

EXERCISE 11.4

Question 1:

Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus

rectum of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$

Solution:

The given equation is $\frac{x^2}{16} - \frac{y^2}{9} = 1$ or $\frac{x^2}{4^2} - \frac{y^2}{3^2} = 1$

On comparing this equation with the standard equation of hyperbola i.e., $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we obtain
 $a = 4$ and $b = 3$

We know that, $c^2 = a^2 + b^2$

Hence,

$$\Rightarrow c^2 = 4^2 + 3^2$$

$$\Rightarrow c^2 = 25$$

$$\Rightarrow c = 5$$

Therefore,

The coordinates of the foci are $(\pm 5, 0)$

The coordinates of the vertices are $(\pm 4, 0)$

Eccentricity, $e = \frac{c}{a} = \frac{5}{4}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 9}{4} = \frac{9}{2}$

Question 2:

Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus

rectum of the hyperbola $\frac{y^2}{9} - \frac{x^2}{27} = 1$

Solution:

The given equation is $\frac{y^2}{9} - \frac{x^2}{27} = 1$ or $\frac{y^2}{3^2} - \frac{x^2}{(\sqrt{27})^2} = 1$

On comparing this equation with the standard equation of hyperbola i.e., $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, we obtain
 $a = 3$ and $b = \sqrt{27}$

We know that, $c^2 = a^2 + b^2$

Hence,

$$\Rightarrow c^2 = 3^2 + (\sqrt{27})^2$$

$$\Rightarrow c^2 = 9 + 27$$

$$\Rightarrow c^2 = 36$$

$$\Rightarrow c = 6$$

Therefore,

The coordinates of the foci are $(0, \pm 6)$

The coordinates of the vertices are $(0, \pm 3)$

Eccentricity, $e = \frac{c}{a} = \frac{6}{3} = 2$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 27}{3} = 18$

Question 3:

Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola $9y^2 - 4x^2 = 36$

Solution:

The given equation is $9y^2 - 4x^2 = 36$

It can be written as

$$9y^2 - 4x^2 = 36$$

$$\Rightarrow \frac{y^2}{4} - \frac{x^2}{9} = 1$$

$$\Rightarrow \frac{y^2}{2^2} - \frac{x^2}{3^2} = 1 \quad \dots(1)$$

On comparing equation (1) with the standard equation of hyperbola, i.e., $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, we obtain
 $a = 2$ and $b = 3$

We know that, $c^2 = a^2 + b^2$

Hence,

$$\Rightarrow c^2 = 2^2 + 3^2$$

$$\Rightarrow c^2 = 4 + 9$$

$$\Rightarrow c^2 = 13$$

$$\Rightarrow c = \sqrt{13}$$

Therefore,

The coordinates of the foci are $(0, \pm\sqrt{13})$

The coordinates of the vertices are $(0, \pm 2)$

Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{13}}{2}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 9}{2} = 9$

Question 4:

Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola $16x^2 - 9y^2 = 576$

Solution:

The given equation is $16x^2 - 9y^2 = 576$

It can be written as

$$16x^2 - 9y^2 = 576$$

$$\Rightarrow \frac{x^2}{36} - \frac{y^2}{64} = 1$$

$$\Rightarrow \frac{x^2}{6^2} - \frac{y^2}{8^2} = 1 \quad \dots(1)$$

On comparing equation (1) with the standard equation of hyperbola, i.e., $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we obtain $a = 6$ and $b = 8$

We know that, $c^2 = a^2 + b^2$

Hence,

$$\Rightarrow c^2 = 6^2 + 8^2$$

$$\Rightarrow c^2 = 36 + 64$$

$$\Rightarrow c^2 = 100$$

$$\Rightarrow c = 10$$

Therefore,

The coordinates of the foci are $(\pm 10, 0)$

The coordinates of the vertices are $(\pm 6, 0)$

Eccentricity, $e = \frac{c}{a} = \frac{10}{6} = \frac{5}{3}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 64}{6} = \frac{64}{3}$

Question 5:

Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola $5y^2 - 9x^2 = 36$

Solution:

