Q1. $\lim_{x\to 3} x+3$

A.1. $\lim_{x\to 3} x+3 = 3+3 = 6.$

Q2. $\lim_{x\to\pi}\left(x-\frac{22}{7}\right)$

A.2. $\lim_{x\to\pi}\left(x-\frac{22}{7}\right) = \pi - \frac{22}{7}$

Q3. $\lim_{r\to 1}\pi r^2$

A.3. $\lim_{r\to 1}\pi r^2 = \pi\cdot(1)^2 = \pi$

Q4. $\lim_{x\to 4}\frac{4x+3}{x-2}$

A4. $\lim_{x\to 4}\frac{4x+3}{x-2} = \frac{4\times 4+3}{4-2} = \frac{16+3}{2} = \frac{19}{2}$

Q5. $\lim_{x\to -1}\frac{x^{10}+x^5+1}{x-1}$

A.5. $\lim_{x\to -1}\frac{x^{10}+x^5+1}{x-1} = \frac{(-1)^{10}+(-1)^5+1}{(-1)-1} = \frac{1-1+1}{-2} = -\frac{1}{2}$

Q6. $\lim_{x\to 0}\frac{(x+1)^5-1}{x}$

A.6. $\lim_{x\to 0}\frac{(x+1)^5-1}{x} = \lim_{x\to 0}\frac{x^5+5x^4+10x^3+10x^2+5x+1-1}{x}$

$= \lim_{x\to 0}\frac{x^5+5x^4+10x^3+10x^2+5x}{x}$

$= \lim_{x\to 0} x^4+5x^3+10x^2+10x+5.$

$\Rightarrow (0)^4 + 5(0)^3 + 10(0)^2 + 10(0) + 5$

$= 5.$

Q7. $\lim_{x\to 2}\frac{3x^2-x-10}{x^2-4}$

A.7. $\lim_{x\to 2}\frac{3x^2-x-10}{x^2-4} = \lim_{x\to 2}\frac{3x^2+5x-6x-10}{x^2-4}$

$= \lim_{x\to 2}\frac{x(3x+5)-2(3x+5)}{(x-2)(x+2)}$

$$= \lim_{x \to 2} \frac{(x-2)(3x+5)}{(x-2)(x+2)}$$

$$= \lim_{x \to 2} \frac{3x+5}{x+2}$$

$$= \frac{3 \times 2 + 5}{2+2}$$

$$= \frac{6+5}{4}$$

$$= \frac{11}{4}.$$

Q8. $\lim_{x \to 3} \frac{x^4 - 81}{2x^2 - 5x - 3}$

A.8. $\lim_{x \to 3} \frac{x^4 - 81}{2x^2 - 5x - 3} = \frac{\left(x^2\right)^2 - \left(3^2\right)}{2x^2 - 6x + x - 3} = \frac{\left(x^2 - 3^2\right)\left(x^2 + 3^2\right)}{2x(x-3) + (x-3)}$

$$= \lim_{x \to 3} \frac{(x-3)(x+3)\left(x^2+9\right)}{(x-3)(2x+1)}$$

$$= \lim_{x \to 3} \frac{(x+3)(x^2+9)}{2x+1.}$$

$$= \frac{(3+3)\left(3^2+9\right)}{2 \times 3 + 1}$$

$$= \frac{6 \times 18}{6+1}$$

$$= \frac{108}{7}$$

Q9. $\text{Lim}_{x \to 0} \frac{ax+b}{cx+1}$

A.9. $\text{Lim}_{x \to 0} \frac{ax+b}{cx+1} = \frac{a \times 0 + b}{c \times 0 + 1} = \frac{b}{1} = b$

Q.10 $\lim_{z \to 1} \frac{z^{1/3} - 1}{z^{1/6} - 1}$

A.10. $\lim_{z \to 1} \frac{z^{1/3} - 1}{z^{1/6} - 1} = \lim_{z \to 1} \left[\frac{z^{1/3} - 1^{1/3}}{z - 1} \div \frac{z^{1/6} - 1^{1/6}}{z^{-1}} \right]$

$$= \lim_{z \to 1} \left[\frac{z^{1/3} - 1^{1/3}}{z-1} \right] \div \lim_{z \to 1} \left[\frac{z^{1/6} - 1^{1/6}}{z-1} \right]$$

