

Exercise 2.2

Q1. Let $A = \{1, 2, 3, \dots, 14\}$. Define a relation R from A to A by $R = \{(x, y) : 3x - y = 0, \text{ where } x, y \in A\}$. Write down its domain, co domain and range.

A.1. Given, $A = \{1, 2, 3, \dots, 14\}$

$$R = \{(x, y) : 3x - y = 0; x, y \in A\}$$

$$= \{(x, y) : 3x = y; x, y \in A\}.$$

$$= \{(1, 3), (2, 6), (3, 9), (4, 12)\}$$

Domain of R is the set of all the first elements of the ordered pairs in R

So, domain of $R = \{1, 2, 3, 4\}$

Codomain of R is the whole set A .

So, codomain of $R = \{1, 2, 3, \dots, 14\}$

Range of R is the set of all the second elements of the ordered pairs in R .

So, range of $R = \{3, 6, 9, 12\}$

Q2. Define a relation R on the set N of natural numbers by $R = \{(x, y) : y = x + 5, x \text{ is a natural number less than } 4; x, y \in N\}$. Depict this relationship using roster form. Write down the domain and the range.

A.2. Given, $R = \{(x, y) : y = x + 5, x \text{ is a natural number less than } 4; x, y \in N\}$

$$= \{(x, y) : y = x + 5; x, y \in N \text{ and } x < 4\}.$$

$$= \{(1, 1+5), (2, 2+5), (3, 3+5)\}$$

$$= \{(1, 6), (2, 7), (3, 8)\}$$

So, domain of $R = \{1, 2, 3\}$

range of $R = \{6, 7, 8\}$

Q3. $A = \{1, 2, 3, 5\}$ and $B = \{4, 6, 9\}$. Define a relation R from A to B by $R = \{(x, y) : \text{the difference between } x \text{ and } y \text{ is odd; } x \in A, y \in B\}$. Write R in roster form.

A.3. Given, $A = \{1, 2, 3, 5\}$

$$B = \{4, 6, 9\}$$

$$R = \{(x, y) : \text{the difference of } x \text{ \& } y \text{ is odd; } x \in A, y \in B\}.$$

$$= \{(x, y) : |x - y| \text{ is odd and } x \in A, y \in B\}$$

$$= \{(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)\}.$$

Q4. The Fig 2.7 shows a relationship between the sets P and Q . Write this relation

(i) in set-builder form (ii) roster form. What is its domain and range?

A.4. As R is a relation from set P to Q .

$$(i) \quad R = \{(x, y) : x - 2 = y; 5 \leq x \leq 7\}$$

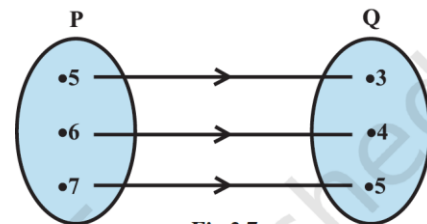


Fig 2.7

$$(ii) \quad R = \{(5,3), (6,4), (7,5)\}$$

Domain of $R = \{5, 6, 7\}$

range of $R = \{3, 4, 5\}$

Q5. Let $A = \{1, 2, 3, 4, 6\}$. Let R be the relation on A defined by $\{(a, b) : a, b \in A, b \text{ is exactly divisible by } a\}$.

(i) Write R in roster form

(ii) Find the domain of R

(iii) Find the range of R .

A.5. Given, $A = \{1, 2, 3, 4, 6\}$

$R = \{(a, b) : a, b \in A, b \text{ is exactly divisible by } a\}$

$$(i) \quad R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (2,2), (2,4), (2,6), (3,3), (3,6), (4,4), (6,6)\}$$

$$(ii) \quad \text{Domain of } R = \{1, 2, 3, 4, 6\}$$

$$(iii) \quad \text{Range of } R = \{1, 2, 3, 4, 6\}$$

Q6. Determine the domain and range of the relation R defined by $R = \{(x, x+5) : x \in \{0, 1, 2, 3, 4, 5\}\}$.

A.6. Given, $R = \{(x, x+5) : x \in \{0, 1, 2, 3, 4, 5\}\}$

$$= \{(0, 0+5), (1, 1+5), (2, 2+5), (3, 3+5), (4, 4+5), (5, 5+5)\}$$

$$= \{(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)\}$$

So, domain of $R = \{0, 1, 2, 3, 4, 5\}$

range of $R = \{5, 6, 7, 8, 9, 10\}$

Q7. Write the relation $R = \{(x, x^3) : x \text{ is a prime number less than } 10\}$ in roster form.

A.7. Given $R = \{(x, x^3) : x \text{ is a prime number less than } 10\}$

$$R = \{(x, x^3) : x = 2, 3, 5, 7\}$$

$$= \{(2, 2^3), (3, 3^3), (5, 5^3), (7, 7^3)\}$$

$$= \{(2, 8), (3, 27), (5, 125), (7, 343)\}$$

Q8. Let $A = \{x, y, z\}$ and $B = \{1, 2\}$. Find the number of relations from A to B .

A.8. Given, $A = \{x, y, z\}$ so, $n(A) = 3$

$$B = \{1, 2\} \quad \text{so } n(B) = 2$$

$$\therefore n(A \times B) = n(A) \times n(B) = 3 \times 2 = 6$$

Hence, no. of relation from A to B = Number of subsets of $A \times B$

$$= 2^6$$

$$= 64.$$

Q9. Let R be the relation on Z defined by $R = \{(a, b) : a, b \in Z, a - b \text{ is an integer}\}$. Find the domain and range of R .

A.9. Given, $R = \{(a, b) : a, b \in Z \text{ and } a - b \text{ is an integer}\}$

We know that, the difference of two integers is also an integer.

$$R = \{(a, b) : a - b \in Z \text{ \& } a, b \in Z\}$$

Domain of $R = Z$.

Range of $R = Z$.