Question 1:

Which of the following cannot be valid assignment of probabilities for outcomes of sample space $S=\{\omega_1,\omega_2,\omega_3,\omega_4,\omega_5,\omega_6,\omega_7\}$

Assignment	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
(a)	0.1	0.01	0.05	0.03	0.01	0.2	0.6
(b)	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$
(c)	0.1	0.2	0.3	0.4	0.5	0.6	0.7
(d)	-0.1	0.2	0.3	0.4	-0.2	0.1	0.3
(e)	$\frac{1}{14}$	$\frac{2}{14}$	$\frac{3}{14}$	$\frac{4}{14}$	$\frac{5}{14}$	$\frac{6}{14}$	$\frac{15}{14}$

Solution:

(a)

ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
0.1	0.01	0.05	0.03	0.01	0.2	0.6

Here, each of the numbers $p(\omega_i)$ is positive and less than 1.
Sum of probabilities

$$\sum_{i=1}^{7} p(\omega_i) = p(\omega_1)+p(\omega_2)+p(\omega_3)+p(\omega_4)+p(\omega_5)+p(\omega_6)+p(\omega_7)$$
$$=0.1+0.01+0.05+0.03+0.01+0.2+0.6$$
$$=1$$

Thus, the assignment is valid.

(b)

ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$

Here, each of the numbers $p(\omega_i)$ is positive and less than 1.
Sum of probabilities

$$\sum_{i=1}^{7} p(\omega_i) = p(\omega_1)+p(\omega_2)+p(\omega_3)+p(\omega_4)+p(\omega_5)+p(\omega_6)+p(\omega_7)$$
$$= \frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}$$
$$= \frac{7}{7}$$
$$= 1$$

Thus, the assignment is valid.

(c)

ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
0.1	0.2	0.3	0.4	0.5	0.6	0.7

Here, each of the numbers $p(\omega_i)$ is positive and less than 1.
Sum of probabilities

$$\sum_{i=1}^{7} p(\omega_i) = p(\omega_1)+p(\omega_2)+p(\omega_3)+p(\omega_4)+p(\omega_5)+p(\omega_6)+p(\omega_7)$$
$$= 0.1+0.2+0.3+0.4+0.5+0.6+0.7$$
$$= 2.8$$
$$\neq 1$$

Thus, the assignment is not valid.

(d)

ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
– 0.1	0.2	0.3	0.4	– 0.2	0.1	0.3

Here, $p(\omega_1)$ and $p(\omega_5)$ are negative.
Hence, the assignment is not valid.

(e)

ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
$\frac{1}{14}$	$\frac{2}{14}$	$\frac{3}{14}$	$\frac{4}{14}$	$\frac{5}{14}$	$\frac{6}{14}$	$\frac{15}{14}$

Here, $p(\omega_7) = \frac{15}{14} > 1$
Hence, the assignment is not valid.

Question 2:
A coin is tossed twice, what is the probability that at least one tail occurs?

Solution:

When a coin is tossed twice, the sample space is given by

$$S=\{HH,HT,TH,TT\};n(S)=4$$

Let A be the event of the occurrence of the least one tail.

Accordingly, $A=\{HT,TH,TT\};n(A)=3$

Therefore,

$$P(A)=\frac{n(A)}{n(S)}$$
$$=\frac{3}{4}$$

Question 3:

A die is thrown, find the probability of following events:

(i) A prime number will appear,

(ii) A number greater than or equal to 3 will appear,

(iii) A number less than or equal to one will appear,

(iv) A number more than 6 will appear,

(v) A number less than 6 will appear.

Solution:

The sample space of the given experiment is given by

$$S=\{1,2,3,4,5,6\};n(S)=6$$

(i) Let A be the event of the occurrence of a prime number.

Accordingly, $A=\{2,3,5\};n(A)=3$

$$P(A)=\frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$$

Therefore,

$$P(A)=\frac{n(A)}{n(S)}$$
$$=\frac{3}{6}$$
$$=\frac{1}{2}$$

(ii) Let B be the event of the occurrence of a number greater than or equal to 3.

Accordingly, $B=\{3,4,5,6\};n(B)=4$

$$P(B)=\frac{\text{Number of outcomes favourable to B}}{\text{Total number of possible outcomes}}$$

Therefore,

$$P(B)=\frac{n(B)}{n(S)}$$
$$=\frac{4}{6}$$
$$=\frac{2}{3}$$

(iii) Let C be the event of the occurrence of a number less than or equal to one. Accordingly, $C=\{1\};n(C)=1$

$$P(C)=\frac{\text{Number of outcomes favourable to C}}{\text{Total number of possible outcomes}}$$

Therefore,

$$P(C)=\frac{n(C)}{n(S)}$$
$$=\frac{1}{6}$$

(iv) Let D be the event of the occurrence of a number greater than 6.