We know that,

$$\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$$

So,

$$\lim_{z \to 1} \frac{z^{1/3} - 1}{z^{1/6} - 1} = \frac{1}{3}(1)^{1/3 - 1} \div \frac{1}{6}(1)^{1/6 - 1}$$

$$= \frac{1}{3} \times 6 = 2$$

Q11. $\lim_{x \to 1} \frac{ax^2 + bx + c}{cx^2 + bx + a} = a + b + c \neq 0$

A.11. $\lim_{x \to 1} \frac{ax^2 + bx + c}{cx^2 + bx + a} = \frac{a + b + c}{c + b + a} = 1$

Q12. $\lim_{x \to -2} \frac{\frac{1}{x} + \frac{1}{2}}{x + 2}$

A.12. $\lim_{x \to -2} \frac{1/x + 1/2}{x + 2} \lim_{x \to -2} \left(\frac{2 + x}{2x} \right) \times \frac{1}{x + 2} = \lim_{x \to 2} \frac{1}{2x}$

$$= \frac{1}{2 \times (-2)} = -\frac{1}{4}.$$

Q13. $\lim_{x \to 0} \frac{\sin ax}{bx}$

A.13. $\lim_{x \to 0} \frac{\sin ax}{bx} = \frac{1}{b} \lim_{x \to 0} \frac{\sin ax}{ax} \times a$

$$= \frac{a}{b} \lim_{x \to 0} \frac{\sin ax}{ax} \qquad \left(\because \lim_{x \to 0} \frac{\sin x}{x} = 1 \right)$$

$$= \frac{a}{b}$$

Q14. $\lim_{x \to 0} \frac{\sin ax}{\sin bx}, a, b \neq 0$

A.14. $\lim_{x \to 0} \frac{\sin ax}{\sin bx} = \lim_{x \to 0} \frac{\frac{\sin ax}{ax} \times ax}{\frac{\sin bx}{bx} \times bx}$

$$= \frac{ax \times \lim_{x \to 0} \frac{\sin ax}{ax}}{bx \times \lim_{x \to 0} \frac{\sin bx}{bx}}$$

$$= \frac{ax}{bx} \qquad \left(\because \sin_{x \to 0} \frac{\sin x}{x} = 1 \right)$$

$= \frac{a}{b}$

Q15. $\lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$

A15. $\lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)} = \frac{1}{\pi} \times \lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi - x}$

$= \frac{1}{\pi}.$

Q16. $\lim_{x \to 0} \frac{\cos x}{\pi - x}$

A.16. $\lim_{x \to 0} \frac{\cos x}{\pi - x} = \frac{\cos 0}{\pi - 0} = \frac{1}{\pi}$

Q17. $\lim_{x \to 0} \frac{\cos 2x - 1}{\cos x - 1}$

A17. $\lim_{x \to 0} \frac{\cos 2x - 1}{\cos x - 1} = \lim_{x \to 0} \frac{\left(1 - 2\sin^2 x\right) - 1}{\left(1 - 2\sin^2 \frac{x}{2}\right) - 1} \quad \left(\begin{array}{l} \because \cos 2x = 1 - \sin^2 x \\ \cos x = 1 - \sin^2 {}^{x}\!/_{2} \end{array}\right)$

$= \lim_{x \to 0} \frac{2\sin^2 x}{2\sin^2 {}^{x}\!/_{2}}$

$= \frac{\lim_{x \to 0} \left(\frac{\sin x}{x}\right)^2 \times x^2}{\lim_{x \to 0} \left(\frac{\sin {}^{x}\!/_{2}}{{}^{x}\!/_{2}}\right)^2 {}^{x}\!/_{2}{}^{2}}$

$= \frac{(1)^2 \times x^2 \times 4}{(1)^2 \times x^2}$

$= 4$

Q18. $\lim_{x \to 0} \frac{ax + x\cos x}{b\sin x}$

A18. $\lim_{x \to 0} \frac{ax + x\cos x}{b\sin x} = \lim_{x \to 0} \frac{x(a + \cos x)}{b\sin x}$

$= \frac{1}{b} \frac{\lim_{x \to 0}(a + \cos x)}{\lim_{x \to 0} \frac{\sin x}{x}}$

$= \frac{1}{b} \times \frac{(a + \cos 0)}{1}$

$= \frac{a+1}{b}$

Q19. $\lim_{x\to 0} x \sec x$

A.19. $\lim_{x\to 0} x \sec x = \lim_{x\to 0} \frac{x}{\cos x}$

$= \frac{0}{\cos 0}$

$= \frac{0}{1}$

= 0.

