Q1. Find the derivative

A.1. Given, $f(x)=x^2-2$., $f'(10)=?$

We have,

$$f'(10)=\lim_{h\to 0}\frac{f(10+h)-f(10)}{h}$$

$$=\lim_{h\to 0}\frac{\left[(10+h)^2-2\right]-\left[10^2-2\right]}{h}$$

$$=\lim_{h\to 0}\frac{10^2+h^2+20h-2-10^2+2}{h}$$

$$=\lim_{h\to 0}\frac{h(h+20)}{h}$$

$$\Rightarrow \lim_{h\to 0} h+20$$

=20

Q2. Find the derivative of x at $x = 1$.

A.2. Given, $f(x)=x, f'(1)=?$

We have,

$$f'(1)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$$

$$=\lim_{h\to 0}\frac{1+h-1}{h}$$

$$=\lim_{h\to 0}\frac{h}{h}$$

$$=\lim_{h\to 0}1$$

=1

Q3. Find the derivative of $99x$ at $x = 100$

A.3. Given, $f(x)= 99x, f'(100)= ?$

So, $f(100)=\lim_{h\to 0}\frac{f(100+h)-f(100)}{h}$

$$=\lim_{h\to 0}\frac{99(100+h)-99(100)}{h}$$

$$=\lim_{h\to 0}\frac{99\times 100+99\times h-99\times 100}{h}$$

$$=\lim_{h\to 0}\frac{99h}{h}$$

$$=\lim_{h\to 0}99$$

=99

Q4. Find the derivative of the following functions from first principle.

(i) x^3-27 (ii) $(x-1)(x-2)$

(iii) $\dfrac{1}{x^2}$ (iv) $\dfrac{x+1}{x-1}$

A.4. (i) Given, $f(x) = x^3 - 27.$

So, $f'(x) = \dfrac{\lim\limits_{h\to 0} f(x+h) - f(x)}{h}$

$$= \lim_{h\to 0} \frac{\left[(x+h)^3 - 27\right] - \left[x^3 - 27\right]}{h}$$

$$= \lim_{h\to 0} \frac{x^3 + h^3 + 3xh(x+h) - 27 - x^3 + 27}{h}$$

$$= \lim_{h\to 0} \frac{h\left(h^2 + 3x(x+h)\right)}{h}$$

$$= \lim_{h\to 0} h^2 + 3x(x+h)$$

$=0+3\,x(x+0)$

$=3x^2$

(ii) Given, $f(x) =(x-1)(x-2)$

$=x^2- 3x+2$

So, $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

$$\lim_{h\to 0} \frac{\left[(x+h)^2 - 3(x+h) + 2\right] - \left[x^2 - 3x + 2\right]}{h}$$

$$= \lim_{h\to 0} \frac{x^2 + h^2 + 2xh - 3x - 3h + 2 - x^2 + 3x - 2}{h}$$

$$= \lim_{h\to 0} \frac{h(h + 2x - 3)}{h}$$

$$= \lim_{h\to 0} h + 2x - 3$$

$= 2x - 3.$

(iii) Given, $f(x)=\dfrac{1}{x^2}$

So, $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

$$= \lim_{h\to 0} \frac{\dfrac{1}{(x+h)^2} - \dfrac{1}{x^2}}{h}$$

$$= \frac{\lim_{h \to 0} \frac{x^2 - (x+h)^2}{(x+h)^2 - x^2}}{h}$$

$$= \lim_{h \to 0} \frac{x^2 - x^2 - h^2 - 2xh}{hx^2 (x+h)^2}$$

$$= \lim_{h \to 0} \frac{h(-h-2x)}{hx^2 (x+h)^2}$$

$$= \lim_{h \to 0} \frac{-h-2x}{x^2 (x+h)^2}$$

$$= \frac{-0-2x}{x^2 (x+0)^2}$$

$$= \frac{-2x}{x^4}$$

$$= \frac{-2}{x^3}$$

(iv) Given, $f(x) = \frac{x+1}{x-1}$

$$f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{1}{h} \left[\frac{fx+h+1}{x+h-1} - \frac{x+1}{x-1} \right]$$

$$= \lim_{h \to 0} \frac{1}{h} \left[\frac{(x+h+1)(x-1) - (x+1)(x+h-1)}{(x+h-1)(x-1)} \right]$$

$$= \lim_{h \to 0} \frac{1}{h} \left[\frac{x^2 - x + hx - h + x - 1 - x^2 - hx + x - x - h + 1}{(x-1)(x+h-1)} \right]$$

$$= \lim_{h \to 0} \frac{1}{h} \left[\frac{-2h}{(x-1)(x+h-1)} \right]$$

$$= \lim_{h \to 0} \frac{-2}{(x-1)(x+h-1)}$$

$$= \frac{-2}{(x-1)(x-1)} = \frac{-2}{(x-1)^2}$$

Q5. For the function

$$f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \cdots + \frac{x^2}{2} + x + 1$$

