Question 1: Exercise 11.2

Find the distance between the following pairs of points:

(i) $(2,3,5)$ and $(4,3,1)$

(ii) $(-3,7,2)$ and $(2,4,-1)$

(iii) $(-1,3,-4)$ and $(1,-3,4)$

(iv) $(2,-1,3)$ and $(-2,1,3)$

Solution:

(i) $(2,3,5)$ and $(4,3,1)$

Let P be $(2,3,5)$ and Q be $(4,3,1)$

By using the formula,

Distance $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

So here,

$x_1=2,\ y_1=3,\ z_1=5$

$x_2=4,\ y_2=3,\ z_2=1$

$$PQ=\sqrt{(4-2)^2+(3-3)^2+(1-5)^2}$$
$$=\sqrt{2^2+0^2+(-4)^2}$$
$$=\sqrt{4+16}$$
$$=\sqrt{20}$$
$$=2\sqrt{5}$$

Therefore, the required distance is $2\sqrt{5}$ units.

(ii) $(-3,7,2)$ and $(2,4,-1)$

Let P be $(-3,7,2)$ and Q be $(2,4,-1)$

By using the formula,

Distance $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

So here,

$x_1=-3,\ y_1=7,\ z_1=2$

$x_2=2,\ y_2=4,\ z_2=-1$

$$PQ = \sqrt{\left(2-(-3)\right)^2 + (4-7)^2 + (-1-2)^2}$$
$$= \sqrt{5^2 + (-3)^2 + (-3)^2}$$
$$= \sqrt{25+9+9}$$
$$= \sqrt{43}$$

Therefore, the required distance is $\sqrt{43}$ units.

(iii) $(2,-1,3)$ and $(-2,1,3)$

Let P be $(-1,3,-4)$ and Q be $(1,-3,4)$

By using the formula,

Distance $= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

So here,

$x_1 = -1,\ y_1 = 3,\ z_1 = -4$

$x_2 = 1,\ y_2 = -3,\ z_2 = 4$

$$PQ = \sqrt{\left(1-(-1)\right)^2 + (-3-3)^2 + \left(4-(-4)\right)^2}$$
$$= \sqrt{2^2 + (-6)^2 + 8^2}$$
$$= \sqrt{4+36+64}$$
$$= \sqrt{104}$$
$$= 2\sqrt{26}$$

Therefore, the required distance is $2\sqrt{26}$ units.

(iv) $(2,-1,3)$ and $(-2,1,3)$

Let P be $(2,-1,3)$ and Q be $(-2,1,3)$

By using the formula,

Distance $= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

So here,

$x_1 = 2,\ y_1 = -1,\ z_1 = 3$

$x_2 = -2,\ y_2 = 1,\ z_2 = 3$

$$PQ=\sqrt{(-2-2)^2+(1-(-1))^2+(3-3)^2}$$
$$=\sqrt{(-4)^2+2^2+0^2}$$
$$=\sqrt{16+4}$$
$$=\sqrt{20}$$
$$=2\sqrt{5}$$

Therefore, the required distance is $2\sqrt{5}$ units.

Question 2:

Show that the points $(-2,3,5),(1,2,3)$ and $(7,0,-1)$ are collinear.

Solution:

If three points are collinear, then they lie on a line.
Firstly, let us calculate distance between the 3 points i.e. PQ, QR and PR.

Calculating PQ:

$P=(-2,3,5)$ and $Q=(1,2,3)$
By using the formula,

Distance $PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$
So here,
$x_1=-2,\ y_1=3,\ z_1=5$
$x_2=1,\ y_2=2,\ z_2=3$

$$PQ=\sqrt{\left[(1-(-2))^2+(2-3)^2+(3-5)^2\right]}$$
$$=\sqrt{(3)^2+(-1)^2+(-2)^2}$$
$$=\sqrt{9+1+4}$$
$$=\sqrt{14}$$

Calculating QR:

$Q=(1,2,3)$ and $R=(7,0,-1)$
By using the formula,

Distance $QR=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$
So here,

$x_1 = 1, y_1 = 2, z_1 = 3$
$x_2 = 7, y_2 = 0, z_2 = -1$

$$QR = \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2}$$
$$= \sqrt{6^2 + (-2)^2 + (-4)^2}$$
$$= \sqrt{36+4+16}$$
$$= \sqrt{56}$$
$$= 2\sqrt{14}$$

<u>Calculating PR</u>:
$P = (-2,3,5)$ and $R = (7,0,-1)$
By using the formula,

Distance $PR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
So here,
$x_1 = -2, y_1 = 3, z_1 = 5$
$x_2 = 7, y_2 = 0, z_2 = -1$

$$PR = \sqrt{(7-(-2))^2 + (0-3)^2 + (-1-5)^2}$$
$$= \sqrt{9^2 + (-3)^2 + (-6)^2}$$
$$= \sqrt{81+9+36}$$
$$= \sqrt{126}$$
$$= 3\sqrt{14}$$

Thus, $PQ = \sqrt{14}$, $QR = 2\sqrt{14}$ and $PR = 3\sqrt{14}$
So,
$$PQ + QR = \sqrt{14} + 2\sqrt{14}$$
$$= 3\sqrt{14}$$
$$= PR$$

Therefore, the points P, Q and R are collinear.

Question 3:
Verify the following:
(i) $(0,7,-10),(1,6,-6)$ and $(4,9,-6)$ are the vertices of an isosceles triangle.
(ii) $(0,7,10),(-1,6,6)$ and $(-4,9,6)$ are the vertices of a right angled triangle.

(iii) $(-1,2,1),(1,-2,5),(4,-7,8)$ and $(2,-3,4)$ are the vertices of a parallelogram.

Solution:

(i) $(0,7,-10),(1,6,-6)$ and $(4,9,-6)$ are the vertices of an isosceles triangle.

Let us consider the points be $P(0,7,-10), Q(1,6,-6)$ and $R(4,9,-6)$
If any 2 sides are equal, hence it will be an isosceles triangle.
So, firstly let us calculate the length of the sides.

Calculating PQ:

$P(0,7,-10)$ and $Q(1,6,-6)$
By using the formula,

Distance $PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$
So here,

$x_1 = 0,\ y_1 = 7,\ z_1 = -10$
$x_2 = 1,\ y_2 = 6,\ z_2 = -6$

$$\begin{aligned} PQ &= \sqrt{(1-0)^2+(6-7)^2+(-6-(-10))^2} \\ &= \sqrt{1^2+(-1)^2+4^2} \\ &= \sqrt{1+1+16} \\ &= \sqrt{18} \end{aligned}$$

Calculating QR:

$Q(1,6,-6)$ and $R(4,9,-6)$
By using the formula,

Distance $QR = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$
So here,

$x_1 = 1,\ y_1 = 6,\ z_1 = -6$
$x_2 = 4,\ y_2 = 9,\ z_2 = -6$

$$\begin{aligned} QR &= \sqrt{(4-1)^2+(9-6)^2+(-6-(-6))^2} \\ &= \sqrt{3^2+3^2+0^2} \\ &= \sqrt{9+9} \\ &= \sqrt{18} \end{aligned}$$

Here, $PQ = QR = \sqrt{18}$

Since, two sides are equal, ΔPQR is an isosceles triangle.

Thus, $(0,7,-10),(1,6,-6)$ and $(4,9,-6)$ are the vertices of an isosceles triangle.

(ii) $(0,7,10),(-1,6,6)$ and $(-4,9,6)$ are the vertices of a right angled triangle.

Let the points be $P(0,7,10), Q(-1,6,6)$ and $R(-4,9,6)$
Firstly, let us calculate the length of sides PQ, QR and PR.

