Question 1:

Exercise 10.3

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$

Solution:

The given equation is $\frac{x^2}{36}+\frac{y^2}{16}=1$

Here, the denominator of $\frac{x^2}{36}$ is greater than the denominator of $\frac{y^2}{16}$.

Therefore, the major axis is along the *x*-axis, while the minor axis is along the *y*-axis.

On comparing the given equation with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, we obtain $a=6$ and $b=4$

Hence,

$$\begin{aligned} c &= \sqrt{a^2-b^2} \\ &= \sqrt{36-16} \\ &= \sqrt{20} \\ &= 2\sqrt{5} \end{aligned}$$

Therefore,

The coordinates of the foci are $\left(\pm 2\sqrt{5},0\right)$

The coordinates of the vertices are $\left(\pm 6,0\right)$

Length of major axis $=2a=12$

Length of minor axis $=2b=8$

Eccentricity, $e=\frac{c}{a}=\frac{2\sqrt{5}}{6}=\frac{\sqrt{5}}{3}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 16}{6}=\frac{16}{3}$

Question 2:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{4}+\frac{y^2}{25}=1$

Solution:

The given equation is $\frac{x^2}{4}+\frac{y^2}{25}=1$ or $\frac{x^2}{2^2}+\frac{y^2}{5^2}=1$

Here, the denominator of $\frac{y^2}{25}$ is greater than the denominator of $\frac{x^2}{4}$.

Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing the given equation with $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, we obtain $b=2$ and $a=5$

Hence,

$$\begin{aligned} c &= \sqrt{a^2-b^2} \\ &= \sqrt{25-4} \\ &= \sqrt{21} \end{aligned}$$

Therefore,

The coordinates of the foci are $(0, \pm\sqrt{21})$

The coordinates of the vertices are $(0, \pm 5)$

Length of major axis $=2a=10$

Length of minor axis $=2b=4$

Eccentricity, $e=\frac{c}{a}=\frac{\sqrt{21}}{5}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 4}{5}=\frac{8}{5}$

Question 3:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$

Solution:

The given equation is $\frac{x^2}{16}+\frac{y^2}{9}=1$ or $\frac{x^2}{4^2}+\frac{y^2}{3^2}=1$

Here, the denominator of $\frac{x^2}{16}$ is greater than the denominator of $\frac{y^2}{9}$.

Therefore, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, we obtain $a=4$ and $b=3$

Hence,

$$\begin{aligned} c &= \sqrt{a^2-b^2} \\ &= \sqrt{16-9} \\ &= \sqrt{7} \end{aligned}$$

Therefore,

The coordinates of the foci are $\left(\pm\sqrt{7},0\right)$

The coordinates of the vertices are $\left(\pm 4,0\right)$

Length of major axis $=2a=8$

Length of minor axis $=2b=6$

Eccentricity, $e=\frac{c}{a}=\frac{\sqrt{7}}{4}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 9}{4}=\frac{9}{2}$

Question 4:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{25}+\frac{y^2}{100}=1$

Solution:

The given equation is $\frac{x^2}{25}+\frac{y^2}{100}=1$ or $\frac{x^2}{5^2}+\frac{y^2}{10^2}=1$

Here, the denominator of $\frac{y^2}{100}$ is greater than the denominator of $\frac{x^2}{25}$.

Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing the given equation with $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, we obtain $b=5$ and $a=10$
Hence,

$$\begin{aligned} c &= \sqrt{a^2-b^2} \\ &= \sqrt{100-25} \\ &= \sqrt{75} \\ &= 5\sqrt{3} \end{aligned}$$

Therefore,

The coordinates of the foci are $(0, \pm 5\sqrt{3})$

The coordinates of the vertices are $(0, \pm 10)$

Length of major axis $=2a=20$

Length of minor axis $=2b=10$

Eccentricity, $e=\frac{c}{a}=\frac{5\sqrt{3}}{10}=\frac{\sqrt{3}}{2}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 25}{10}=5$

Question 5:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{49}+\frac{y^2}{36}=1$

Solution:

The given equation is $\frac{x^2}{49}+\frac{y^2}{36}=1$ or $\frac{x^2}{7^2}+\frac{y^2}{6^2}=1$

Here, the denominator of $\frac{x^2}{49}$ is greater than the denominator of $\frac{y^2}{36}$.

Therefore, the major axis is along the *x*-axis, while the minor axis is along the *y*-axis.