Accordingly, $D=\phi;n(D)=0$

$$P(D)=\frac{\text{Number of outcomes favourable to D}}{\text{Total number of possible outcomes}}$$

Therefore,

$$P(D)=\frac{n(D)}{n(S)}$$
$$=\frac{0}{6}$$
$$=0$$

(v) Let E be the event of the occurrence of a number less than 6.

Accordingly, $E=\{1,2,3,4,5\};n(E)=5$

$$P(E)=\frac{\text{Number of outcomes favourable to E}}{\text{Total number of possible outcomes}}$$

Therefore,

$$P(E)=\frac{n(E)}{n(S)}$$
$$=\frac{5}{6}$$

Question 4:

A card is selected from a pack of 52 cards.

(a) How many points are there in the sample space?

(b) Calculate the probability that the card is an ace of spades.

(c) Calculate the probability that the card is (i) an ace (ii) black card.

Solution:

(a) When a card is selected from a pack of 52 cards, the number of possible outcomes is 52.
i.e., 52 sample space contains 52 elements.
Therefore, there are 52 points in the sample space.

(b) Let A be the event in which the card drawn is an ace of spades.

Accordingly, $n(A)=1$

$$P(A)=\frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$$
$$=\frac{n(A)}{n(S)}$$
$$=\frac{1}{52}$$

(c) (i) Let E be the event in which the card drawn is an ace.

Since there are 4 ace in a pack of 52 cards, $n(E)=4$

$$P(E)=\frac{\text{Number of outcomes favourable to E}}{\text{Total number of possible outcomes}}$$
$$=\frac{n(E)}{n(S)}$$
$$=\frac{4}{52}$$
$$=\frac{1}{13}$$

(ii) Let F be the event in which the card drawn is black.

Since there are 26 black cards in a pack of 52 cards, $n(F)=26$

$$P(F)=\frac{\text{Number of outcomes favourable to F}}{\text{Total number of possible outcomes}}$$

$$=\frac{n(F)}{n(F)}$$

$$=\frac{26}{52}$$

$$=\frac{1}{2}$$

Question 5:

A fair coin with 1 marked on one face and 6 on the other and a fair die are both tossed. Find the probability that the sum of numbers that turn up is (i) 3 (ii) 12

Solution:

Since the fair coin has 1 marked on one face and 6 on the other, and the die has six faces that are numbered 1, 2, 3, 4, 5 and 6, the sample space is given by

$$S=\begin{Bmatrix}(1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{Bmatrix}$$

Accordingly, $n(S)=12$

(i) Let A be the event in which the sum of numbers that turn up is 3.

Accordingly, $A=\{(1,2)\}$

$$P(A)=\frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$$

$$=\frac{n(A)}{n(S)}$$

$$=\frac{1}{12}$$

(ii) Let B be the event in which the sum of numbers that turn up is 12

Accordingly, $B=\{(6,6)\}$

$$P(B)=\frac{\text{Number of outcomes favourable to B}}{\text{Total number of possible outcomes}}$$
$$=\frac{n(B)}{n(S)}$$
$$=\frac{1}{12}$$

Question 6:

There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that it is a women?

Solution:

There are four men and six women on the city council.

As one council member is to be selected for a committee at random, the sample space contains $(4+6)=10$ elements. i.e., $n(S)=10$

Let W be the event in which the selected council member is a woman.

Accordingly, $n(W)=6$

$$P(W)=\frac{\text{Number of outcomes favourable to W}}{\text{Total number of possible outcomes}}$$
$$=\frac{n(W)}{n(S)}$$
$$=\frac{6}{10}$$
$$=\frac{3}{5}$$

Question 7:

A fair coin is tossed four times, and a person win ₹ 1 for each head and loss ₹ 1.50 for each tail that turns up. From the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

Solution:

Since the coin is tossed four time, there can be a maximum of 4 heads and tails.

When 4 heads turn up, ₹$(1+1+1+1)=4$, i.e., ₹ 4 is the gain.

When 3 heads and 1 tail turn up, ₹$(1+1+1-1.50)=3-1.50=1.50$, i.e., ₹1.50 is the gain.