The given equation is $5y^2 - 9x^2 = 36$

It can be written as

$$\begin{aligned} 5y^2 - 9x^2 &= 36 \\ \Rightarrow \frac{y^2}{\left(\frac{36}{5}\right)} - \frac{x^2}{4} &= 1 \\ \Rightarrow \frac{y^2}{\left(\frac{6}{\sqrt{5}}\right)^2} - \frac{x^2}{2^2} &= 1 \quad \dots(1) \end{aligned}$$

On comparing equation (1) with the standard equation of hyperbola, i.e., $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, we obtain

$$a = \frac{6}{\sqrt{5}} \text{ and } b = 2$$

We know that, $a^2 + b^2 = c^2$

Hence,

$$\Rightarrow c^2 = \left(\frac{6}{\sqrt{5}}\right)^2 + (2)^2$$

$$\Rightarrow c^2 = \frac{36}{5} + 4$$

$$\Rightarrow c^2 = \frac{56}{5}$$

$$\Rightarrow c = \sqrt{\frac{56}{5}}$$

$$\Rightarrow c = \frac{2\sqrt{14}}{\sqrt{5}}$$

Therefore,

The coordinates of the foci are $\left(0, \pm \frac{2\sqrt{14}}{\sqrt{5}}\right)$

The coordinates of the vertices are $\left(0, \pm \frac{6}{\sqrt{5}}\right)$

$$e = \frac{c}{a} = \frac{\left(\frac{2\sqrt{14}}{\sqrt{5}}\right)}{\left(\frac{6}{\sqrt{5}}\right)} = \frac{\sqrt{14}}{3}$$

Eccentricity,

$$= \frac{2b^2}{a} = \frac{2 \times 4}{\left(\frac{6}{\sqrt{5}}\right)} = \frac{4\sqrt{5}}{3}$$

Length of latus rectum

Question 6:

Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola $49y^2 - 16x^2 = 784$

Solution:

The given equation is $49y^2 - 16x^2 = 784$

It can be written as

$$49y^2 - 16x^2 = 784$$

$$49y^2 - 16x^2 = 784$$

$$\Rightarrow \frac{y^2}{16} - \frac{x^2}{49} = 1$$

$$\Rightarrow \frac{y^2}{4^2} - \frac{x^2}{7^2} = 1 \quad \dots(1)$$

On comparing equation (1) with the standard equation of hyperbola, i.e., $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, we obtain $a = 4$ and $b = 7$

We know that, $a^2 + b^2 = c^2$

Hence,

$$\Rightarrow c^2 = (4)^2 + (7)^2$$

$$\Rightarrow c^2 = 16 + 49$$

$$\Rightarrow c^2 = 65$$

$$\Rightarrow c = \sqrt{65}$$

Therefore,

The coordinates of the foci are $(0, \pm\sqrt{65})$

The coordinates of the vertices are $(0, \pm 4)$

Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{65}}{4}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 49}{4} = \frac{49}{2}$

Question 7:

Find the equation of the hyperbola satisfying the given conditions: Vertices $(\pm 2, 0)$, Foci $(\pm 3, 0)$

Solution:

Vertices $(\pm 2, 0)$, Foci $(\pm 3, 0)$

Here, the vertices are on the ix-axis.

Therefore, the equation of the hyperbola is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Since the vertices are $(\pm 2, 0)$, $a = 2$

Since the foci are $(\pm 3, 0)$, $c = 3$

We know that, $a^2 + b^2 = c^2$

Hence,

$$\Rightarrow 2^2 + b^2 = 3^2$$

$$\Rightarrow b^2 = 9 - 4$$

$$\Rightarrow b^2 = 5$$

Thus, the equation of the hyperbola is $\frac{x^2}{4} - \frac{y^2}{5} = 1$

Question 8:

Find the equation of the hyperbola satisfying the given conditions: Vertices $(0, \pm 5)$, Foci $(0, \pm 8)$

Solution:

Vertices $(0, \pm 5)$, Foci $(0, \pm 8)$

Here, the vertices are on the y -axis.

