Q1. Find the mean and variance for each of the data in Exercies 1 to 5.
6, 7, 10, 12, 13, 4, 8, 12

A.1. The given data can be tabulated as.

x_i	$x_i - \overline{x}$	$\left(x_i - \overline{x}\right)^2$
6	-3	9
7	-2	4
10	1	1
12	3	9
13	4	16
4	5	26
8	-1	1
12	3	9
Total→ 72		74

we have,

$$\text{mean}, \overline{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{6+7+10+12+13+4+8+12}{8} = \frac{72}{8} = 9.$$

So, variance, $a^2 = \frac{1}{8}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2$

$$= \frac{1}{8} \times 74 = 9.25.$$

Q2. First *n* natural numbers

A.2. We know that,

$$\text{Sum of first}'n'\text{ natural no } = \frac{n(n+1)}{2}$$

So, mean, $\overline{x} = \frac{\text{Sum of first } (n) \text{ natural no.}}{\text{no of observations}} = \frac{n(n+1)/2}{n}$

$$= \frac{n+1}{2}$$

So, Variance, $a^2 = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2 = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \left(\frac{n+1}{2}\right)\right)^2$

$$a^2 = \frac{1}{n}\left[\sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n} 2x_i\left(\frac{n+1}{2}\right) + \sum_{i=1}^{n}\left(\frac{n+1}{2}\right)^2\right]$$

________(1)

So, $\sum_{i=1}^{n} x_i^2 = (1)^2 + (2)^2 + (3)^2 + \ldots + (n)^2$

$$= \frac{n(n+1)(2n+1)}{6} \quad \_\_\_\_\_ (2).$$

$$\sum_{i=1}^{n} 2x_i \frac{(n+1)}{2} = \frac{2(n+1) \cdot n}{2} \frac{(n+1)}{2} = \frac{n(n+1)^2}{2}. \quad \_\_\_\_\_ (3)$$

And $\sum_{i=1}^{n} \left(\frac{n+1}{2}\right)^2 = \frac{(n+1)^2}{4} \sum_{i=1}^{n} 1. = \frac{n(n+1)^2}{4 \cdot} \quad \_\_\_\_ (4).$

Putting (2), (3) and (4) in (1) we get,

$$a^2 = \frac{1}{n}\left[\frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)^2}{2} + \frac{n(n+1)^2}{4}\right]$$

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{2} + \frac{(n+1)^2}{4}$$

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}.$$

$$= (n+1)\left[\frac{2n+1}{6} - \frac{(n+1)}{4}\right]$$

$$= (n+1)\left[\frac{4n+2-3n-3}{12}\right]$$

$$= \frac{(n+1)(n-1)}{12} = \frac{n^2-1}{12}.$$

Q3. First 10 multiples of 3

A.3. We have, first 10 multiples of 3=3,6,9,12,15,18,21,24,27,30.

So, $\bar{x} = \frac{3+6+9+12+15+18+21+24+27+30}{10} = \frac{165}{10} = 16.5$

We can now tabulate the given data as following.

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
3	-13.5	182.25
6	-10.5	110.25
9	-7.5	56.25
12	-4.5	20.25
15	-1.5	2.25
18	1.5	2.25
21	4.5	20.25
24	7.5	56.25
27	10.5	110.25
30	13.5	182.25

Total		742.25

Therefore, variance, $a^2 = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$

$= \frac{1}{10} \times 742.5$

= 74.25.

Q4.

x_i	6	10	14	18	24	28	30
f_i	2	4	7	12	8	4	3

A.4. The given data can be tabulated as follow.

x_i	f_i	$f_i x_i$	$\left(x_i - \bar{x}\right)$	$\left(x_i - \bar{x}\right)^2$	$f_i\left(x_i - \bar{x}\right)^2$
6	2	12	-13	169	338
10	4	40	-9	81	324
14	7	98	-5	25	175
18	12	216	-1	1	12
24	8	192	5	25	200
28	4	112	9	81	324
30	3	90	11	121	363
Total	40	760			1736

We have, $N = \sum_{i=1}^{n} f_i = 40$.