Prove that $f'(1) = 100 f'(0)$

A.5. Given, $f(x)=\frac{x^{100}}{100}+\frac{x^{99}}{99}+\cdots+\frac{x^2}{2}+x+1.$

$$\lim_{h\to0}\frac{(x+h)^{100}-x^{100}}{100\cdot h}+\lim_{h\to0}\frac{(x+h)^{99}-x^{99}}{99\cdot h}+\ldots+\lim_{x\to0}\frac{(x+h)^2-x^2}{2h}+\lim_{x\to0}x+\lim_{x\to0}1$$

$$=\frac{100\cdot x^{99}}{100}+\frac{99x^{98}}{99}+\cdots+\frac{2x}{2}+0+1$$

$$=x^{99}+x^{98}+\cdots+x+1$$

At x=0,

$f'(x)$ =1.

and at x=1,

$f'(1)\ =1^{99}+1^{98}+\ldots+1^2+1+1$

=100 × 1

=$100\times f'(0)$

Hence, $f'(1)=100f'(0)$

Q6. Find the derivative of $x^n+ax^{n-1}+a^2x^{n-2}+\cdots+a^{n-1}x+a^n$ for some fixed real number a.

A.6. Given, $f(x)=x^n+ax^{n-1}+a^2x^{n-2}+\cdots+a^{n-1}x+a^n$.

We know that,

$$\frac{d}{dx}\left(x^x\right)=nx^{x-1}$$

So,

$$f'(x)=\frac{d}{dx}x^x+\frac{d}{dx}ax^{x-1}+\frac{d}{dx}a^2x^{x-2}+\cdots+\frac{d}{dx}$$

$$a^{x-1}x+\frac{d}{dx}a^x=nx^{n-1}+a(n-1)x^{n-2}+a^2(n-2)x^{n-3}+\cdots+a^{n-1}+0.$$

$\left(\because\frac{dax}{dx}=a\frac{dx}{dx}\right.$ and $\frac{da}{dx}=0$ where a is constant)

Q7. For some constants *a* and *b*, find the derivative of

(i)(*x*- *a*) + (*x* - *b*) **(ii)** $\left(ax^2+b\right)^2$ **(iii)** $\frac{x-9}{x-b}$

A.7. (c) Given, $f(x)=(x-a)(x-b)$

where a and b are constants.

So,

$$f'(x)\ =(x-a)\frac{d}{dx}(x-b)+(x-b)\frac{d}{dx}(x-a)$$

=(x-a)+(x-b)

= 2x– a- b.

(ii) Given $f(x)=\left(ax^2+b\right)^2$. where ab are constant

So, $f^{'}(x)=\frac{d}{dx}\left(ax^2+b\right)^2$

$=\frac{d}{dx}\left(a^2x^4+b^2+2ax^2b\right)$

$=\frac{d}{dx}a^2x^4+\frac{d}{dx}b^2+\frac{d}{dx}2ax^2b$

$=4a^2x^3+0+4axb$

$=4ax\left(ax^2+b\right).$

(iii) Given, $f(x)=\frac{x-9}{x-b}$ where a and $b$are constants

So, $f(x)\ =\frac{(x-b)\frac{d}{dx}(x-a)-(x-a)\frac{d}{dx}(x-b)}{(x-b)^2}$

$=\frac{(x-b)-(x-a)}{(x-b)^2}$

$=\frac{a-b}{(x-b)^2}$

Q8. Find the derivative of $\frac{x^n-a^n}{x-a}$ for some constant a.

A.8. Given, $f(x)=\frac{x^n-a^n}{x-a}$.