Therefore, the equation of the hyperbola is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Since the vertices are $(0, \pm 5)$, $a = 5$

Since the foci are $(0, \pm 8)$, $c = 8$

We know that, $a^2 + b^2 = c^2$

Hence,

$$\Rightarrow 5^2 + b^2 = 8^2$$

$$\Rightarrow b^2 = 64 - 25$$

$$\Rightarrow b^2 = 39$$

Thus, the equation of the hyperbola is $\frac{y^2}{25} - \frac{x^2}{39} = 1$

Question 9:

Find the equation of the hyperbola satisfying the given conditions: Vertices $(0, \pm 3)$, Foci $(0, \pm 5)$

Solution:

Vertices $(0, \pm 3)$, Foci $(0, \pm 5)$

Here, the vertices are on the y -axis.

Therefore, the equation of the hyperbola is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Since the vertices are $(0, \pm 3)$, $a = 3$

Since the foci are $(0, \pm 5)$, $c = 5$

We know that, $a^2 + b^2 = c^2$

Hence,

$$\Rightarrow 3^2 + b^2 = 5^2$$

$$\Rightarrow b^2 = 25 - 9$$

$$\Rightarrow b^2 = 16$$

Thus, the equation of the hyperbola is $\frac{y^2}{9} - \frac{x^2}{16} = 1$

Question 10:

Find the equation of the hyperbola satisfying the given conditions: Foci $(\pm 5, 0)$, the transverse axis is of length 8.

Solution:

Foci $(\pm 5, 0)$, the transverse axis is of length 8.

Here, the foci are on the x -axis.

Therefore, the equation of the hyperbola is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Since the foci are $(\pm 5, 0)$, $c = 5$

Since the length of the transverse axis is 8,

Then,

$$\Rightarrow 2a = 8$$

$$\Rightarrow a = 4$$

We know that, $a^2 + b^2 = c^2$

Hence,

$$\Rightarrow 4^2 + b^2 = 5^2$$

$$\Rightarrow b^2 = 25 - 16$$

$$\Rightarrow b^2 = 9$$

Thus, the equation of the hyperbola is $\frac{x^2}{16} - \frac{y^2}{9} = 1$

Question 11:

Find the equation of the hyperbola satisfying the given conditions: Foci $(0, \pm 13)$, the conjugate axis is of length 24.

Solution:

Foci $(0, \pm 13)$, the conjugate axis is of length 24

Here, the foci are on the y -axis.

Therefore, the equation of the hyperbola is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Since the foci are $(0, \pm 13)$, $c = 13$

Since the length of the transverse axis is 24,

Then,

$$\Rightarrow 2b = 24$$

$$\Rightarrow b = 12$$

We know that, $a^2 + b^2 = c^2$

Hence,

$$\Rightarrow a^2 + 12^2 = 13^2$$

$$\Rightarrow a^2 = 169 - 144$$

$$\Rightarrow a^2 = 25$$

Thus, the equation of the hyperbola is $\frac{y^2}{25} - \frac{x^2}{144} = 1$

Question 12:

Find the equation of the hyperbola satisfying the given conditions: Foci $(\pm 3\sqrt{5}, 0)$, the latus rectum is of length 8.

Solution:

Foci $(\pm 3\sqrt{5}, 0)$, the latus rectum is of length 8

Here, the foci are on the x -axis.

Therefore, the equation of the hyperbola is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Since the foci are $(\pm 3\sqrt{5}, 0)$, $c = \pm 3\sqrt{5}$

Since the length of latus rectum is 8

Then,

$$\Rightarrow \frac{2b^2}{a} = 8$$

$$\Rightarrow b^2 = 4a$$

We know that, $a^2 + b^2 = c^2$

Hence,

$$\Rightarrow a^2 + 4a = 45$$

$$\Rightarrow a^2 + 4a - 45 = 0$$

$$\Rightarrow a^2 + 9a - 5a - 45 = 0$$

$$\Rightarrow a(a+9) - 5(a+9) = 0$$

$$\Rightarrow (a+9)(a-5) = 0$$

$$\Rightarrow a = -9, 5$$

Since a is non-negative, $a = 5$

Therefore,

$$\Rightarrow b^2 = 4a$$

$$\Rightarrow b^2 = 4 \times 5$$

$$\Rightarrow b^2 = 20$$

Thus, the equation of the hyperbola is $\frac{x^2}{25} - \frac{y^2}{20} = 1$

Question 13:

Find the equation of the hyperbola satisfying the given conditions: Foci $(\pm 4, 0)$, the latus rectum is of length 12.