When 2 heads and 2 tail turn up, ₹$(1+1-1.50-1.50)=2-3=-1$, ₹ 1 is the loss.

When 1 heads and 3 tail turn up, ₹$(1-1.50-1.50-1.50)=1-4.50=-3.50$, i.e., ₹$3.50$ is the loss.

When 4 tails turn up, ₹$(-1.50-1.50-1.50-1.50)=-6$, i.e., ₹6 is the loss.

There are $2^4=16$ elements in the sample space S, which is given by:

$$S=\begin{Bmatrix} HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTTH, TTHH, \\ HTHT, THTH, THHT, HTTT, THTT, TTHT, TTTH, TTTT \end{Bmatrix}$$

Therefore, $n(S)=16$

The person wins Rs4.00 when 4 heads turn up, i.e., when the event $\{HHHH\}$ occurs.
Hence,

Probability (winning ₹ 4) $=\frac{1}{16}$

The person wins ₹1.50 when 3 heads and 1 tail turns up, i.e., when the event $\{HHHT, HHTH, HTHH, THHH\}$ occurs.
Hence,

Probability (winning ₹1.50) $=\frac{4}{16}=\frac{1}{4}$

The person loses ₹ 1 when 2 heads and 2 tails turns up, i.e., when the event $\{HHTT, HTTH, TTHH, HTHT, THTH, THHT\}$ occurs.
Hence,

Probability (losing ₹1) $=\frac{6}{16}=\frac{3}{8}$

The person losses ₹3.50 when 1 head and 3 tails turn up, i.e., when the event $\{HTTT, THTT, TTHT, TTTH\}$ occurs.
Hence,

Probability (losing ₹3.50) $=\frac{4}{16}=\frac{1}{4}$

The person losses ₹ 6 when 4 tails turn up, i.e., when the event $\{TTTT\}$ occurs.
Hence,

Probability (losing ₹ 6) $=\frac{1}{16}$

Question 8:

Three coins are tossed once. Find the probability of getting

(i) 3 heads (ii) 2 heads (iii) atleast 2 heads

(iv) atmost 2 heads (v) no head (vi) 3 tails

(vii) exactly two tails (vii) no tail (ix) atmost two tails.

Solution:

When three coins are tossed once, the sample space is given by

$$S=\{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

Hence, $n(S)=8$

It is known that the probability of an event A is given by

$$P(A)=\frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$$

$$=\frac{n(A)}{n(S)}$$

(i) Let B be the event of the occurrence of 3 heads.

Accordingly, $B=\{HHH\}$

$$P(B)=\frac{n(B)}{n(S)}$$

$$=\frac{1}{8}$$

(ii) Let C be the event of the occurrence of 2 heads.

Accordingly, $C=\{HHT, HTH, THH\}$

$$P(C)=\frac{n(C)}{n(S)}$$

$$=\frac{3}{8}$$

(iii) Let D be the event of the occurrence of at least 2 heads.

Accordingly, $D=\{HHH, HHT, HTH, THH\}$

$$P(D)=\frac{n(D)}{n(S)}$$

$$=\frac{4}{8}$$

$$=\frac{1}{2}$$

(iv) Let E be the event of the occurrence of atmost 2 heads.

Accordingly, $E=\{HHT,HTH,THH,HTT,THT,TTH,TTT\}$

$$P(E)=\frac{n(E)}{n(S)}$$

$$=\frac{7}{8}$$

(v) Let F be the event of the occurrence of no head.

Accordingly, $F=\{TTT\}$

$$P(F)=\frac{n(F)}{n(S)}$$

$$=\frac{1}{8}$$

(vi) Let G be the event of the occurrence of 3 tails.

Accordingly, $G=\{TTT\}$

$$P(G)=\frac{n(G)}{n(S)}$$

$$=\frac{1}{8}$$

(vii) Let H be the event of the occurrence of exactly 2 tails.

Accordingly, $H=\{HTT,THT,TTH\}$

$$P(H)=\frac{n(H)}{n(S)}$$

$$=\frac{3}{8}$$

(viii) Let I be the event of the occurrence of no tail.

Accordingly, $I=\{HHH\}$

$$P(I)=\frac{n(I)}{n(S)}$$

$$=\frac{1}{8}$$

(ix) Let J be the event of the occurrence of at most 2 tails.