So, $f^{'}(x)=\frac{(x-a)\frac{d}{dx}\left(x^n-a^3\right)-\frac{d}{dx}(x-a)\cdot\left(x^n-a^n\right)}{(x-a)^2}=\frac{(x-a)\cdot nx^{n-1}-\left(x^n-a^n\right)}{(x-a)^2}$

$=\frac{(x-a)\cdot nx^{n-1}-\left(x^n-a^n\right)}{(x-a)^2}$

$=\frac{nx^{n-1}\cdot x-nax^{n-1}-x^n+a^n}{(x-a)^2}$

$=\frac{nx^n-x^x-nax^{n-1}+a^n}{(x-a)^2}$

Q9. Find the derivative of

(i) $2x-\frac{3}{4}$ **(ii)** $\left(5x^3+3x-1\right)(x-1)$

(iii) $x^{-3}(5+3x)$ **(iv)** $x^5\left(3-6x^{-9}\right)$

(v) $x^{-4}\left(3-4x^{-5}\right)$ **(vi)** $\frac{2}{x+1}-\frac{x^2}{3x-1}$

A.9. (i) $f(x) = 2x - \frac{3}{4}$

$f'(x) = \frac{d}{dx}\left(2x - \frac{3}{4}\right)$

$= 2\frac{dx}{dx} - 0$

=2.

(ii) Given, $f(x) = \left(5x^3 + 3x - 1\right)(x-1)$

So, $f(x) = \left(5x^3 + 3x - 1\right)\frac{d}{dx}(x-1) + (x-1)\frac{d}{dx}\left(5x^3 + 3x - 1\right)$

$= \left(5x^3 + 3x - 1\right).1 + (x-1)\left(15x^2 + 3\right)$

$= 5x^3 + 3x - 1 + i5x^3 + 3x - 15x^2 - 3$

$= 20x^3 - 15x^2 + 6x - 4.$

(iii) Given, $f(x) = x^{-3}(5+3x)$

So, $f'(x) = x^{-3}\frac{d}{dx}(5+3x) + (5+3x)\frac{dx^{-3}}{dx}$

$= x^{-3}\cdot 3 + (5+3x)\cdot(-3)x^{-4}$

$= \frac{3}{x^3} - \frac{15}{x^4} - \frac{9}{x^3}$

$= \frac{-15}{x^4} - \frac{6}{x^3}$

$= -\frac{3}{x^4}(5+2x).$

(iv) Given, $f(x) = x^5\left(3 - 6x^{-9}\right).$

So, $f'(x) = x^5\frac{d}{dx}\left(3 - 6x^{-9}\right) + \left(3 - 6x^{-9}\right)\frac{d}{dx}x^5.$

$= x^5\left(-6x - 9x^{-10}\right) + \left(3 - 6x^{-9}\right)\cdot 5x^4$

$= x^5\left(54x^{-10}\right) + 15x^4 - 30x^{-5}$

$= 54x^{-5} - 30x^{-5} + 15x^4$

$= \frac{24}{x^5} + 15x^4$

(v) Given, $f(x) = x^{-4}\left(3 - 4x^{-5}\right).$

So, $f'(x) = x^{-4}\cdot\frac{d}{dx}\left(3 - 4x^{-5}\right) + \left(3 - 4x^{-5}\right)\frac{d}{dx}x^{-4}$

$$= x^{-4}\cdot\left(-4x-5\times x^{-6}\right)+\left(3-4x^{-5}\right)\cdot\left(-4x^{-5}\right)$$

$$= 20x^{-10}-12x^{-5}+16x^{-10}.$$

$$= 36x^{-10}-12x^{-5}.$$

$$= \frac{36}{x^{10}}-\frac{12}{x^{5}}.$$

(vi) Given, $f(x)=\dfrac{2}{x+1}-\dfrac{x^2}{3x-1}$

So, $f^{'}(x)=\dfrac{d}{dx}\left(\dfrac{2}{x+1}\right)-\dfrac{d}{dx}\left(\dfrac{x^2}{3x-1}\right)$

$$=\frac{(x+1)\frac{d}{dx}-2\frac{d}{dx}(x+1)}{(x+1)^2}-\frac{(3x-1)\frac{dx^2}{dx}-x^2\frac{d}{dx}(3x-1)}{(3x-1)^2}$$

$$=\frac{-2}{(x+1)^2}-\frac{2x(3x-1)-3x^2}{(3x-1)^2}$$

$$=-\frac{2}{(x+1)^2}-\frac{6x^2-2x-3x^2}{(3x-1)^2}$$

$$=-\frac{2}{(x+1)^2}-\frac{3x^2-2x}{(3x-1)^2}$$

$$=-\frac{2}{(x+1)^2}-\frac{x(3x-2)}{(3x-1)^2}$$

Q10. Find the derivative of cos *x* from first principle.

