Question 1:

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $y^2 = 12x$.

Solution:

The given equation is $y^2 = 12x$
Here, the coefficient of x is positive.
Hence, the parabola opens towards the right.

On comparing this equation with $y^2 = 4ax$, we obtain

$$4a = 12 \Rightarrow a = 3$$

Therefore,

Coordinates of the focus $F = (a, 0) \Rightarrow (3, 0)$

Since the given equation involves y^2, the axis of the parabola is the x-axis.

Equation of directrix, $x = -a$, i.e., $x = -3$

Length of latus rectum $= 4a = 4 \times 3 = 12$

Question 2:

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $x^2 = 6y$.

Solution:

The given equation is $x^2 = 6y$
Here, the coefficient of y is positive.
Hence, the parabola opens upwards.

On comparing this equation with $x^2 = 4ay$, we obtain

$$4a = 6 \Rightarrow a = \frac{3}{2}$$

Therefore,

Coordinates of the focus $F = (0, a) \Rightarrow \left(0, \frac{3}{2}\right)$

Since the given equation involves x^2, the axis of the parabola is the y-axis.

Equation of directrix, $y = -a$, i.e., $y = -\frac{3}{2}$

Length of latus rectum $= 4a = 4 \times \frac{3}{2} = 6$

Question 3:

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $y^2 = -8x$

Solution:

The given equation is $y^2 = -8x$
Here, the coefficient of x is negative.
Hence, the parabola opens towards the left.

On comparing this equation with $y^2 = -4ax$, we obtain

$$-4a = 8 \Rightarrow a = -2$$

Therefore,

Coordinates of the focus $F = (-a, 0) \Rightarrow (-2, 0)$

Since the given equation involves y^2, the axis of the parabola is the x-axis.

Equation of directrix, $x = a$, i.e., $x = 2$

Length of latus rectum $= 4a = 4 \times 2 = 8$

Question 4:

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $x^2 = -16y$.

Solution:

The given equation is $x^2 = -16y$
Here, the coefficient of y is negative.
Hence, the parabola opens downwards.

On comparing this equation $x^2 = -4ay$, we obtain

$$-4a = -16 \Rightarrow a = 4$$

Therefore,

Coordinates of the focus $F = (0, -a) = (0, -4)$

Since the given equation involves x^2, the axis of the parabola is the y-axis.

Equation of directrix, $y = a$ i.e., $y = 4$

Length of latus rectum $4a = 4 \times 4 = 16$

Question 5:

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $y^2 = 10x$.

Solution:

The given equation is $y^2 = 10x$
Here, the coefficient of x is positive.
Hence, the parabola opens towards the right.
On comparing this equation with $y^2 = 4ax$, we obtain

$$4a = 10 \Rightarrow a = \frac{5}{2}$$

Therefore,

Coordinates of the focus $= (a, 0) = \left(\frac{5}{2}, 0\right)$

Since the given equation involves y^2, the axis of the parabola is the x-axis.

Equation of directrix, $x = -a$, i.e., $x = -\frac{5}{2}$

Length of latus rectum $= 4a = 4 \times \frac{5}{2} = 10$

Question 6:

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $x^2 = -9y$.

Solution:

The given equation is $x^2 = -9y$
Here, the coefficient of y is negative.
Hence, the parabola opens downwards.

On comparing this equation $x^2 = -4ay$, we obtain

$$-4a = -9 \Rightarrow a = \frac{9}{4}$$

Therefore,

Coordinates of the focus $= (0, -a) = \left(0, -\frac{9}{4}\right)$

Since the given equation involves x^2, the axis of the parabola is the $y-$axis.

Equation of directrix, $y = a$, i.e., $y = \frac{9}{4}$

Length of latus rectum $= 4a = 4 \times \frac{9}{4} = 9$

Question 7:

Find the equation of the parabola that satisfies the following conditions: Focus $(6, 0)$; Directrix $x = -6$.

Solution:

Focus $(6, 0)$; Directrix $x = -6$
Since the focus lies on the x-axis, the x-axis is the axis of the parabola.