Accordingly, $J=\{HHH, HHT, HTH, THH, HTT, THT, TTH\}$

$$P(J)=\frac{n(J)}{n(S)}$$
$$=\frac{7}{8}$$

Question 9:

If $\frac{2}{11}$ is the probability of an event, what is the probability of the event 'not A'.

Solution:

It is given that

$$P(A)=\frac{2}{11}$$

As we know that

$$P(A)+P(not\ A)=1$$

Therefore,

$$P(not\ A)=1-P(A)$$
$$=1-\frac{2}{11}$$
$$=\frac{9}{11}$$

Question 10:

A letter is chosen at random from the word 'ASSASSINATION'. Find the probability that letter is (i) a vowel (ii) a consonant

Solution:

There are 13 letters in the word 'ASSASSINATION'.

Hence, $n(S)=13$

(i) There are 6 vowels in the given word.

$V=\{A, A, I, A, I, O\}; n(V)=6$

$$P(V)=\frac{n(V)}{n(S)}$$
$$=\frac{6}{13}$$

(ii) There are 7 consonants in the given word.

$C=\{S,S,S,S,N,T,N\}; n(C)=7$

$$P(C)=\frac{n(C)}{n(S)}$$
$$=\frac{7}{13}$$

Question 11:

In a lottery, person choses six different natural numbers at random from 1 to 20, and if these six numbers match with the six numbers already fixed by lottery committee, he wins the price. What is the probability of winning the price in the games?
[Hint: order of the numbers is not important.]

Solution:

Total number of ways in which one can choose six different numbers from 1 to 20 is $^{20}C_6$
Therefore,

$$^{20}C_6=\frac{(20)!}{(6)!(20-6)!}$$
$$=\frac{20\times19\times18\times17\times16\times15\times(14)!}{(6)!(14)!}$$
$$=\frac{20\times19\times18\times17\times16\times15}{1\times2\times3\times4\times5\times6}$$
$$=38760$$

Hence, there are 38760 combinations of 6 numbers.
Out of these combinations, one combination is already fixed by the lottery committee.

Hence, the required probability of winning the prize in the game

$$P(w)=\frac{1}{38760}$$

Question 12:

Check whether the following probabilities P(A) and P(B) are consistently defined

(i) $P(A)=0.5,\ P(B)=0.7,\ P(A\cap B)=0.6$

(ii) $P(A)=0.5,\ P(B)=0.4,\ P(A\cup B)=0.8$

Solution:

(i) $P(A)=0.5,\ P(B)=0.7,\ P(A\cap B)=0.6$

It is known that if E and F are two events such that $E\subset F$, then $P(E)\le P(F)$

However, $P(A\cap B)>P(A)$

Hence, $P(A)$ and $P(B)$ are not consistently defined.

(ii) $P(A)=0.5,\ P(B)=0.4,\ P(A\cup B)=0.8$

It is known that if E and F are two events such that $E\subset F,$ then $P(E)\le P(F)$
Here, we can see that

$P(A\cup B)>P(A)$ and $P(A\cup B)>P(B)$

Hence, $P(A)$ and $P(B)$ are consistently defined.

Question 13:

Fill in the blank in the following table:

	$P(A)$	$P(B)$	$P(A\cap B)$	$P(A\cup B)$
(i)	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{15}$	…
(ii)	0.35	…	0.25	0.6
(iii)	0.5	0.35	…	0.7

Solution:

(i) Here, $P(A)=\frac{1}{3}, P(B)=\frac{1}{5}, P(A\cap B)=\frac{1}{15}$

We know that; $P(A\cup B)=P(A)+P(B)-P(A\cap B)$
Therefore,

$$P(A\cup B)=\frac{1}{3}+\frac{1}{5}-\frac{1}{15}$$
$$=\frac{5+3-1}{15}$$
$$=\frac{7}{15}$$

(ii) Here, $P(A)=0.35, P(A\cap B)=0.25, P(A\cup B)=0.6$

We know that; $P(A\cup B)=P(A)+P(B)-P(A\cap B)$
Therefore,

$$P(B)=P(A\cup B)-P(A)+P(A\cap B)$$
$$=0.6-0.35+0.25$$
$$=0.85-0.35$$
$$=0.5$$

(iii) Here, $P(A) = 0.5$, $P(B) = 0.35$, $P(A\cup B) = 0.7$

We know that; $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

Therefore,

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$
$$P(A\cap B)=P(A)+P(B)-P(A\cup B)$$
$$=0.5+0.35-0.7$$
$$=0.85-0.7$$
$$=0.15$$

Question 14:

Given $P(A)=\frac{3}{5}$ and $P(B)=\frac{1}{5}$. Find $P(A\ or\ B)$, if A and B are mutually exclusive events.