On comparing the given equation with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, we obtain $a=7$ and $b=6$
Hence,

$$\begin{aligned} c &= \sqrt{a^2-b^2} \\ &= \sqrt{49-36} \\ &= \sqrt{13} \end{aligned}$$

Therefore,

The coordinates of the foci are $\left(\pm\sqrt{13}, 0\right)$

The coordinates of the vertices are $\left(\pm 7, 0\right)$

Length of major axis $= 2a = 14$

Length of minor axis $= 2b = 12$

Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{13}}{7}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2\times 36}{7} = \frac{72}{7}$

Question 6:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{100} + \frac{y^2}{400} = 1$

Solution:

The given equation is $\frac{x^2}{100} + \frac{y^2}{400} = 1$ or $\frac{x^2}{10^2} + \frac{y^2}{20^2} = 1$

Here, the denominator of $\frac{y^2}{400}$ is greater than the denominator of $\frac{x^2}{100}$.

Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing the given equation with $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, we obtain $b = 10$ and $a = 20$

Hence,

$$\begin{aligned} c &= \sqrt{a^2 - b^2} \\ &= \sqrt{400 - 100} \\ &= \sqrt{300} \\ &= 10\sqrt{3} \end{aligned}$$

Therefore,

The coordinates of the foci are $\left(0, \pm 10\sqrt{3}\right)$

The coordinates of the vertices are $\left(0, \pm 20\right)$

Length of major axis $= 2a = 40$

Length of minor axis $= 2b = 20$

Eccentricity, $e = \frac{c}{a} = \frac{10\sqrt{3}}{20} = \frac{\sqrt{3}}{2}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 100}{20} = 10$

Question 7:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $36x^2 + 4y^2 = 144$

Solution:

The given equation is $36x^2 + 4y^2 = 144$
It can be written as,

$$36x^2 + 4y^2 = 144$$
$$\Rightarrow \frac{x^2}{4} + \frac{y^2}{36} = 1$$
$$\Rightarrow \frac{x^2}{2^2} + \frac{y^2}{6^2} = 1 \qquad \ldots(1)$$

Here, the denominator of $\frac{y^2}{6^2}$ is greater than the denominator of $\frac{x^2}{2^2}$.

Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing equation (1) with $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, we obtain $a = 6$ and $b = 2$
Hence,

$$c = \sqrt{a^2 - b^2}$$
$$= \sqrt{36 - 4}$$
$$= \sqrt{32}$$
$$= 4\sqrt{2}$$

Therefore,

The coordinates of the foci are $\left(0, \pm 4\sqrt{2}\right)$

The coordinates of the vertices are $(0, \pm 6)$
Length of major axis $= 2a = 12$
Length of minor axis $= 2b = 4$

Eccentricity, $e=\frac{c}{a}=\frac{4\sqrt{2}}{6}=\frac{2\sqrt{2}}{3}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 4}{6}=\frac{4}{3}$

Question 8:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $16x^2+y^2=16$

Solution:

The given equation is $16x^2+y^2=16$

It can be written as,

$$16x^2+y^2=16$$

$$\Rightarrow \frac{x^2}{1}+\frac{y^2}{16}=1$$

$$\Rightarrow \frac{x^2}{1^2}+\frac{y^2}{4^2}=1 \qquad \ldots(1)$$

Here, the denominator of $\frac{y^2}{4^2}$ is greater than the denominator of $\frac{x^2}{1^2}$.

Therefore, the major axis is along the *y*-axis, while the minor axis is along the *x*-axis.

On comparing equation (1) with $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, we obtain $a=4$ and $b=1$

Hence,

$$c = \sqrt{a^2 - b^2}$$
$$= \sqrt{16 - 1}$$
$$= \sqrt{15}$$

Therefore,

The coordinates of the foci are $\left(0, \pm\sqrt{15}\right)$

The coordinates of the vertices are $(0, \pm 4)$

Length of major axis $= 2a = 8$

Length of minor axis $= 2b = 2$

Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{15}}{4}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 1}{4} = \frac{1}{2}$

Question 9:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $4x^2 + 9y^2 = 36$

Solution:

The given equation is $4x^2 + 9y^2 = 36$

It can be written as,

$$4x^2 + 9y^2 = 36$$
$$\Rightarrow \frac{x^2}{9} + \frac{y^2}{4} = 1$$
$$\Rightarrow \frac{x^2}{3^2} + \frac{y^2}{2^2} = 1 \qquad \ldots(1)$$

Here, the denominator of $\frac{x^2}{3^2}$ is greater than the denominator of $\frac{y^2}{2^2}$.