A.10. Given, $f(x) = \cos x$

So, $f(x+h)=\cos(x+h)$.

By first principle,

$$f^{'}(x) = \lim_{x\to h}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{x\to h}\frac{1}{h}[\cos(x+h)-\cos x]$$

$$=\lim_{x\to h}\frac{1}{h}\left[-2\sin\left(\frac{x+h+x}{2}\right)\sin\left(\frac{x+h-x}{2}\right)\right]^{'}$$

$$=\lim_{x\to h}\frac{1}{h}\left[-2\sin\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)\right]$$

$$=\lim_{x\to h}\frac{1}{h}\left[-2\sin\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)\right]$$

$$= \lim_{x \to h} - \sin\left(\frac{2x+h}{2}\right) \times \lim_{x \to h} \frac{\sin h/2}{h/2}$$

$$= -\sin\left(\frac{2x+0}{2}\right) \times 1 = -\sin x.$$

Q11. **Find the derivative of the following functions:**

(i)sin x cos x (ii) $\sec x$ **(iii)5 sec x + 4 cos$x$**

(iv) $\operatorname{cosec} x$ **(v)3cot x + 5 cosec x**

(vi) $5\sin x - 6\cos x + 7$ **(vii)** $2\tan x - 7\sec x$

A.11. (i) $f(x)$=sin x cos x

So, $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{\sin(x+h)\cos(x+h) - \sin x \cos x}{h}$$

$$= \lim_{h \to 0} \frac{1}{2h} \times [2\sin(x+h)\cos(x+h) - 2\sin x \cos x]$$

$$= \lim_{h \to 0} \frac{1}{2h} [\sin 2(x+h) - \sin 2x]$$

$$= \lim_{h \to 0} \frac{1}{2h} \left[2\cos\frac{2(x+h)+2x}{2} \sin\frac{2(x+h)-2x}{2}\right]$$

$$= \lim_{h \to 0} \frac{1}{h} \left[\cos\left(2x+h\right) \sin h\right]$$

$$= \lim_{h \to 0} \cos(2x+h) \times \lim_{h \to 0} \frac{\sin h}{h}$$

$$= \cos(2x+0)\text{x}1$$

$$= \cos 2x$$

(ii) $f(x) = \sec x.$

So, $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{1}{h} [\sec(x+h) - \sec x]$$

$$= \lim_{h \to 0} \frac{1}{h} \left[\frac{1}{\cos(x+h)} - \frac{1}{\cos x}\right]$$

$$= \lim_{h \to 0} \frac{1}{h} \left[\frac{\cos x - \cos(x+h)}{\cos(x+h)\cos x}\right]$$

$$= \lim_{h \to 0} \frac{1}{h} \left[\frac{-2\sin\left(\frac{x+x+h}{2}\right)\sin\left(\frac{x-(x+h)}{2}\right)}{\cos(x+h)\cos x} \right]$$

$$= \lim_{h \to 0} \frac{1}{h} \left[-\frac{2\sin\left(\frac{2x+h}{2}\right)\sin(-h/2)}{\cos(x+h)\cos x} \right]$$

$$= \lim_{h \to 0} \frac{\left(-1\sin\left(\frac{2x+h}{2}\right)\right)}{\cos(x+h)\cos x} \times \lim_{h \to 0}(-1)\frac{\sin h/2}{h/2}$$

$$= \frac{\sin x}{\cos x \cdot \cos x} \times 1$$

$$= \tan x \cdot \sec x.$$

(iii) Given $f(x)$=5 sec x+4 cos$x$.

So, $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{5\sec(x+h) + 4\cos(x+h) - [5\sec x + 4\cos x]}{h}.$$

$$= \lim_{h \to 0} \frac{5}{h}[\sec(x+h) - \sec x] + \lim_{h \to 0} \frac{4}{h}[\cos(x+h) - \cos x]$$

$$= \lim_{h \to 0} \frac{5}{h}\left[\frac{1}{\cos(x+h)} - \frac{1}{\cos x}\right] + \lim_{h \to 0} \frac{4}{h}\left[-2\sin\left(\frac{x+h+x}{2}\right)\sin\left(\frac{x+h-x}{2}\right)\right]$$