Solution:

Here, $P(A)=\frac{3}{5}$ and $P(B)=\frac{1}{5}$

For mutually exclusive events A and B

$$P(A\ or\ B)=P(A)+P(B)$$
$$=\frac{3}{5}+\frac{1}{5}$$
$$=\frac{4}{5}$$

Question 15:

If E and F are events such that $P(E)=\frac{1}{4}$, $P(F)=\frac{1}{2}$, and $P(E\ and\ F)=\frac{1}{8}$, find

(i) $P(E\ or\ F)$, (ii) $P(not\ E\ and\ not\ F)$

Solution:

Here, $P(E)=\frac{1}{4}$, $P(F)=\frac{1}{2}$, and $P(E\ and\ F)=\frac{1}{8}$

(i) We know that

$P(E\ and\ F)=P(E)+P(F)-P(E\ and\ F)$

Therefore,

$$P(E\ or\ F)=\frac{1}{4}+\frac{1}{2}-\frac{1}{8}$$
$$=\frac{2+4-1}{8}$$
$$=\frac{5}{8}$$

(ii) From (i), $P(E\ or\ F)=P(E\cup F)=\frac{5}{8}$

By De Morgan's law,

$$(E'\cap F')=(E\cup F)'$$

Therefore,

$$P(E'\cap F')=P(E\cup F)'$$

Now,

$$P(E\cup F)'=1-P(E\cup F)$$
$$=1-\frac{5}{8}$$
$$=\frac{3}{8}$$

Hence,

$$P(E'\cap F')=\frac{3}{8}$$

Therefore, $P(not\ E\ and\ not\ F)=\frac{3}{8}$

Question 16:

Events E and F are such that $P(not\ E\ or\ not\ F)=0.25$. State whether E and F are mutually exclusive.

Solution:

It is given that; $P(not\ E\ or\ not\ F)=0.25$

i.e., $P(E'\cup F')=0.25$

By De Morgan's law

$$(E'\cup F')=(E\cap F)'$$

Therefore,

$$P(E'\cup F')=P(E\cap F)'=0.25$$

Now,

$$P(E \cap F) = 1 - P(E \cap F)'$$
$$= 1 - 0.25$$
$$= 0.75$$
$$\neq 0$$

Hence,

$$E \cap F \neq 0$$

Thus, E and F are not mutually exclusive.

Question 17:

A and B are events such that $P(A) = 0.42, P(B) = 0.48$ and $P(A \text{ and } B) = 0.16$. Determine (i) $P(\text{not } A)$, (ii) $P(\text{not } B)$ and (iii) $P(A \text{ or } B)$.

Solution:

It is given that $P(A) = 0.42, P(B) = 0.48$ and $P(A \text{ and } B) = 0.16$

(i) Wek now that

$$P(\text{not } A) = 1 - P(A)$$
$$= 1 - 0.42$$
$$= 0.58$$

(ii) We know that

$$P(\text{not } B) = 1 - P(B)$$
$$= 1 - 0.48$$
$$= 0.52$$

(iii) We know that

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$
$$= 0.42 + 0.48 - 0.16$$
$$= 0.74$$

Question 18:

In class XI of a school 40% of the students study Mathematics and 30% study Biology 10% of the class study both Mathematics and Biology. If a student is selected at random from the class, find the probability that he will be studying Mathematics or Biology.

Solution:

Let A be the event in which the selected student studies Mathematics and B be the event in which the selected student studies Biology.
Accordingly,

$$\begin{aligned} P(A) &= 40\% \\ &= \frac{40}{100} \\ &= \frac{2}{5} \end{aligned}$$

$$\begin{aligned} P(B) &= 30\% \\ &= \frac{30}{100}\frac{3}{10} \end{aligned}$$

$$\begin{aligned} P(A \text{ and } B) &= 10\% \\ &= \frac{10}{100} \\ &= \frac{1}{10} \end{aligned}$$

We know that

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\ &= \frac{2}{5} + \frac{3}{10} - \frac{1}{10} \\ &= \frac{4+3-1}{10} \\ &= \frac{6}{10} \\ &= 0.6 \end{aligned}$$

Thus, the probability that the selected student will be studying Mathematics or Biology is 0.6.

