Exercise 10.1

Question 1:

Find the equation of the circle with centre $(0, 2)$ and radius 2.

Solution:

The equation of a circle with centre (h, k) and radius r is given as

$$(x-h)^2+(y-k)^2=r^2$$

It is given that centre $(h, k)=(0, 2)$ and radius $(r)=2$.

Therefore, the equation of the circle is

$$(x-0)^2+(y-2)^2=2^2$$
$$x^2+y^2-4y+4=4$$
$$x^2+y^2-4y=0$$

Question 2:

Find the equation of the circle with centre $(-2, 3)$ and radius 4.

Solution:

The equation of a circle with centre (h, k) and radius r is given as

$$(x-h)^2+(y-k)^2=r^2$$

It is given that centre $(h, k)=(-2, 3)$ and radius $(r)=4$.

Therefore, the equation of the circle is

$$(x+2)^2+(y-3)^2=4^2$$
$$x^2+4x+4+y^2-6y+9=16$$
$$x^2+y^2+4x-6y-3=0$$

Question 3:

Find the equation of the circle with centre $\left(\frac{1}{2}, \frac{1}{4}\right)$ and radius $\left(\frac{1}{12}\right)$

Solution:

The equation of a circle with centre (h, k) and radius r is given as

$$(x-h)^2+(y-k)^2=r^2$$

It is given that centre $(h, k)=\left(\frac{1}{2}, \frac{1}{4}\right)$ and radius $(r)=\frac{1}{12}$.

Therefore, the equation of the circle is

$$\left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{4}\right)^2=\left(\frac{1}{12}\right)^2$$

$$x^2-x+\frac{1}{4}+y^2-\frac{y}{2}+\frac{1}{16}=\frac{1}{144}$$

$$x^2-x+\frac{1}{4}+y^2-\frac{y}{2}+\frac{1}{16}-\frac{1}{144}=0$$

$$144x^2-144x+36+144y^2-72y+9-1=0$$

$$144x^2-144x+144y^2-72y+44=0$$

$$36x^2-36x+36y^2-18y+11=0$$

$$36x^2+36y^2-36x-18y+11=0$$

Question 4:

Find the equation of the circle with centre $(1, 1)$ and radius $\sqrt{2}$

Solution:

The equation of a circle with centre (h, k) and radius r is given as

$$(x-h)^2+(y-k)^2=r^2$$

It is given that centre $(h, k)=(1, 1)$ and radius $(r)=\sqrt{2}$.

Therefore, the equation of the circle is

$$(x-1)^2+(y-1)^2=\left(\sqrt{2}\right)^2$$

$$x^2-2x+1+y^2-2y+1=2$$

$$x^2+y^2-2x-2y=0$$

Question 5:

Find the equation of the circle with centre $(-a, -b)$ and radius $\sqrt{a^2-b^2}$

Solution:

The equation of a circle with centre (h, k) and radius r is given as

$$(x-h)^2+(y-k)^2=r^2$$

It is given that centre $(h, k)=(-a, -b)$ and radius $(r)=\sqrt{a^2-b^2}$

Therefore, the equation of the circle is

$$(x+a)^2+(y+b)^2=\left(\sqrt{a^2-b^2}\right)^2$$
$$x^2+2ax+a^2+y^2+2by+b^2=a^2-b^2$$
$$x^2+y^2+2ax+2by+2b^2=0$$

Question 6:

Find the centre and radius of the circle $(x+5)^2+(y-3)^2=36$

Solution:

The equation of the given circle is $(x+5)^2+(y-3)^2=36$

$$\Rightarrow (x+5)^2+(y-3)^2=36$$
$$\Rightarrow [x-(-5)]^2+(y-3)^2=6^2$$

which is of the form $(x-h)^2+(y-k)^2=r^2$

Therefore, on comparing both equations we get

$h=-5, k=3$ and $r=6$

Thus, the centre of the given circle is $(-5, 3)$ while its radius is 6.