$$= \lim_{h \to 0} \frac{5}{h}\left[\frac{\cos x - \cos(x+h)}{\cos(x+h)(\cos x)}\right] + \lim_{h \to 0} \frac{4}{h}\left[-2\sin\left(\frac{2x+h}{2}\right)\sin\frac{h}{2}\right]$$

$$= \lim_{h \to 0} \frac{5}{h}\left[-\frac{2\sin\left(\frac{2x+h}{2}\right)\sin(-h/2)}{\cos(x+h)\cos x}\right] - 4\lim_{h \to 0}\sin\left(\frac{2x+h}{2}\right)\lim_{h \to 0}\frac{\sin h/2}{h/2}$$

$$= \frac{\sin\left(\frac{2x+0}{2}\right)}{\cos(x+0)\cos x} \times 1 - 4\sin\left(\frac{2x}{2}\right)$$

$$= 5\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} - 4\sin x$$

$$= 5\tan x \cdot \sec x - 4\sin x$$

(iv) Given, $f(x) = \operatorname{cosec} x.$

$$\text{So}, f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{1}{h}[\operatorname{cosec}(x+h) - \operatorname{cosec} x]$$

$$= \lim_{h\to 0}\frac{1}{h}\left[\frac{1}{\sin(x+h)} - \frac{1}{\sin x}\right]$$

$$= \lim_{h\to 0}\frac{1}{h}\left[\frac{\sin x - \sin(x+h)}{\sin(x+h)\sin x}\right]$$

$$= \lim_{h\to 0}\frac{1}{h}\left[\frac{2\cos\left(\frac{x+x+h}{2}\right)\sin\left(\frac{x-(x+h)}{2}\right)]}{\sin(x+h)\sin x}\right]$$

$$= \lim_{h\to 0}\frac{1}{h}\left[\frac{2\cos\left(\frac{x+x+h}{2}\right)\sin\left(\frac{x-(x+h)}{2}\right)}{\sin(x+h)\sin x}\right]$$

$$= \lim_{h\to 0}\frac{1}{h}\left[\frac{2\cos\left(\frac{2x+h}{2}\right)\sin(-h/2)]}{\sin(x+h)\sin x}\right]$$

$$= \lim_{h\to 0}\frac{\cos\left(\frac{2x+h}{2}\right)}{\sin(x+h)\sin x}\times\frac{(-1)\sin(2)}{h/2}\Bigg)$$

$$= \frac{\cos\left(\frac{2x+0}{2}\right)}{\sin(x+0)\sin x}\times(-1)$$

$$= -\frac{\cos x}{\sin x}\times\frac{1}{\sin x}$$

$$= -\cot x\cdot\operatorname{cosec} x$$

(v) Given, $f(x)=3\cot x+5\operatorname{cosec} x$.

$$\text{So, } f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \frac{2}{\cos(x+0)\cos x}\times 1 + 7\sin\left(\frac{2x+0}{2}\right)\times(-1)$$

$$= \lim_{h\to 0} 3\cot(x+h) + 5\operatorname{cosec}(x+h) - [3\cot x + 5\cos x)$$

$$\lim_{h\to 0}\frac{3}{h}[\cot(x+h) - \cot x] + \lim_{h\to 0}$$

$$=\frac{5}{h}[\operatorname{cosec}(x+h)-\operatorname{cosec} x]$$

$$=\lim_{h\to 0}\frac{3}{h}\left[\frac{\cos(x+h)}{\sin(x+h)}-\frac{\cos x}{\sin x}\right]+\lim_{h\to 0}\frac{5}{h}\left[\frac{1}{\sin(x+h)}-\frac{1}{\sin x}\right]$$

$$=\lim_{h\to 0}\frac{3}{h}\left[\frac{\cos(x+h)\sin x-\cos x\sin(x+h)}{\sin(x+h)\sin x}\right]+\lim_{h\to 0}\frac{5}{h}\left[\frac{\sin x-\sin(x+h)}{\sin(x+h)\sin x}\right]$$

$$=\lim_{h\to 0}\frac{3}{h}\left[\frac{\sin(x-(x+h))}{\sin(x+h)\sin x}+\lim_{h\to 0}\frac{5}{h}\left[\frac{2\cos\left(\frac{x+x+h}{2}\right)\sin\left(\frac{x-(x+h)}{2}\right)}{\sin(x+h)\sin x}\right.\right.$$