Question 19:

In an entrance test that is graded on the basis of two examinations, the probability of a randomly chosen student passing the first examination is 0.8 and the probability of passing the second examination is 0.7 The probability of passing at least one of them is 0.95. What is the probability of passing both?

Solution:
Let A and B be the events of passing first and second examinations respectively.

Accordingly, $P(A)=0.8, P(B)=0.7$ and $P(A \text{ or } B)=0.95$
We know that

$$\begin{aligned} P(A \text{ or } B) &= P(A)+P(B)-P(A \text{ and } B) \\ &= 0.8+0.7-0.95 \\ &= 1.5-0.95 \\ &= 0.55 \end{aligned}$$

Thus, the probability of passing both the examinations is 0.55

Question 20:
The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Hindi examination?

Solution:
Let A and B be the events of passing English and Hindi examination respectively.

Accordingly,

$$P(A)=0.75, P(A \text{ and } B)=0.5, P(\text{not } A \text{ and } B)=P(A' \cap B')=0.1$$

By De Morgan's law

$$(A \cup B)' = (A' \cap B')$$

Therefore,

$$P(A \cup B)' = P(A' \cap B') = 0.1$$

Now,

$$\begin{aligned} P(A \cup B) &= 1-P(A \cup B)' \\ &= 1-0.1 \\ &= 0.9 \end{aligned}$$

Hence,

$$P(A \text{ or } B)=0.9$$

We know that,

$$\begin{aligned} P(A \text{ or } B) &= P(A)+P(B)-P(A \text{ and } B) \\ P(B) &= P(A \text{ or } B)-P(A)+P(A \text{ and } B) \\ &= 0.9-0.75+0.5 \\ &= 1.40-0.75 \\ &= 0.65 \end{aligned}$$

Thus, the probability of passing the Hindi examination is 0.65.

Question 21:

In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that

(i) The student opted for NCC or NSS.

(ii) The student has opted neither NCC nor NSS.

(iii) The student has opted NSS but not NCC.

Solution:

Let A be the event in which the selected student has opted for NCC and B be the event in which the selected student has opted for NSS.

Total number of students $= 60$

Number of students who have opted for NCC $= 30$

Therefore,

$$P(A) = \frac{30}{60}$$
$$= \frac{1}{2}$$

Number of students who have opted for NSS $= 32$

Therefore,

$$P(B) = \frac{32}{60}$$
$$= \frac{8}{15}$$

Number of students who have opted for both NCC and NSS $= 24$

Therefore,

$$P(A \text{ and } B) = \frac{24}{60}$$
$$= \frac{2}{5}$$

(i) We know that

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$
$$= \frac{1}{2} + \frac{8}{15} - \frac{2}{5}$$
$$= \frac{15+16-12}{30}$$
$$= \frac{19}{30}$$

Thus, the probability that the selected student has opted for NCC or NSS is $= \frac{19}{30}$

(ii) We know that

$$P(\text{not } A \text{ and not } B) = P(A' \text{ and } B')$$
$$= P(A' \cap B')$$

By De Morgan's law

$$(A' \cap B') = (A \cup B)'$$

Therefore,

$$P(A' \cap B') = P(A \cup B)'$$

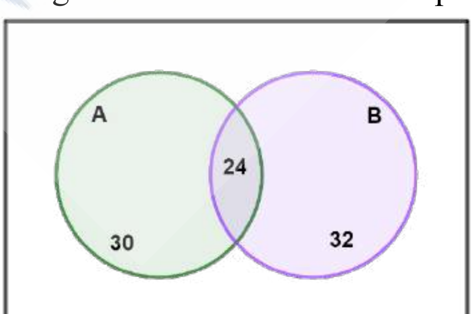
Now,

$$P(A \cup B)' = 1 - P(A \cup B)$$
$$= 1 - P(A \text{ or } B)$$
$$= 1 - \frac{19}{30}$$
$$= \frac{11}{30}$$

Thus, the probability that the selected students has neither opted for NCC nor NSS is $\frac{11}{30}$

(iii) We know that

The given information can be represented by a Venn diagram as



From the diagram It can be seen clearly that

Number of students who have opted for NSS but not NCC

$$\begin{aligned} n(B-A) &= n(B) - n(A \cap B) \\ &= 32 - 24 \\ &= 8 \end{aligned}$$

Thus, the probability that the selected student has opted for NSS but not for NCC

$$\begin{aligned} P(B-A) &= \frac{8}{60} \\ &= \frac{2}{15} \end{aligned}$$