Calculating PQ:

$P(0,7,10)$ and $Q(-1,6,6)$
By using the formula,

Distance $PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$
So here,
$x_1=0, y_1=7, z_1=10$
$x_2=-1, y_2=6, z_2=6$

$$\begin{aligned}PQ&=\sqrt{(-1-0)^2+(6-7)^2+(6-10)^2}\\&=\sqrt{(-1)^2+(-1)^2+(-4)^2}\\&=\sqrt{1+1+16}\\&=\sqrt{18}\end{aligned}$$

Calculating QR:

$Q(-1,6,6)$ and $R(-4,9,6)$
By using the formula,

Distance $QR=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

So here,
$x_1=-1, y_1=6, z_1=6$
$x_2=-4, y_2=9, z_2=6$

$$\begin{aligned}QR&=\sqrt{(-4-(-1))^2+(9-6)^2+(6-6)^2}\\&=\sqrt{(-3)^2+3^2+0^2}\\&=\sqrt{9+9}\\&=\sqrt{18}\end{aligned}$$

Calculating PR:

$P(0,7,10)$ and $R(-4,9,6)$
By using the formula,

Distance $PR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
So here,

$x_1 = 0, y_1 = 7, z_1 = 10$

$x_2 = -4, y_2 = 9, z_2 = 6$

$$\begin{aligned} PR &= \sqrt{(-4-0)^2 + (9-7)^2 + (6-10)^2} \\ &= \sqrt{(-4)^2 + 2^2 + (-4)^2} \\ &= \sqrt{16+4+16} \\ &= \sqrt{36} \end{aligned}$$

Now,

$$\begin{aligned} PQ^2 + QR^2 &= 18+18 \\ &= 36 \\ &= PR^2 \end{aligned}$$

By using Converse of Pythagoras theorem,
The given vertices P, Q and R are the vertices of a right-angled triangle at Q.

Thus, $(0,7,10), (-1,6,6)$ and $(-4,9,6)$ are the vertices of a right angled triangle.

(iii) $(-1,2,1), (1,-2,5), (4,-7,8)$ and $(2,-3,4)$ are the vertices of a parallelogram.

Let the points be $A(-1,2,1), B(1,-2,5), C(4,-7,8)$ and $D(2,-3,4)$
if pairs of opposite sides are equal then only ABCD can be a parallelogram.
i.e., $AB = CD$ and $BC = AD$.

Firstly, let us calculate the lengths of the sides

Calculating AB:

$A(-1,2,1)$ and $B(1,-2,5)$
By using the distance formula,

Distance $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
So here,

$x_1 = -1, y_1 = 2, z_1 = 1$

$x_2 = 1, y_2 = -2, z_2 = 5$

$$AB = \sqrt{(1-(-1))^2 + (-2-2)^2 + (5-1)^2}$$
$$= \sqrt{2^2 + (-4)^2 + 4^2}$$
$$= \sqrt{4+16+16}$$
$$= \sqrt{36}$$
$$= 6$$

Calculating BC:

$B(1,-2,5)$ and $C(4,-7,8)$

By using the distance formula,

Distance $BC = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

So here,

$x_1 = 1, y_1 = -2, z_1 = 5$

$x_2 = 4, y_2 = -7, z_2 = 8$

$$BC = \sqrt{(4-1)^2 + (-7-(-2))^2 + (8-5)^2}$$
$$= \sqrt{3^2 + (-5)^2 + 3^2}$$
$$= \sqrt{9+25+9}$$
$$= \sqrt{43}$$

Calculating CD:

$C(4,-7,8)$ and $D(2,-3,4)$

By using the distance formula,

Distance $CD = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

So here,

$x_1 = 4, y_1 = -7, z_1 = 8$

$x_2 = 2, y_2 = -3, z_2 = 4$

$$CD = \sqrt{\left[(2-4)^2 + (-3-(-7))^2 + (4-8)^2\right]}$$
$$= \sqrt{(-2)^2 + 4^2 + (-4)^2}$$
$$= \sqrt{4+16+16}$$
$$= \sqrt{36}$$
$$= 6$$

Calculating DA:

$D(2,-3,4)$ and $A(-1,2,1)$

By using the formula,

Distance $DA=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

So here,

$x_1=2,\ y_1=-3,\ z_1=4$

$x_2=-1,\ y_2=2,\ z_2=1$

$$\begin{aligned}DA&=\sqrt{(-1-2)^2+(2-(-3))^2+(1-4)^2}\\&=\sqrt{(-3)^2+5^2+(-3)^2}\\&=\sqrt{9+25+9}\\&=\sqrt{43}\end{aligned}$$

Since, in quadrilateral ABCD both the pairs of opposite sides are equal i.e., $AB=CD$ and $BC=AD$, ABCD is a parallelogram.

