Exercise 1.5

Q1. Let U = { 1, 2, 3, 4, 5, 6, 7, 8, 9 }, A = { 1, 2, 3, 4}, B = { 2, 4, 6, 8 } andC = { 3, 4, 5, 6 }.
Find (i) A′ (ii) B′ (iii) (A ∪C)′ (iv) (A ∪B)′
(v) (A′)′ (vi) (B – C)′

A.1. (i) A′ = U – A = {1,2,3,4,5,6,7,8,9} – {1,2,3,4}
= {5,6,7,8,9}

(ii) B′ = U – B = {1,2,3,4,5,6,7,8,9} – {2,4,6,8}
= {1,3,5,7,9}.

(iii) (A ∪ C)′ = A′ ∩ C′
= {5,6,7,8,9} ∩ [U – C] [∵ (i)]
= {5,6,7,8,9} ∩ [{1,2,3,4,5,6,1,8,9} – {3,4,5,6}]
= {5,6,7,8,9} ∩ {1,2,7,8,9}
= {7,8,9}

(iv) (A ∪ B)′ = A′ ∩ B′ [By demorgan's law]
= {5,6,7,8,9} ∩ {1,3,5,7,9} [∵ (i) and (ii)]
= {5,7,9}.

(v) (A′)′ = U – A′ = {1,2,3,4,5,6,7,8,9} – {5,6,7,8,9} [∵ (1)]
= {1,2,3,4} = A
(A′)′ = A.

(vi) (B – C)′ = U – (B – C) = {1,2,3,4,5,6,7,8,9} – [{2, 4, 6, 8} – {3, 4, 5, 6}]
= {1,2,3,4,5,6,7,8,9} – {2,8}
= {1,3,4, 5, 6,7,9}.

Q2. If U = { *a, b, c, d, e, f, g, h*}, find the complements of the following sets :
(i) A = {*a, b, c*} (ii) B = {*d, e, f, g*}
(iii) C = {*a, c, e, g*} (iv) D = { *f, g, h, a*}

A.2. (i) A′ = U – A = {a, b, c, d, e, f, g, h} – {a, b, c}
= {d, e, f, g, h}

(ii) B′ = U – B = {a, b, c, d, e, f, g, h} – {d, e, f, g}
= {a, b, c, h}.

(iii) C′ = U – C = {a, b, c, d, e, f, g, h} – {a, c, e, g}.
= {b, d, f, h}

(iv) D′ = U – D = {a, b, c, d, e, f, g, h} – {f, g, h, a}
= {b, c, d, e}

Q3. Taking the set of natural numbers as the universal set, write down the complements of the following sets:

(i) {*x* : *x* is an even natural number} (ii) { *x* : *x* is an odd natural number }
(iii) {*x* : *x* is a positive multiple of 3} (iv) { *x* : *x* is a prime number }
(v) {*x* : *x* is a natural number divisible by 3 and 5}
(vi) {*x* : *x* is a perfect square } (vii) { *x* : *x* is a perfect cube}
(viii) {*x* : *x* + 5 = 8 } (ix) { *x* : 2*x* + 5 = 9}
(x) {*x* : *x* ≥ 7} (xi) { *x* : *x* ∈ N and 2*x* + 1 > 10 }

A.3. (i) {*x* : *x* is an odd natural number}

(ii) {*x* : *x* is an even natural number}

(iii) {*x* : *x* is not a multiple of 3}

(iv) {*x* : *x* is a positive composite number and x = 1}

(v) $\{x : x$ is a natural number not divisible by 3 and 5$\}$.

(vi) $\{x : x$ is not a perfect square$\}$

(vii) $\{x : x$ is not a perfect cube$\}$

(viii) We have, $x + 5 = 8$.

$\Rightarrow \quad x = 8 - 5 = 3$

$\Rightarrow \quad x = 3$

$\therefore \ \{x : x \neq 3, x \in N\}$

(ix) We have,

$2x + 5 = 9$

$\Rightarrow \quad 2x = 9 - 5$

$\Rightarrow \quad 2x = 4$

$\Rightarrow \quad x = 2$

$\therefore \quad \{x : x \in N \text{ and } x \neq 2\}$

(x) $\{x : x < 7\} = \{1,2,3,4,5,6\}$

(xi) We have,

$2x + 1 > 10$

$\Rightarrow \quad 2x > 10 - 1$

$\Rightarrow \quad x > \dfrac{9}{2}$

$\therefore \quad \left\{x : x \in N \text{ and } x < \dfrac{9}{2}\right\} = \{1,2,3,4\}$