Therefore, the equation of the parabola is either of the form $y^2 = 4ax$ or $y^2 = -4ax$.

It is also seen that the directrix, $x = -6$ is to the left of the y-axis while the focus $(6, 0)$ is to right of the y-axis.
Hence, the parabola is of the form $y^2 = 4ax$.

Here, $a = 6$

Thus, the equation of the parabola is $y^2 = 24x$.

Question 8:

Find the equation of the parabola that satisfies the following conditions: Focus $(0,-3)$; Directrix $y=3$

Solution:

Focus $(0,-3)$; Directrix $y=3$
Since the focus lies on the $y-$axis, the $y-$axis is the axis of the parabola.

Therefore, the equation of the parabola is either of the form $x^2=4ay$ or $x^2=-4ay$.

It is also seen that the directrix, $y=3$ is above the $x-$axis while the focus $(0,-3)$ is below the $x-$axis.
Hence, the parabola is of the form $x^2=-4ay$.

Here, $a=3$

Thus, the equation of the parabola is $x^2=-12y$.

Question 9:

Find the equation of the parabola that satisfies the following conditions: Vertex $(0,0)$; Focus $(3,0)$

Solution:

Vertex $(0,0)$; Focus $(3,0)$

Since the vertex of the parabola is $(0,0)$ and the focus lies on the positive *x*-axis, *x*-axis is the axis of the parabola, while the equation of the parabola is of the form $y^2=4ax$

Since the focus is $(3,0)$, $a=3$

Thus, the equation of the parabola is $y^2=4\times3\times x$, i.e., $y^2=12x$

Question 10:

Find the equation of the parabola that satisfies the following conditions: Vertex $(0,0)$; Focus $(-2,0)$

Solution:

Vertex $(0,0)$; Focus $(-2,0)$

Since the vertex of the parabola is $(0,0)$ and the focus lies on the negative x-axis, x-axis is the axis of the parabola, while the equation of the parabola is of the form $y^2 = -4ax$

Since the focus is $(-2,0)$, $a = 2$

Thus, the equation of the parabola is $y^2 = -4 \times 2 \times x$, i.e., $y^2 = -8x$

Question 11:

Find the equation of the parabola that satisfies the following conditions: Vertex $(0,0)$ passing through $(2,3)$ and axis is along x-axis.

Solution:

Since the vertex is $(0,0)$ and the axis of the parabola is the x-axis, the equation of the parabola is either of the form $y^2 = 4ax$ or $y^2 = -4ax$

The parabola passes through point $(2,3)$, which lies in the first quadrant.

Therefore, the equation of the parabola is of the form $y^2 = 4ax$, while point $(2,3)$ must satisfy the equation $y^2 = 4ax$

Hence,

$$3^2 = 4a \times 2 \Rightarrow a = \frac{9}{8}$$

Thus, the equation of the parabola is

$$\Rightarrow y^2 = 4 \times \frac{9}{8} \times x$$

$$\Rightarrow y^2 = \frac{9}{2}x$$

$$\Rightarrow 2y^2 = 9x$$

Question 12:

Find the equation of the parabola that satisfies the following conditions: Vertex $(0,0)$ passing through $(5,2)$ and symmetric with respect to $y-$axis.

Solution:

Since the vertex is $(0,0)$ and the parabola is symmetric about the $y-$axis, the equation of the parabola is either of the form $x^2 = 4ay$ or $x^2 = -4ay$

The parabola passes through point $(5,2)$, which lies in the first quadrant.

Therefore, the equation of the parabola is of the form $x^2 = 4ay$, while point $(5,2)$ must satisfy the equation $x^2 = 4ay$

Hence,

$$5^2 = 4a \times 2 \Rightarrow 25 = 8a \Rightarrow a = \frac{25}{8}$$

Thus, the equation of the parabola is

$$\Rightarrow x^2 = 4 \times \frac{25}{8} \times y$$

$$\Rightarrow x^2 = \frac{25}{2} y$$

$$\Rightarrow 2x^2 = 25y$$