Question 7:

Find the centre and radius of the circle $x^2+y^2-4x-8y-45=0$

Solution:

The equation of the given circle is $x^2+y^2-4x-8y-45=0$.

$$\Rightarrow x^2+y^2-4x-8y-45=0$$
$$\Rightarrow \left(x^2-4x\right)+\left(y^2-8y\right)=45$$
$$\Rightarrow \left\{x^2-2(x)(2)+2^2\right\}+\left\{y^2-2(y)(4)+4^2\right\}-4-16=45$$
$$\Rightarrow (x-2)^2+(y-4)^2=65$$
$$\Rightarrow (x-2)^2+(y-4)^2=\left(\sqrt{65}\right)^2$$

which is of the form $(x-h)^2+(y-k)^2=r^2$

Therefore, on comparing both equations we get

$h=2, k=4$ and $r=\sqrt{65}$

Thus, the centre of the given circle is $(2,4)$ while its radius is $\sqrt{65}$.

Question 8:

Find the centre and radius of the circle $x^2+y^2-8x+10y-12=0$

Solution:

The equation of the given circle is $x^2+y^2-8x+10y-12=0$.

$$\Rightarrow x^2+y^2-8x+10y-12=0$$
$$\Rightarrow \left(x^2-8x\right)+\left(y^2+10y\right)=12$$
$$\Rightarrow \left\{x^2-2(x)(4)+4^2\right\}+\left\{y^2+2(y)(5)+5^2\right\}-16-25=12$$
$$\Rightarrow (x-4)^2+(y+5)^2=53$$
$$\Rightarrow (x-4)^2+\left[y-(-5)\right]^2=\left(\sqrt{53}\right)^2$$

which is of the form $(x-h)^2+(y-k)^2=r^2$

Therefore, on comparing both equations we get

$h=4, k=-5$ and $r=\sqrt{53}$

Thus, the centre of the given circle is $(4,-5)$ while its radius is $\sqrt{53}$.

Question 9:

Find the centre and radius of the circle $2x^2+2y^2-x=0$

Solution:

The equation of the given circle is $2x^2+2y^2-x=0$

$$\Rightarrow 2x^2+2y^2-x=0$$
$$\Rightarrow (2x^2-x)+2y^2=0$$
$$\Rightarrow 2\left[\left(x^2-\frac{x}{2}\right)+y^2\right]=0$$
$$\Rightarrow \left\{x^2-2(x)\left(\frac{1}{4}\right)+\left(\frac{1}{4}\right)^2\right\}+y^2-\left(\frac{1}{4}\right)^2=0$$
$$\Rightarrow \left(x-\frac{1}{4}\right)^2+(y-0)^2=\left(\frac{1}{4}\right)^2$$

which is of the form $(x-h)^2+(y-k)^2=r^2$

Therefore, on comparing both equations we get

$$h=\frac{1}{4}, k=0 \text{ and } r=\frac{1}{4}$$

Thus, the centre of the given circle is $\left(\frac{1}{4},0\right)$ while its radius is $\frac{1}{4}$.

Question 10:

Find the equation of the circle passing through the points $(4,1)$ and $(6,5)$ and whose centre is on the line $4x+y=16$.

Solution:

Let the equation of the required circle be $(x-h)^2+(y-k)^2=r^2$.

Since the circle passes through the points $(4,1)$ and $(6,5)$

$$(4-h)^2+(1-k)^2=r^2 \qquad \ldots(1)$$
$$(6-h)^2+(5-k)^2=r^2 \qquad \ldots(2)$$

Since the centre (h,k) of the circle lies on the line $4x+y=16$

$$4h+k=16 \qquad \ldots(3)$$

From equations (1) and (2), we obtain

$$\Rightarrow (4-h)^2+(1-k)^2=(6-h)^2+(5-k)^2$$
$$\Rightarrow 16-8h+h^2+1-2k+k^2=36-12h+h^2+25-10k+k^2$$
$$\Rightarrow 16-8h+1-2k=36-12h+25-10k$$
$$\Rightarrow 4h+8k=44$$
$$\Rightarrow h+2k=11 \quad \ldots(4)$$

On solving equations (3) and (4), we obtain

$$h=3 \text{ and } k=4$$

On substituting the values of h and k in equation (1), we obtain

$$(4-3)^2+(1-4)^2=r^2$$
$$\Rightarrow 1^2+(-3)^2=r^2$$
$$\Rightarrow 1+9=r^2$$
$$\Rightarrow r^2=10$$
$$\Rightarrow r=\sqrt{10}$$

Thus, the equation of the required circle is

$$(x-3)^2+(y-4)^2=\left(\sqrt{10}\right)^2$$
$$x^2-6x+9+y^2-8y+16=10$$
$$x^2+y^2-6x-8y+15=0$$

Question 11:

Find the equation of the circle passing through the points $(2,3)$ and $(-1,1)$ and whose centre is on the line $x-3y-11=0$.