Therefore, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing equation (1) with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we obtain $a = 3$ and $b = 2$

Hence,

$$c = \sqrt{a^2 - b^2}$$
$$= \sqrt{9 - 4}$$
$$= \sqrt{5}$$

Therefore,

The coordinates of the foci are $\left(\pm\sqrt{5}, 0\right)$

The coordinates of the vertices are $(\pm 3, 0)$

Length of major axis $= 2a = 6$

Length of minor axis $= 2b = 4$

Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{5}}{3}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 4}{3} = \frac{8}{3}$

Question 10:

Find the equation for the ellipse that satisfies the given conditions: Vertices $(\pm 5, 0)$, Foci $(\pm 4, 0)$.

Solution:

Vertices $(\pm 5, 0)$, Foci $(\pm 4, 0)$

Here, the vertices are on the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a is the semi major axis.

Accordingly, $a = 5$ and $c = 4$

It is known that $a^2 = b^2 + c^2$

Hence,

$$\Rightarrow 5^2 = b^2 + 4^2$$
$$\Rightarrow 25 = b^2 + 16$$
$$\Rightarrow b^2 = 25 - 16$$
$$\Rightarrow b^2 = 9$$
$$\Rightarrow b = 3$$

Thus, the equation of the ellipse is $\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1$ or $\frac{x^2}{25} + \frac{y^2}{9} = 1$

Question 11:

Find the equation for the ellipse that satisfies the given conditions: Vertices $(0, \pm 13)$, Foci $(0, \pm 5)$

Solution:

Vertices $(0, \pm 13)$, Foci $(0, \pm 5)$
Here, the vertices are on the y-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, where a is the semi major axis.

Accordingly, $a = 13$ and $c = 5$

It is known that $a^2 = b^2 + c^2$

Hence,

$$\Rightarrow 13^2 = b^2 + 5^2$$
$$\Rightarrow 169 = b^2 + 25$$
$$\Rightarrow b^2 = 169 - 25$$
$$\Rightarrow b^2 = 144$$
$$\Rightarrow b = 12$$

Thus, the equation of the ellipse is $\frac{x^2}{12^2} + \frac{y^2}{13^2} = 1$ or $\frac{x^2}{144} + \frac{y^2}{169} = 1$.

Question 12:

Find the equation for the ellipse that satisfies the given conditions: Vertices $(\pm 6, 0)$, Foci $(\pm 4, 0)$
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Solution:

Vertices $(\pm 6, 0)$, Foci $(\pm 4, 0)$
Here, the vertices are on the $x-$axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a is the semi major axis.

Accordingly, $a=6$ and $c=4$

It is known that $a^2=b^2+c^2$

Hence,

$$\Rightarrow 6^2=b^2+4^2$$
$$\Rightarrow 36=b^2+16$$
$$\Rightarrow b^2=36-16$$
$$\Rightarrow b^2=20$$
$$\Rightarrow b=\sqrt{20}$$

Thus, the equation of the ellipse is $\frac{x^2}{6^2}+\frac{y^2}{\left(\sqrt{20}\right)^2}=1$ or $\frac{x^2}{36}+\frac{y^2}{20}=1$.

Question 13:

Find the equation for the ellipse that satisfies the given conditions: Ends of major axis $(\pm 3,0)$, Ends of minor axis $(0,\pm 2)$

Solution:

Ends of major axis $(\pm 3,0)$, Ends of minor axis $(0,\pm 2)$
Here, the major axis is along the *x*-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a is the semi major axis.

Accordingly, $a=3$ and $b=2$.

Thus, the equation of the ellipse is $\frac{x^2}{3^2}+\frac{y^2}{2^2}=1$ or $\frac{x^2}{9}+\frac{y^2}{4}=1$

Question 14:

Find the equation for the ellipse that satisfies the given conditions: Ends of major axis $\left(0,\pm\sqrt{5}\right)$, Ends of minor axis $(\pm 1,0)$.

Solution:

Ends of major axis $\left(0,\pm\sqrt{5}\right)$, Ends of minor axis $(\pm 1,0)$

Here, the major axis is along the y-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, where a is the semi major axis.

Accordingly, $a=\sqrt{5}$ and $b=1$.

Thus, the equation of the ellipse is $\frac{x^2}{1^2}+\frac{y^2}{\left(\sqrt{5}\right)^2}=1$ or $\frac{x^2}{1}+\frac{y^2}{5}=1$

Question 15:

Find the equation for the ellipse that satisfies the given conditions: Length of major axis 26, Foci $(\pm 5,0)$

Solution:

Length of major axis $=26$, Foci $=(\pm 5,0)$
Since the foci are on the x-axis, the major axis is along the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a is the semi major axis.