So, mean, $\bar{x} = \frac{1}{N}\sum_{i=1}^{N} f_i x_i^{\sigma} = \frac{1}{40} \times 160 = 19$

$\Rightarrow \bar{x} = 19.$

And variance, $a^2 = \frac{1}{N}\sum_{i=1}^{n} f_i \left(x_i - \bar{x}\right)^2.$

$= \frac{1}{40} \times 1736 = 43.4$

Q5.

x_i	92	93	97	98	102	104	109
f_i	3	2	3	2	6	3	3

A.5. The given data can be tabulated as follow

x_i	f_i	$f_i x_i$	$\left(x_i - \bar{x}\right)$	$\left(x_i - \bar{x}\right)^2$	$f_i\left(x_i - \bar{x}\right)^2$
92	3	276	-8	64	192
93	2	186	-7	49	98
97	3	291	-3	9	27
98	2	196	-2	4	8
102	6	612	2	4	24
104	3	312	4	16	48
109	3	327	9	81	243

Total	22	2200			640

we have,

$$\mathrm{N} = \sum_{i=1}^{n} f_i = 22.$$

So, mean, $\bar{x} = \frac{1}{\mathrm{N}} \times \sum_{i=1}^{n} f_i x_i = \frac{1}{22} \times 2200 = 100.$

And, Variance, $a^2 = \frac{1}{\mathrm{N}} \times \sum_{i=1}^{n} f_i \left(x_i - \bar{x}\right)^2$

$$= \frac{1}{22} \times 640 = 29.09.$$

Q6.Find the mean and standard deviation using short-cut method.

x_i	60	61	62	63	64	65	66	67	68
f_i	2	1	12	29	25	12	10	4	5

A.6. Let the assumed mean be A=64 and it the width, h=1.

x_i	f_i	$y_i = \left(\frac{x_i - \mathrm{A}}{n}\right)$	y_i^2	$f_i y_i$	$f_i\, y_i^2$
6	2	-4	16	-8	32
61	1	-3	9	-3	9
62	12	-2	4	-24	48
63	29	-1	1	-29	29
64	25	0	0	0	0
65	12	1	1	12	12
66	10	2	4	20	40
67	4	3	9	12	36
68	5	4	16	20	80
Total	100			0	286

Therefore, $\bar{x} = \mathrm{A} + \frac{\sum f_i y_i}{\mathrm{N}} \times h$

$$= 64 + \frac{0}{100} \times 1$$

$$\bar{x} = 64$$

And variance, $a^2 = \frac{h^2}{\mathrm{N}^2}\left[\mathrm{N}\sum fi\, y_i^2 - \left(\sum f_i\, y_i\right)^2\right]$

$$= \frac{1}{10000} \cdot \left[100 \times 286 - 0^2\right]$$

= 2. 86.

So, standard deviation $= \sqrt{a^2} = \sqrt{2.86} = 1.69$.

Q7. Find the mean and variance for the following frequency distributions in Exercises7 and 8.

Classes	**0-30**	**30-60**	**60-90**	**90-120**	**120-150**	**150-180**	**180-210**
Frequencies	**2**	**3**	**5**	**10**	**3**	**5**	**2**

A.7. Let the assumed mean be A=105 and class width, *h*=30. The given data can be tabulated as

Classes	Frequencies (fi)	mid-point (xi)	$y_i = \frac{x_i - A}{n}$	yi^2	$fiyi$	$fiyi^2$.
0-30	2	15	-3	9	-6	18
30-60	3	45	-2	4	-6	12
60-90	5	75	-1	1	-5	5
90-120	10	105	0	0	0	0
120-150	3	135	1	1	3	3
150-180	5	165	2	4	10	20
180-210	2	195	3	9	6	18
Total	30				2	76

Therefore, $\bar{x} = A + \frac{\sum fi\, yi}{N} \times h$

$= 105 + \frac{2}{30} \times 30$

$\bar{x} = 107.$

Variance, $a^2 = \frac{h^2}{N^2}\left[N\sum fi\, yi^2 - \left(\sum fi\, yi\right)^2\right]$

$= \frac{30 \times 30}{30 \times 30}\left[30 \times 76 - (2)^2\right]$

= 2280 – 4
= 2276.

Q8.

Classes	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**
Frequencies	**5**	**8**	**15**	**16**	**6**

A.8.