Q20. $\lim_{x\to 0} \frac{\sin ax + bx}{ax + \sin bx} a, b, a+b \neq 0$

A.20. $\lim_{x\to 0} \frac{\sin ax + bx}{ax + \sin bx} = \frac{\lim_{x\to 0} \frac{\sin ax}{ax} \times ax + bx}{\lim_{x\to 0} ax + \frac{\sin bx}{bx} \times bx}.$

$\frac{\lim_{x\to 0} \frac{\sin ax}{ax} \times \lim_{x\to 0} ax + \lim_{x\to 0} bx}{\lim_{x\to 0} ax + \lim_{x\to 0} \frac{\sin bx}{bx} \times \lim_{x\to 0} bx}$

$= \frac{\lim_{x\to 0} ax + \lim_{x\to 0} bx}{\lim_{x\to 0} ax + \lim_{x\to 0} bx}$

$= \lim_{x\to 0} \frac{ax+bx}{ax+bx}$

$= \lim_{x\to 0} 1$

= 1.

Q21. $\lim_{x\to 0} (\operatorname{cosec} x - \cot x)$

A.21. $\lim_{x\to 0} (\operatorname{cosec} x - \cot x) = \lim_{x\to 0} \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x} \right)$

$= \lim_{x\to 0} \left(\frac{1 - \cos x}{\sin x} \right)$

$= \lim_{x\to 0} \frac{2 \sin^2 x/2}{2 \sin x/2 \cos x/2}$

$$\left\{ \begin{array}{l} \because \cos 2x = 1 - 2\sin^2 x \\ \Rightarrow 1 - \cos 2x = 2\sin^2 x \\ \text{and } 1 - \cos x = 2\sin^2 x/2 \\ \because \sin 2x = 2\sin x \cos x \\ \sin x = 2 \sin x/2 \cos x/2 \end{array} \right.$$

$$= \lim_{x\to 0} \frac{\sin x/2}{\cos x/2}$$

$$= \lim_{x\to 0} \tan x/2$$

$$= \tan 0/2 = 0.$$

Q.22. $\lim_{x\to\frac{\pi}{2}} \frac{\tan 2x}{x-\frac{\pi}{2}}$

A.22. $\lim_{x\to x/2} \frac{\tan 2x}{x-\pi/2}$

Put $y = x-\pi/2$. So that as $y\to 0$ cos $\to \pi/2$

Then,

$$\lim_{x\to\pi/2} \frac{\tan 2x}{x-\pi/2} = \lim_{y\to 0} \frac{\tan 2\left(y+\pi/2\right)}{y}$$

$$= \lim_{y\to 0} \frac{\tan(\pi+2y)}{y} = \lim_{y\to 0} \frac{\tan 2y}{y}$$

$$= \lim_{y\to 0} \frac{\sin 2y}{y\cdot\cos 2y} = \lim_{y\to 0} \frac{\sin 2y}{2y} \times 2 \times \lim_{y\to 0} \frac{1}{\cos 2y}$$

$$= 1\times 2\times \frac{1}{1}$$

$$= 2.$$

Q23. Find $\lim_{x\to 0} f(x)$ and $\lim_{x\to 1} f(x)$, where $f(x) \begin{Bmatrix} 2x+3, & x\le 0 \\ 3(x+1), & x>0 \end{Bmatrix}$

A.23. Given $f(x) \begin{Bmatrix} 2x+3, & x\le 0 \\ 3(x+1), & x>0 \end{Bmatrix}$

for $\lim_{x\to 0} f(x)$,

left hand limit, L.H.S = $\lim_{x\to 0} f(x) = \lim_{x\to 0}(2x+3)$

$= 2 \times 0 + 3 = 3.$

Right hand limit, R.H.L = $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 3(x+1)$

$= (0 + 1) = 3 \times 1 = 3.$

Thus, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = 3$

For $\lim_{x\to 1} f(x)$,

L.H.L = $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 3(x+1) = 3(1+1) = 3\times 2 = 6$

R.H.L = $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 3(x+1) = 3(1+1) = 3\times 2 = 6.$

Thus, $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = \lim_{x\to 1} f(x) = 6.$

