8,4),(10,5),(12,6),(14,7)\}$

(iii) $\{(1,3),(1,5),(2,5)\}$

Solution:

(i) $\{(2,1),(5,1),(8,1),(11,1),(14,1),(17,1)\}$

Since $2,5,8,11,14,$ and 17 are the elements of the domain of the given relation having their unique images, this relation is a function.

Here, domain $=\{2,5,8,11,14,17\}$ and range $=\{1\}$

(ii) $\{(2,1),(4,2),(6,3),(8,4),(10,5),(12,6),(14,7)\}$

Since $2,4,6,8,10,12$ and 14 are the elements of the domain of the given relation having their unique images, this relation is a function.

Here, domain $=\{2,4,6,8,10,12,14\}$ and range $=\{1,2,3,4,5,6,7\}$

(iii) $\{(1,3),(1,5),(2,5)\}$

Since the same first element i.e., 1 corresponds to two different images i.e., 3 and 5, this relation is not a function.

Question 2:

Find the domain and range of the following real function:

(i) $f(x)=-|x|$ (ii) $f(x)=\sqrt{9-x^2}$

Solution:

(i) $f(x)=-|x|, x \in \mathbf{R}$

We know that, $|x|=\begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x<0 \end{cases}$

Therefore, $f(x)=|-x|=\begin{cases} -x, & \text{if } x \geq 0 \\ x, & \text{if } x<0 \end{cases}$

Since $f(x)$ is defined for $x \in \mathbf{R}$, the domain of $f=\mathbf{R}$

It can be observed that the range of $f(x) = -|x|$ is all real numbers except positive real numbers.

Therefore, the range of is $f = (-\infty, 0]$

(ii) $f(x) = \sqrt{9 - x^2}$

Since $\sqrt{9 - x^2}$ is defined for all real numbers that are greater than or equal to -3 and less than or equal to 3, the domain of $f(x)$ is $\{x : -3 \le x \le 3\}$ or $[-3, 3]$.

For any value of x such that $-3 \le x \le 3$, the value of $f(x)$ will lie between 0 and 3.

Therefore, the range of $f(x)$ is $\{x : 0 \le x \le 3\}$ or $[0, 3]$

Question 3:

A function f is defined by $f(x) = 2x - 5$. Write down the values of

(i) $f(0)$ (ii) $f(7)$ (iii) $f(-3)$

Solution:

The given function is $f(x) = 2x - 5$.
Therefore,

(i) $f(0) = 2 \times 0 - 5 = 0 - 5 = -5$

(ii) $f(7) = 2 \times 7 - 5 = 14 - 5 = 9$

(iii) $f(-3) = 2 \times (-3) - 5 = -6 - 5 = -11$

Question 4:

The function 't' which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(C) = \frac{9C}{5} + 32$.

Find (i) $t(0)$ (ii) $t(28)$ (iii) $t(-10)$ (iv) The value of C, when $t(C) = 212$.

Solution:

The given function is $t(C) = \frac{9C}{5} + 32$
Therefore,

(i) $t(0)=\frac{9\times 0}{5}+32=0+32=32$

(ii) $t(28)=\frac{9\times 28}{5}+32=\frac{252+160}{5}=\frac{412}{5}=82.4$

(iii) $t(-10)=\frac{9\times(-10)}{5}+32=9\times(-2)+32=-18+32=14$

(iv) It is given that

$$t(C)=212$$
$$\Rightarrow \frac{9C}{5}+32=212$$
$$\Rightarrow \frac{9C}{5}=212-32$$
$$\Rightarrow C=\frac{180\times 5}{9}=100$$

Thus, the value of '*t*', when $t(C)=212$ is 100.

Question 5:

Find the range of each of the following functions.

(i) $f(x)=2-3x, x\in R, x>0.$

(ii) $f(x)=x^2+2, x$ is a real number.

(iii) $f(x)=x, x$ is a real number.

Solution:

(i) $f(x)=2-3x, x\in \mathbf{R}, x>0.$

The values of $f(x)$ for various values of real numbers $x>0$ can be written in the tabular form as

x	0.01	0.1	0.9	1	2	2.5	4	5	…
$f(x)$	1.97	1.7	-0.7	-1	-4	-5.5	-10	-13	…

Thus, it can be clearly observed that the range of f is the set of all real numbers less than 2. i.e., range of $f=(-\infty,2)$

Alternative Method:

Let $x>0$

$\Rightarrow 3x > 0$

$\Rightarrow 2 - 3x < 2$

$\Rightarrow f(x) < 2$

Therefore, Range of $f = (-\infty, 2)$

(ii) $f(x) = x^2 + 2, x$ is a real number.

The values of f(x) for various values of real numbers x can be written in the tabular form as

x	0	±0.3	±0.8	±1	±2	±3	…
$f(x)$	2	2.09	2.64	3	6	11	…

Thus, it can be clearly observed that the range of f is the set of all real numbers greater than 2. i.e., range of $f = [2, \infty)$

Alternative Method:

Let x be any real number i.e., $x^2 \geq 0$.

Accordingly,

$x^2 \geq 0$

$\Rightarrow x^2 + 2 \geq 0 + 2$

$\Rightarrow x^2 + 2 \geq 2$

$\Rightarrow f(x) \geq 2$

Therefore, Range of $f = [2, \infty)$

(iii) $f(x) = x, x$ is a real number

It is clear that, the range of f is the set of all real numbers.

Therefore, Range of $f = \mathbf{R}$.