CHAPTER – 2: Relations and Functions.

Exercise 2.1

Q1. **If $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3},\frac{1}{3}\right)$, find the values of *x* and *y*.**

A.1. Since the ordered pairs are equal, the corresponding elements are equal.

$\therefore \frac{x}{3}+1=\frac{5}{3}.$ and $y-\frac{2}{3}=\frac{1}{3}$

$\Rightarrow \frac{x}{3}=\frac{5}{3}-1$ $\qquad \Rightarrow y=\frac{1}{3}+\frac{2}{3}$

$\Rightarrow \frac{x}{3}=\frac{5-3}{3}$ $\qquad \Rightarrow y=\frac{1+2}{3}$

$\Rightarrow \frac{x}{3}=\frac{2}{3}$ $\qquad \Rightarrow y=\frac{3}{3}$

$\Rightarrow x=2$ $\qquad \Rightarrow y = 1.$

Q2. **If the set A has 3 elements and the set B = {3, 4, 5}, then find the number of elements in (A×B).**

A.2. Given, n(A) = 3

n(B) = 3 or B = {3,4,5}

So, number of elements in A × B = n(A× B) = n(A) ×n(B) = 3 × 3 = 9.

Q3. **If G = {7, 8} and H = {5, 4, 2}, find G × H and H × G.**

A.3. Given, G = {7, 8} and H = {5, 4, 2}

By the definition of the Cartesian product,

G × H = {(x, y): $x \in$ G and y = ∈ H}

= {((7, 5), (7, 4), (7, 2), (8, 5),(8,4), (8,2)}

H × G = {(x, y): $x \in$ H and $y \in$G}

= {(5, 7), (5, 8), (4,7), (4, 8), (2, 7), (2,8)}

Q4. **State whether each of the following statements are true or false. If the statement is false, rewrite the given statement correctly.**

(i) If P = {m, n} and Q = { n, m}, then P × Q = {(m, n),(n, m)}.

(ii) If A and B are non-empty sets, then A × B is a non-empty set of ordered pairs (x, y) such that $x \in$A and $y \in$B.

(iii) If A = {1, 2}, B = {3, 4}, then A × $(\mathrm{B}\cap\phi)=\phi$.

A.4. (i) False. Here P = {m, n}, $n(p)$=2

Q = {n, m}, n(Q)=2

n(P × Q) = n(P) × n(Q) = 2 × 2 = 4.

So, P ×Q = {((m, n),(m, m),(n, n),(n, m)}

(ii) True.

(iii) True. { ∴ A × (B ∩ ϕ) = A ×ϕ $\qquad$ { ∵ B ∩ ϕ = ϕ}

= n(A) × 0 $\qquad$ {∵ ϕ is empty set}

= ϕ

Q5. **If A = {–1, 1}, find A × A × A.**

A.5. Given, A = {–1,1}

So, A × A = {(–1,–1),(–1,1),(1,–1),(1,1)}

∴ A × A × A = {(–1,–1),(–1,1),(1,–1),(1,1)} × {–1,1}

$= \{(-1,-1.-1),(-1, 1, -1),(1, -1, -1,),(1,1,-1), (-1,-1,1), (-1, 1,1), (1, -1,1), (1,1,1)\}$

Q6. **If A × B = {(*a*, *x*),(*a* , *y*), (*b*, *x*), (*b*, *y*)}. Find A and B.**

A.6. Given,

$A\times B = \{(a, x),(a, y),(b, x),(b, y)\}$

We know that,

$A \times B = \{(p, q); p \in A \text{ and } q \in B\}$

So, $A = \{a, b\}$ and $B = \{x, y\}$.

Q7. **Let A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}. Verify that (i) A × (B ∩ C) = (A × B) ∩ (A × C). (ii) A × C is a subset of B × D.**

A.7. Given,

A={1, 2}, B = {1,2,3,4}, C={5,6} and D={5,6,7,8}

(i) L.H. S $= A \times(B \cap C) = \{1,2\} \times [\{1,2,3,4\} \cap\{5,6\}]$

$=\{1,2\} \times\phi$

$= \phi$.

R.H.S $= (A \times B) \cap (A \times C)=[\{1,2\}\times\{1,2,3,4\}]\cap[\{1,2\} \times\{5,6\}]$

$=[\{(1 , 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)] \cap [\{1,5),(1,6),(2,5), (2,6)\}]$

$= \phi$

Hence, L.H.S= R.H.S.

(ii)

$A \times C =\{1, 2\} \times\{5,6\}$

$=\{(1,5),(1,6),(2,5),(2,6)\}$

$B \times D =\{1,2,3,4\} \times\{5,6,7,8\}$

$=\{(1,5),(1,6),(1,7),(1,8),(2,5),(2,6),(2,7),(2,8),(3,5),(3,6),(3,7),(3,8),$
$(4,5),(4,6),(4,7),(4,8)\}$

As every element of A × C is also an element of B ×D.

$\therefore A\times C \subset B \times D$

Q8. **Let A = {1, 2} and B = {3, 4}. Write A × B. How many subsets will A × B have? List them.**

A.8. A Given, A={1,2}

B={3,4}

So, $A \times B=\{(1,3),(1,4),(2,3),(2, 4)\}$

i.e., $n(A \times B)=4$

$\therefore A \times B$ will have subset $=2^4=16$.They are,

$\phi,\{(1,3)\},\{(1,4)\},\{(2, 3)\},\{(2,4)\},\{(1,3),(1,4)\},\{(1,3),(2,3)\},$

$\{(1,3),(2, 4)\},\{(1,4),(2, 3)\},\{(1,4),(2, 4)\}, \{(2,3),(2, 4)\},$

$\{(1,3),(1,4),(2, 3)\},\{(1,3),(1,4),(2,4)\}, \{(1,3),(2,3),(2, 4)\},\{(1,4),(2,3),(2,4)\},$

and $\{(1,3),(1,4),(2,3),(2,4)\}$

Q9. **Let A and B be two sets such that *n*(A) = 3 and *n*(B) = 2. If (*x*, 1), (*y*, 2), (*z*, 1) are in A × B, find A and B, where *x*, *y* and *z* are distinct elements.**

A.9. Given, $n(A)=3$

$n(B)= 2$

So, $n(A \times B)=n(A).n(B)=3 \times 2=6$

as $(x, 1),(y, 2),(z, 1) \in A \times B=\{(x, y), x\in A \text{and } y\in B\}$.

$\therefore A=\{x, y, z\}$ and $B=\{1,2\}$

As $n(A) = 3$ as $n(B) = 2$

Q10. The Cartesian product A × A has 9 elements among which are found (–1, 0) and (0,1). Find the set A and the remaining elements of A × A.

A.10. Given, $n(A \times A)=9$

$\Rightarrow \quad n(A) \times n(A) = 9.$

$\Rightarrow \quad n(A)^2 = 3^2.$

$\Rightarrow \quad n(A) = 3 .$

And $(-1,0),(0,1) \in A \times A$ i.e., $A \times A = \{(x, y), x \in A, y \in B\}$

$\therefore \quad A=\{-1,0,1\}$

And $A \times A=\{-1,0,1\} \times \{-1,0,1\}$

$=\{(-1, -1),(-1,0),(-1,1),(0, -1),(0,0),(0,1),(1, -1),(1,0),(1,1)\}$