Solution:

Let the equation of the required circle be $(x-h)^2+(y-k)^2=r^2$.

Since the circle passes through the points $(2,3)$ and $(-1,1)$

$$(2-h)^2+(3-k)^2=r^2 \qquad \ldots(1)$$

$$(-1-h)^2+(1-k)^2=r^2 \qquad \ldots(2)$$

Since the centre (h,k) of the circle passes lies on the line $x-3y-11=0$,

$$h-3k=11 \qquad \ldots(3)$$

From equations (1) and (2), we obtain

$$(2-h)^2+(3-k)^2=(-1-h)^2+(1-k)^2$$

$$\Rightarrow 4-4h+h^2+9-6k+k^2=1+2h+h^2+1+k^2-2k$$

$$\Rightarrow 4-4h+9-6k=1+2h+1-2k$$

$$\Rightarrow 6h+4k=11 \qquad \ldots(4)$$

On solving equations (3) and (4), we obtain

$h=\frac{7}{2}$ and $k=\frac{-5}{2}$

On substituting the values of h and k in equation (1), we obtain

$$\Rightarrow \left(2-\frac{7}{2}\right)^2+\left(3+\frac{5}{2}\right)^2=r^2$$

$$\Rightarrow \left(\frac{4-7}{2}\right)^2+\left(\frac{6+5}{2}\right)^2=r^2$$

$$\Rightarrow \left(\frac{-3}{2}\right)^2+\left(\frac{11}{2}\right)^2=r^2$$

$$\Rightarrow \frac{9}{4}+\frac{121}{4}=r^2$$

$$\Rightarrow \frac{130}{4}=r^2$$

Thus, the equation of the required circle is

$$\left(x-\frac{7}{2}\right)^2+\left(y+\frac{5}{2}\right)^2=\frac{130}{4}$$

$$\left(\frac{2x-7}{2}\right)^2+\left(\frac{2y+5}{2}\right)^2=\frac{130}{4}$$

$$4x^2-28x+49+4y^2+20y+25=130$$

$$4x^2+4y^2-28x+20y-56=0$$

$$4\left(x^2+y^2-7x+5y-14\right)=0$$

$$x^2+y^2-7x+5y-14=0$$

Question 12:

Find the equation of the circle with radius 5 whose centre lies on *x*-axis and passes through the point $(2,3)$.

Solution:

Let the equation of the required circle be $(x-h)^2+(y-k)^2=r^2$.
Since the radius of the circle is 5 and its centre lies on the *x*-axis, $k=0$ and $r=5$.

Now, the equation of the circle becomes $(x-h)^2+y^2=25$.

It is given that the circle passes through the point $(2,3)$.
Therefore,

$$\Rightarrow (2-h)^2+3^2=25$$

$$\Rightarrow (2-h)^2=25-9$$

$$\Rightarrow (2-h)^2=16$$

$$\Rightarrow 2-h=\pm\sqrt{16}$$

$$\Rightarrow 2-h=\pm 4$$

If, $2-h=4$, then $h=-2$
If $2-h=-4$, then $h=6$

When $h=-2$, the equation of the circle becomes

$$(x+2)^2+y^2=25$$

$$x^2+4x+4+y^2=25$$

$$x^2+y^2+4x-21=0$$

When $h=6$, the equation of the circle becomes

$$(x-6)^2+y^2=25$$
$$x^2-12x+36+y^2=25$$
$$x^2+y^2-12x+11=0$$

Question 13:

Find the equation of the circle passing through $(0,0)$ and making intercepts a and b on the coordinate axes.