Thus, $(-1,2,1),(1,-2,5),(4,-7,8)$ and $(2,-3,4)$ are the vertices of a parallelogram.

Question 4:

Find the equation of the set of points which are equidistant from the points $(1,2,3)$ and $(3,2,-1)$.

Solution:

Let $A(1,2,3)$ and $B(3,2,-1)$

Let point P be (x,y,z)

Since it is given that point $P(x,y,z)$ is equidistant from the points $A(1,2,3)$ and $B(3,2,-1)$ i.e., $PA=PB$

Firstly, let us calculate distances PA and PB

Calculating PA:

$P(x,y,z)$ and $A(1,2,3)$

By using the distance formula,

Distance $PA=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

So here,

$x_1=x,\ y_1=y,\ z_1=z$

$x_2=1,\ y_2=2,\ z_2=3$

$$PA=\sqrt{(1-x)^2+(2-y)^2+(3-z)^2}$$

<u>Calculating PB</u>:

$P(x,y,z)$ and $B(3,2,-1)$

By using the distance formula,

Distance $PB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

So here,

$x_1=x,\ y_1=y,\ z_1=z$

$x_2=3,\ y_2=2,\ z_2=-1$

$$PB=\sqrt{(3-x)^2+(2-y)^2+(-1-z)^2}$$

Since, $PA=PB$

On squaring both the sides, we get

$$PA^2=PB^2$$

Therefore,

$$(1-x)^2+(2-y)^2+(3-z)^2=(3-x)^2+(2-y)^2+(-1-z)^2$$
$$(1+x^2-2x)+(4+y^2-4y)+(9+z^2-6z)=(9+x^2-6x)+(4+y^2-4y)+(1+z^2+2z)$$
$$-2x-4y-6z+14=-6x-4y+2z+14$$
$$4x-8z=0$$
$$x-2z=0$$

Thus, the required equation is $x-2z=0$

Question 5:

Find the equation of the set of points P, the sum of whose distances from $A(4,0,0)$ and $B(-4,0,0)$ is equal to 10.

Solution:

Let $A(4,0,0)$ and $B(-4,0,0)$

Let the coordinates of point P be (x,y,z)

<u>Calculating PA</u>:

$P(x,y,z)$ and $A(4,0,0)$

By using the distance formula,

Distance $PA=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

So here,

$x_1=x, y_1=y, z_1=z$

$x_2=4, y_2=0, z_2=0$

Distance $PA=\sqrt{(4-x)^2+(0-y)^2+(0-z)^2}$

<u>Calculating PB</u>:

$P(x,y,z)$ and $B(-4,0,0)$

By using the distance formula,

Distance $PB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

So here,

$x_1=x, y_1=y, z_1=z$

$x_2=-4, y_2=0, z_2=0$

Distance $PB=\sqrt{(-4-x)^2+(0-y)^2+(0-z)^2}$

Now it is given that

$$PA+PB=10$$
$$PA=10-PB$$

On squaring both the sides, we get

$$PA^2=(10-PB)^2$$
$$PA^2=100+PB^2-20PB$$

Therefore,

$$(4-x)^2+(0-y)^2+(0-z)^2=100+(-4-x)^2+(0-y)^2+(0-z)^2-20PB$$
$$(16+x^2-8x)+y^2+z^2=100+(16+x^2+8x)+y^2+z^2-20PB$$
$$20PB=16x+100$$
$$5PB=(4x+25)$$

On squaring both the sides again, we get

$$25PB^2=16x^2+200x+625$$
$$25\left[(-4-x)^2+(0-y)^2+(0-z)^2\right]=16x^2+200x+625$$
$$25\left[x^2+y^2+z^2+8x+16\right]=16x^2+200x+625$$
$$25x^2+25y^2+25z^2+200x+400=16x^2+200x+625$$
$$9x^2+25y^2+25z^2-225=0$$

Thus, the required equation is $9x^2+25y^2+25z^2-225=0$.