Q24. Find $\lim_{x\to 1} f(x)$, where $f(x) = \begin{cases} x^2 - 1, & x \le 1 \\ -x^2 - 1, & x > 1 \end{cases}$

A.24. Given $f(x) = \begin{cases} x^2 - 1, x \le 1 \\ -x^2 - 1, x > 1 \end{cases}, \lim_{x\to 1} f(x) = ?$

Now, L.H.L = $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \left(x^2 - 1\right)$

$1^2 - 1$

$= 0$

And R.H.L = $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \left(-x^2 - 1\right) - (1)^2 - 1 = -1 - 1 = -2.$

Thus, $\lim_{x\to 1^-} f(x) \ne \lim_{x\to 1^+} f(x)$

So, $\lim_{x\to 1} f(x)$ does not exist.

Q25. Evaluate $\lim_{x\to 0} f(x)$, where $f(x) = \begin{cases} \frac{|x|}{x}, x \ne 0 \\ 0, \; x = 0 \end{cases}$

A.25. Given $f(x) = \begin{cases} \frac{|x|}{x}, x \ne 0 \\ 0, \; x = 0 \end{cases}, \lim_{x\to 0} f(x) = ?$

We know that, $|x| = \begin{cases} x, & x \ge 0 \\ -x, & x < 0 \end{cases}$

Now,

L.H.L = $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{-x}{x} = \lim_{x\to 0^-} -1 = -1$

and R.H.L = $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{x}{x} = \lim_{x\to 0^+} 1 = 1$

Thus, $\lim_{x\to 0^-} f(x) \ne \lim_{x\to 0^+} f(x)$

i e, $\lim_{x\to 0} f(x)$ does not exist.

Q26. Find $\lim_{x\to 0} f(x)$, where $f(x) \begin{cases} \frac{x}{|x|}, & x \ne 0 \\ 0, & x = 0 \end{cases}$

A.26. Given, $f(x)\begin{cases}\dfrac{x}{|x|}, & x\neq 0\\ 0, & x=0\end{cases}$

As $|x|=\begin{cases}-x, x<0\\ x, x\geq 0.\end{cases}$

L.H.S = $\lim_{x\to 0^-} f(x)=\lim_{x\to 0^-}\dfrac{x}{-x}=\lim_{x\to 0^-}-1=-1$

R.H.L $\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+}\dfrac{x}{x}=\lim_{x\to 0^+}1=1$

Thus, $\lim_{x\to 0^-} f(x)\neq\lim_{x\to 0^+} f(x)$

i e, $\lim_{x\to 0} f(x)$ does not exist.

Q27. Find $\lim_{x\to 5} f(x)$, where $f(x)=|x|-5$

A27. $\lim_{x\to 5} f(x)=\lim_{x\to 5}|x|-5$

$= |5| - 5$

$= 5 - 5$

$= 0$

Q28. Suppose $f(x)=\begin{cases}a+bx, & x<1\\ 4, & x=1\\ b-ax, & x>1\end{cases}$ and if $\lim_{x\to 1} f(x) = f(1)$ what are possible values of a and b?

A.28. Given, $f(x)=\begin{cases}a+bx, & x<1\\ 4, & x=1\\ b-ax, & x>1\end{cases}$

Since we need $\lim_{x\to 1} f(x)$ we need,

LHL =

$\lim_{x\to 1^-} f(x)=\lim_{x\to 1^-}(a+bx) = a + b\times 1 = a + b$

and RHL =

$\lim_{x\to 1^+} f(x)=\lim_{x\to 1^+}(b-ax) = b-a\times 1 = b-a$

Given, $\lim_{x\to 1} f(x)=f(1)$: we have the following equations

$a + b = 4$ ____ (1)

$b-a = 4$ ____ (2)

Adding (1) and (2) we get,

$2b = 8$

$\Rightarrow \boxed{b = 4}$

Putting $b = 4$ in (1) we get

$a + 4 = 4$

$\Rightarrow \boxed{a = 0}$

Q29. Let $a_1, a_2, \ldots, a_n$ be fixed real numbers and define a function

$f(x) = (x - a_1)(x - a_2)\ldots(x - a_n)$ .What is $\lim_{x \to a_1} f(x)$? For some $a_1, a_1, a_2, \ldots, a_n$, compute $\lim_{x \to a} f(x)$

A.29. Given, $f(x) = (x - a_1)(x - a_2)\ldots(x - a_n)$.