Q4. If U = {1, 2, 3, 4, 5, 6, 7, 8, 9 }, A = {2, 4, 6, 8} and B = { 2, 3, 5, 7}. Verify that
(i) $(A \cup B)' = A' \cap B'$ **(ii)** $(A \cap B)' = A' \cup B'$

A.4. (i) L.H.S $= (A \cup B)' = U - (A \cup B)$

$= \{1,2,3,4,5,6,7,8,9\} - [\{2,4,6,8) \cup \{2,3,5,7\}]$

$= \{1,2,3,4,5,6,7,8,9\} - \{2,3,4,5,6,7,8\}$

$= \{1,9\}$

R.H.S. $= A' \cap B' = [U - A] \cap [U \cup B]$

$= [\{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2, 4, 6, 8\}] \cap [\{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2, 3, 5, 7\}]$

$= \{1,3,5,7,9\} \cap \{1,4,6,8,9\}$

$= \{1,9\}$

$\therefore$ L.H.S. = R.H.S.

$\Rightarrow (A \cup B)' = A' \cap B'$.

(ii) L.H.S. $= (A \cap B)' = U - (A \cap B)$

$= \{1,2,3,4,5,6,7,8,9\} - [\{2,4,6,8\} \cap \{2,3,5,7\}]$

$= \{1,2,3,4,5,6,7,8,9\} - \{2\}$

$= \{1,3,4,5,6,7,8,9\}$

R.H.S. $= A' \cup B'$

$= [U - A] \cup [U - B]$

$= [\{1,2,3,4,5,6,7,8,9\} - \{2,4,6,8\}] \cup [\{1,2,3,4,5,6,7,8,9\} - \{2,3,5,7\}]$

$= \{1,3,5,7,9\} \cup \{1,4,6,8,9\}$

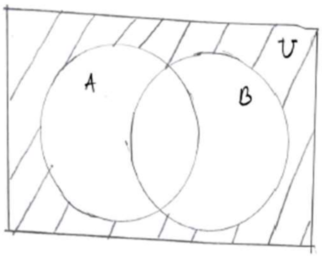
$= \{1,3,4,5,6,7,8,9\}$

$\therefore$ L.H.S. = R.H.S.

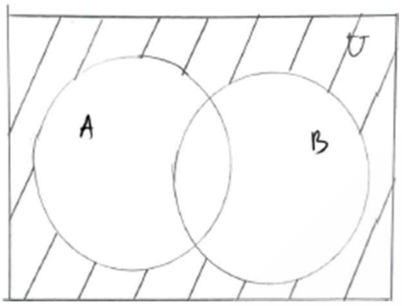
$\Rightarrow (A \cap B)' = A' \cup B'$.

Q5. Draw appropriate Venn diagram for each of the following :
(i) $(A \cup B)'$, (ii) $A' \cup B'$, (iii) $(A \cup B)'$, (iv) $A' \cup B'$

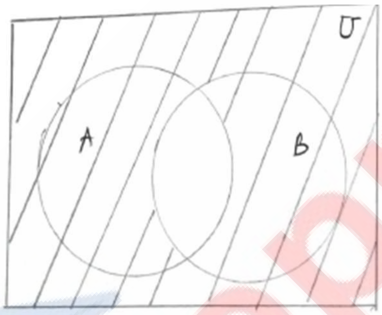
A.5. (i) $(A \cup B)' = U - (A \cup B)$



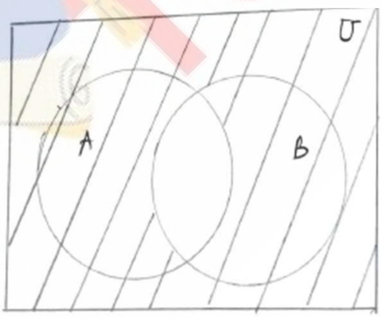
(ii) $A' \cap B' = (A \cup B)' = U - (A \cup B)$



(iii) $(A \cap B)' = U - (A \cap B)$



(iv) $A' \cup B' = (A \cap B)' = U - (A \cap B)'$.



Q.6. Let U be the set of all triangles in a plane. If A is the set of all triangles with at least one angle different from 60°, what is A′?

A. $A' = U - A$

= Set of all triangle in a plane – Set of all triangle with at least the angle different from 60°.

= Set of all triangle with each angle 60°.

A′ = set of all equilateral triangle.

Q7. Fill in the blanks to make each of the following a true statement :

(i) $A \cup A' = \ldots$ **(ii)** $\phi' \cap A = \ldots$

(iii) $A \cap A' = \ldots$ **(iv)** $U' \cap A = \ldots$

A.7. (i) $A \cup A' = U$

(ii) $\phi' \cap A = U \cap A = A$.

(iii) $A \cap A' = \phi$.

(iv) $U' \cap A = \phi \cap A = \phi$.