Solution:

Let the equation of the required circle be $(x-h)^2+(y-k)^2=r^2$.

Since the circle passes through $(0,0)$,

$$(0-h)^2+(0-k)^2=r^2$$
$$\Rightarrow h^2+k^2=r^2$$

The equation of the circle now becomes $(x-h)^2+(y-k)^2=h^2+k^2$.

It is given that the circle makes intercepts a and b on the coordinate axes. This means that the circle passes through points $(a,0)$ and $(0,b)$.

Therefore,

$$(a-h)^2+(0-k)^2=h^2+k^2 \quad \ldots(1)$$
$$(0-h)^2+(b-k)^2=h^2+k^2 \quad \ldots(2)$$

From equation (1), we obtain

$$\Rightarrow a^2+h^2-2ah+k^2=h^2+k^2$$
$$\Rightarrow a^2-2ah=0$$
$$\Rightarrow a(a-2h)=0$$
$$\Rightarrow a=0 \text{ or } (a-2h)=0$$

However, $a\neq 0$;

Hence,

$$(a-2h)=0$$
$$\Rightarrow h=\frac{a}{2}$$

From equation (2), we obtain

$$h^2+b^2-2bk+k^2=h^2+k^2$$
$$\Rightarrow b^2-2bk=0$$
$$\Rightarrow b(b-2k)=0$$
$$\Rightarrow b=0 \text{ or } (b-2k)=0$$

However, $b \neq 0$;
Hence,

$$(b-2k)=0$$
$$\Rightarrow k=\frac{b}{2}$$

Thus, the equation of the required circle is

$$\Rightarrow \left(x-\frac{a}{2}\right)^2+\left(y-\frac{b}{2}\right)^2=\left(\frac{a}{2}\right)^2+\left(\frac{b}{2}\right)^2$$
$$\Rightarrow \left(\frac{2x-a}{2}\right)^2+\left(\frac{2y-b}{2}\right)^2=\frac{a^2+b^2}{4}$$
$$\Rightarrow 4x^2-4ax+a^2+4y^2-4by+b^2=a^2+b^2$$
$$\Rightarrow 4x^2+4y^2-4ax-4by=0$$
$$\Rightarrow 4\left(x^2+y^2-ax-by\right)=0$$
$$\Rightarrow x^2+y^2-ax-by=0$$

Question 14:

Find the equation of a circle with centre $(2,2)$ and passes through the point $(4,5)$.

Solution:

The centre of the circle is given as $(h,k)=(2,2)$

Since the circle passes through the point $(4,5)$, the radius (r) of the circle is the distance between the points $(2,2)$ and $(4,5)$.

Therefore,

$$r=\sqrt{(2-4)^2+(2-5)^2}$$
$$=\sqrt{(-2)^2+(-3)^2}$$
$$=\sqrt{4+9}$$
$$=\sqrt{13}$$

Thus, the equation of the circle is

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-2)^2+(y-2)^2=\left(\sqrt{13}\right)^2$$
$$x^2-4x+4+y^2-4y+4=13$$
$$x^2+y^2-4x-4y-5=0$$

Question 15:

Does the point $(-2.5, 3.5)$ lie inside, outside or on the circle $x^2+y^2=25$?

Solution:

The equation of the given circle is $x^2+y^2=25$

$$\Rightarrow x^2+y^2=25$$
$$\Rightarrow (x-0)^2+(y-0)^2=5^2$$

which is of the form $(x-h)^2+(y-k)^2=r^2$, where $h=0, k=0$ and $r=5$

Therefore, Centre $\Rightarrow (0,0)$ and radius $\Rightarrow 5$

Distance between point $(-2.5, 3.5)$ and centre $(0,0)$

$$=\sqrt{(-2.5-0)^2+(3.5-0)^2}$$
$$=\sqrt{6.25+12.25}$$
$$=\sqrt{18.25}$$
$$=4.272$$

Here, $4.272<5$, this means less than the radius.

Since the distance between the point $(-2.5, 3.5)$ and centre $(0,0)$ of the circle is less than the radius of the circle, hence, the point $(-2.5, 3.5)$ lies inside the circle.