Accordingly, $2a=26 \Rightarrow a=13$ and $c=5$.

It is known that $a^2=b^2+c^2$
Hence,

$$\Rightarrow 13^2=b^2+5^2$$
$$\Rightarrow 169=b^2+25$$
$$\Rightarrow b^2=169-25$$
$$\Rightarrow b^2=144$$
$$\Rightarrow b=12$$

Thus, the equation of the ellipse is $\frac{x^2}{13^2}+\frac{y^2}{12^2}=1$ or $\frac{x^2}{169}+\frac{y^2}{144}=1$

Question 16:

Find the equation for the ellipse that satisfies the given conditions: Length of minor axis 16, Foci $(0,\pm 6)$

Solution:

Length of minor axis $=16$, Foci $=(0,\pm 6)$
Since the foci are on the y-axis, the major axis is along the y-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, where a is the semi major axis.

Accordingly, $2b=16 \Rightarrow b=8$ and $c=6$.

It is known that $a^2=b^2+c^2$

Hence,

$$\Rightarrow a^2=8^2+6^2$$
$$\Rightarrow a^2=64+36$$
$$\Rightarrow a^2=100$$
$$\Rightarrow a=\sqrt{100}=10$$

Thus, the equation of the ellipse is $\frac{x^2}{8^2}+\frac{y^2}{10^2}=1$ or $\frac{x^2}{64}+\frac{y^2}{100}=1$

Question 17:

Find the equation for the ellipse that satisfies the given conditions: Foci $(\pm 3,0)$, $a=4$.

Solution:

Foci $(\pm 3,0)$, $a=4$
Since the foci are on the x-axis, the major axis is along the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a is the semi major axis.

Accordingly, $a=4$ and $c=3$.

It is known that $a^2=b^2+c^2$

Hence,

$$\Rightarrow 4^2 = b^2 + 3^2$$
$$\Rightarrow 16 = b^2 + 9$$
$$\Rightarrow b^2 = 16 - 9$$
$$\Rightarrow b^2 = 7$$

Thus, the equation of the ellipse is $\frac{x^2}{16} + \frac{y^2}{7} = 1$

Question 18:

Find the equation for the ellipse that satisfies the given conditions: $b = 3, c = 4$, centre at the origin; foci on the x-axis.

Solution:

It is given that $b = 3, c = 4$, centre at the origin, foci on the x-axis.
Since the foci are on the x-axis, the major axis is along the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a is the semi major axis.

Accordingly, $b = 3$ and $c = 4$.

It is known that $a^2 = b^2 + c^2$

Hence,

$$\Rightarrow a^2 = 3^2 + 4^2$$
$$\Rightarrow a^2 = 9 + 16$$
$$\Rightarrow a^2 = 25$$
$$\Rightarrow a = 5$$

Thus, the equation of the ellipse is $\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1$ or $\frac{x^2}{25} + \frac{y^2}{9} = 1$

Question 19:

Find the equation for the ellipse that satisfies the given conditions: Centre at $(0,0)$, major axis on the y-axis and passes through the points $(3,2)$ and $(1,6)$.

Solution:

Since the centre is at $(0,0)$ and the major axis is on the y-axis, the equation of the ellipse will be of the form $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, where a is the semi major axis.

The ellipse passes through points $(3,2)$ and $(1,6)$
Hence,

$$\frac{9}{b^2}+\frac{4}{a^2}=1 \qquad \ldots(1)$$
$$\frac{1}{b^2}+\frac{36}{a^2}=1 \qquad \ldots(2)$$

On solving equations (1) and (2), we obtain $a^2=40$ and $b^2=10$.

Thus, the equation of the ellipse is $\frac{x^2}{10}+\frac{y^2}{40}=1$ or $4x^2+y^2=40$

Question 20:

Find the equation for the ellipse that satisfies the given conditions: Major axis on the x-axis and passes through the points $(4,3)$ and $(6,2)$.

Solution:

Since the major axis is on the x-axis, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a is the semi major axis.

The ellipse passes through points $(4,3)$ and $(6,2)$.

Hence,

$$\frac{16}{b^2}+\frac{9}{a^2}=1 \qquad \ldots(1)$$
$$\frac{36}{b^2}+\frac{4}{a^2}=1 \qquad \ldots(2)$$

On solving equations (1) and (2), we obtain $a^2=52$ and $b^2=13$.

Thus, the equation of the ellipse is $\frac{x^2}{52}+\frac{y^2}{13}=1$ or $x^2+4y^2=52$