$$=\lim_{h\to 0}\frac{3}{\sin(x+h)\sin x}\times\lim_{h\to 0}(-1)\frac{\sin h}{h}+\lim_{h\to 0}\frac{5}{h}\left[\frac{2\cos\left(\frac{2x+h}{2}\right)\sin(-h/2)}{\sin(x+h)\sin x}\right.$$

$$=\frac{3}{\sin x\cdot\sin x}\times(-1)+5\lim_{h\to 0}\frac{\cos\left(\frac{2x+h}{2}\right)}{\sin(x+h)\sin x}\times(-1)\lim_{h\to 0}\frac{\sin h/2}{h/2}.$$

$$=-3\operatorname{cosec}^2 x-5\cot x\cdot\operatorname{cosec} x.$$

(vi) Given, $f(x)=5\sin x-6\cos x+7$

So, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$=\lim_{h\to 0}\frac{1}{h}[5\sin(x+h)-6\cos(x+h)+7-5\sin x+6\cos x-7]$$

$$=\lim_{h\to 0}\frac{1}{h}[\sin(x+h)-\sin x]-\lim_{h\to 0}\frac{6}{h}[\cos(x+h)-\cos x]$$

$$=\lim_{h\to 0}\frac{5}{h}\left[2\cos\left(\frac{x+h+x}{2}\right)\sin\left(\frac{x+h-x}{2}\right)\right]-\lim_{h\to 0}\frac{6}{h}\left[-2\sin\left(\frac{x+h+x}{2}\right)\sin\left(\frac{x+h-x}{2}\right)\right]$$

$$=\lim_{h\to 0}5\cdot\cos\left(\frac{2x+h}{2}\right)\times\lim_{h\to 0}\frac{\sin(h/2)}{h/2}+\lim_{h\to 0}\sin\left(\frac{2x+h}{2}\right)\times\lim_{h\to 0}\frac{\sin h/2}{h/2}$$

$$=5\cdot\cos\left(\frac{2x+0}{2}\right)\times 1+6\times\sin\left(\frac{2x+0}{2}\right)\times 1$$

$$=5\cos x+6\sin x.$$

(vii)

Given, $f(x)=2\tan x-7\sec x.$

So, $f(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$=\lim_{h\to 0}\frac{2\tan(x+h)-7\sec(x+h)-(2\tan x-7\sec x)}{h}$$

$$=\lim_{h\to 0}\frac{2}{h}[\tan(x+h)-\tan x]-\lim_{h\to 0}\frac{7}{h}[\sec(x+h)-\sec x]$$

$$=\lim_{h\to 0}\frac{2}{h}\left[\frac{\sin(x+h)}{\cos(x+h)}-\frac{\sin x}{\cos x}\right]-\lim_{h\to 0}\frac{7}{h}\left[\frac{1}{\cos(x+h)}-\frac{1}{x}\right].$$

$$=\lim_{h\to 0}\frac{2}{h}\left[\frac{\sin(x+h)\cos x-\sin x\cos(x+h)}{\cos(x+h)\cos x}\right]-\lim_{h\to 0}\frac{7}{h}\left[\frac{\cos x-\cos(x+h)}{\cos(x+h)\cos x}\right]$$

$$=\lim_{h\to 0}\frac{2}{h}\left[\frac{\sin((x+h)-x)}{\cos(x+h)\cos x}\right]-\lim_{h\to 0}\frac{7}{h}\left[-2\sin\left(\frac{x+h+x}{2}\right)\sin\cos\left(\frac{x-(x+h))}{2}\right]$$

$$=\lim_{h\to 0\cdot h}\frac{2}{\cos(x+h)\cos(x+h)\cos x}\Bigg]+\lim_{h\to 0}\frac{7}{h}\left[\frac{2\sin\left(\frac{2x+h}{2}\right)\sin(-h/2)}{\cos(x+h)\cos x}\right]$$

$$=\lim_{h\to 0}\frac{2}{\cos(x+h)\cos x}\times\lim_{h\to 0}\frac{\sin h}{h}+\lim_{h\to 0}\frac{\sin\left(\frac{2x+h}{2}\right)}{\cos(x+h)\cos x}.(-1)\lim_{h\to 0}\frac{\sin h/2}{h/2}$$

$$=\frac{2}{\cos(x+0)\cos x}\times 1+7\sin\left(\frac{2x+0}{2}\right)\times(-1)$$

$$=2\sec^2 x-7\sec x\tan x$$