So, $\lim_{x \to a_1} f(x) = \lim_{x \to a_1}(x - a_1) \lim_{x \to a_1}(x - a_2) \cdots \lim_{x \to a_1}(x - a^n)$

$= (a_1 - a_1)(a_1 - a_2)\ldots(a_1 - a_n)$

$= 0\,(a_1 - a_2)\ldots(a_1 - a_n)$

$= 0$

And $\lim_{x \to a} f(x) = \lim_{x \to a}(x - a_1) \lim_{x \to a}(x - a_2) \ldots \lim_{x \to a}(x - an)$

$= (a - a_1)(a - a_2)\ldots(a - a_n)$

Q30. If $f(x) = \begin{cases} |x| + 1, & x < 0 \\ 0, & x = 0 \\ |x| - 1, & x > 0 \end{cases}$ For what value (s) of a does $\lim_{x \to a} f(x)$ exists?

A.30. Given, $f(x) = \begin{cases} |x| + 1, & x < 0 \\ 0, & x = 0 \\ |x| - 1, & x > 0 \end{cases}$ and $\lim_{x \to a} f(x) = ?$

As $|x| = \begin{cases} -x, & x < 0 \\ x, & x > 0 \end{cases}$

We can rewrite $f(x) = \begin{cases} -x + 1, & x < 0 \\ 0, & x = 0 \\ x - 1, & x > 0. \end{cases}$

<u>Case</u> 1: when $a<0$,

$\lim_{x \to a} f(x) = \lim_{x \to a}(-x + 1) = -a + 1$

So, $\lim_{x \to a} f(x)$ = exist such that $a < 0$

<u>Case II</u> when $a > 0$,

$\lim_{x \to a} f(x) = \lim_{x \to a}(x - 1) = a - 1$

So, $\lim_{x \to a} f(x)$ exist such that $a>0$.

<u>Case III</u> when $a = 0$.

L.H.L $= \lim_{x \to a^-} f(x) = \lim_{x \to a^-}(-x + 1) = \lim_{x \to 0^-}(-x + 1) = 1$

R.H.L $= \lim_{x \to a^+} f(x) = \lim_{x \to a^+}(x - 1) = \lim_{x \to 0^+}(x - 1) = -1$

Thus, $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$

So, $\lim_{x\to a} f(x)$ does not exist at $a = 0$.

Q31. If the function *f(x)* satisfies $\lim_{x\to 1}\dfrac{f(x)-2}{x^2-1} = \pi$, evaluate $\lim_{x\to 1} f(x)$.

A.31. Given, $\lim_{x\to 1}\dfrac{f(x)-2}{x^2-1} = \pi$

$$\Rightarrow \frac{\lim_{x\to 1} f(x)-2}{\lim_{x\to 1} x^2-1} = \pi$$

$$\Rightarrow \lim_{x\to 1}[f(x)-2] = \pi \lim_{x\to 1}\left(x^2-1\right)$$

$$\Rightarrow \lim_{x\to 1} f(x) - \lim_{x\to 1} 2 = \pi\left(1^2-1\right)$$

$$\Rightarrow \lim_{x\to 1} f(x) - 2 = 0.$$

$$\Rightarrow \lim_{x\to 1} f(x) = 2$$

Q32. If $f(x) = \begin{cases} mx^2+n, & x<0 \\ nx+m, & 0\le x\le 1 \\ nx^3+m, & x>1 \end{cases}$. For what integers m and n does both $\lim_{x\to 0} f(x)$ and $\lim_{x\to 1} f(x)$ exist?

A.32. Given, $f(x) = \begin{cases} mx^2+n, & x<0 \\ nx+m, & 0\le x\le 1 \end{cases}$

$nx^3+m,\ x>1.$

For $\lim_{x\to 0} f(x)$ to exist $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$.

$$\Rightarrow \lim_{x\to 0^-}\left(mx^2+n\right) = \lim_{x\to 0^+}\left(mx^3+m\right)$$

$\Rightarrow n = m$

So, $\lim_{x\to 0} f(x)$ exist for $n = m$.

Again, $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} nx+m = n+m.$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} nx^3+m = n+m$$

So, $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x) = n+m$, Thus, $\lim_{x\to 1} f(x)$ exist for any integral value of m and n.