Classes	Frequency(fi)	mid-point(xi)	$fi\, xi$	$xi - \bar{x}$	$(xi - \bar{x})^2$	$fi(xi - \bar{x})^2$
0-10	5	5	25	-22	484	2420
10-20	8	15	120	-12	144	1152
20-30	15	25	375	-2	4	60
30-40	16	35	560	8	64	1024
40-50	6	45	270	18	324	1944
Total	50		1350			6600

We have, $N = \sum_{i=1}^{n} fi = 50$.

So, mean, $\bar{x} = \frac{1}{N} \times \sum_{i=1}^{n} fi\,xi = \frac{1350}{5} = 27.$

And variance, $\alpha^2 = \frac{1}{N} \sum_{i=1}^{n} fi\left(xi - \bar{x}\right)^2$

$= \frac{1}{50} \times 6600$

= 132.

Q9. Find the mean, variance and standard deviation using short-cut method

Height in cms	**70-75**	**75-80**	**80-85**	**85-90**	**90-95**	**95-100**	**100-125**	**105-110**	**110-115**
Frequencies	**3**	**4**	**7**	**7**	**15**	**9**	**6**	**6**	**3**

A.9. Let the assumed mean be A=92.5 and h=5

Heights in cms	No. of children f_i	Mid-point xi	$y_i = \frac{x_i - A}{h}$	yi^2	$f_i x_i$	$f_i y_i^2$
70-75	3	72.5	-4	16	−12	48
75-80	4	77.5	-3	9	−12	36
80-85	7	82.5	-2	4	−14	28
85-90	7	87.5	-1	1	−7	7
90-95	15	92.5	0	0	0	0
95-100	9	97.5	1	1	9	9
100-105	6	102.5	2	4	12	24
105-110	6	107.5	3	9	18	54
110-115	3	112.5	4	16	12	48
Total	60				6	254

Therefore, $\bar{x} = A + \frac{\sum fi\,yi}{N} \times A$

$= 92.5 + \frac{6}{60} \times 5.$

= 92.5 + 0.5 = 93.

and variance, $a^2 = \frac{h^2}{N^2}\left[N\sum fi\,yi^2 - \left(\sum fi\,yi\right)^2\right]$

$= \frac{25}{3600}[60 \times 254 - 36]$

$= \frac{25}{3600} \times 15204 = 105.58.$

So, Standard deviation $(a) = \sqrt{a^2} = \sqrt{105.58} = 10.27$.

Q10. The diameters of circles (in mm) drawn in a design are given below:

Diameters	33-36	37-40	41-44	45-48	49-52
No. of circles	15	17	21	22	25

Calculate the standard deviation and mean diameter of the circles.
[Hint First make the data continuous by making the classes as 32.5-36.5, 36.5-40.5, 40.5-44.5, 44.5 - 48.5, 48.5 - 52.5 and then proceed.]

A.10. The given data is converted into continuous frequency duration by subtracting and adding 0.5 from lower and upper limit respectively. Lit the assumed mean be A=42.5 and h=4

Diameters (in mm)	No. of circles fi	Mid-point xi	$yi = \frac{xi - A}{h}$	yi^2	$fiyi$	$fi\, yi^2$
32.5-36.5	15	34.5	-2	4	-30	60
36.5-40.5	17	38.5	-1	1	-17	17
40.5-44.5	21	42.5	0	0	0	0
44.5-48.5	22	46.5	1	1	22	22
48.5-52.5	23	50.5	2	4	50	100
Total	100				25	199

Therefore, mean, $\bar{x} = A + \frac{\sum fi\, yi}{N} \times h$

$$= 42.5 + \frac{25}{100} \times 4$$

$$= 43.5.$$

Variance, $a^2 = \frac{h^2}{N^2}\left[N\sum fi\, yi^2 - \left(\sum fi\, yi\right)^2\right]$

$$= \frac{16}{100 \times 100}\left[100 \times 199 - (25)^2\right]$$

$$= \frac{16}{10000}[19900 - 625]$$

$$= \frac{16 \times 19,275}{10010}$$

$\therefore$ Standard deviation, $a = \sqrt{\frac{16}{10000} \times 19275}$

$$= \frac{4}{100} \times 138.83$$

$$= 5.55$$

Ex. 15.3