Solution:

Foci $(\pm 4, 0)$, the latus rectum is of length 12

Here, the foci are on the x -axis.

Therefore, the equation of the hyperbola is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Since the foci are $(\pm 4, 0)$, $c = 4$

Since the length of latus rectum is 12

Then,

$$\Rightarrow \frac{2b^2}{a} = 12$$

$$\Rightarrow b^2 = 6a$$

We know that, $a^2 + b^2 = c^2$

Hence,

$$\Rightarrow a^2 + 6a = 16$$

$$\Rightarrow a^2 + 6a - 16 = 0$$

$$\Rightarrow a^2 + 8a - 2a - 16 = 0$$

$$\Rightarrow a(a+8) - 2(a+8) = 0$$

$$\Rightarrow (a+8)(a-2) = 0$$

$$\Rightarrow a = -8, 2$$

Since a is non-negative, $a = 2$

Therefore,

$$\Rightarrow b^2 = 6a$$

$$\Rightarrow b^2 = 6 \times 2$$

$$\Rightarrow b^2 = 12$$

Thus, the equation of the hyperbola is $\frac{x^2}{4} - \frac{y^2}{12} = 1$

Question 14:

Find the equation of the hyperbola satisfying the given conditions: Vertices $(\pm 7, 0)$, $e = \frac{4}{3}$.

Solution:

Vertices $(\pm 7, 0)$, $e = \frac{4}{3}$

Here, the foci are on the x -axis.

Therefore, the equation of the hyperbola is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Since the vertices are $(\pm 7, 0)$, $a = 7$

It is given that $e = \frac{4}{3}$

Hence,

$$\Rightarrow e = \frac{c}{a} = \frac{4}{3}$$

$$\Rightarrow \frac{c}{7} = \frac{4}{3}$$

$$\Rightarrow c = \frac{28}{3}$$

We know that, $a^2 + b^2 = c^2$

Therefore,

$$\Rightarrow 7^2 + b^2 = \left(\frac{28}{3}\right)^2$$

$$\Rightarrow b^2 = \frac{784}{9} - 49$$

$$\Rightarrow b^2 = \frac{784 - 441}{9}$$

$$\Rightarrow b^2 = \frac{343}{9}$$

Thus, the equation of the hyperbola is $\frac{x^2}{49} - \frac{9y^2}{343} = 1$

Question 15:

Find the equation of the hyperbola satisfying the given conditions: Foci $(0, \pm\sqrt{10})$, passing through $(2, 3)$

Solution:

Foci $(0, \pm\sqrt{10})$, passing through $(2, 3)$

Here, the foci are on the y -axis.

Therefore, the equation of the hyperbola is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Since the foci are $(0, \pm\sqrt{10})$, $c = \sqrt{10}$.

We know that, $a^2 + b^2 = c^2$

Hence,

$$\Rightarrow a^2 + b^2 = 10$$

$$\Rightarrow b^2 = 10 - a^2 \quad \dots(1)$$

Since the hyperbola passes through point $(2, 3)$

$$\frac{9}{a^2} - \frac{4}{b^2} = 1 \quad \dots(2)$$

From equations (1) and (2), we obtain

$$\begin{aligned}
&\Rightarrow \frac{9}{a^2} - \frac{4}{10-a^2} = 1 \\
&\Rightarrow 9(10-a^2) - 4a^2 = a^2(10-a^2) \\
&\Rightarrow 90 - 9a^2 - 4a^2 = 10a^2 - a^4 \\
&\Rightarrow a^4 - 23a^2 + 90 = 0 \\
&\Rightarrow a^4 - 18a^2 - 5a^2 + 90 = 0 \\
&\Rightarrow a^2(a^2 - 18) - 5(a^2 - 18) = 0 \\
&\Rightarrow (a^2 - 18)(a^2 - 5) = 0 \\
&\Rightarrow a^2 = 18 \text{ or } 5
\end{aligned}$$

In hyperbola, $c > a$, i.e., $c^2 > a^2$

Therefore,

$$\begin{aligned}
&\Rightarrow a^2 = 5 \\
&\Rightarrow b^2 = 10 - a^2 \\
&\Rightarrow b^2 = 10 - 5 \\
&\Rightarrow b^2 = 5
\end{aligned}$$

Thus, the equation of the hyperbola is $\frac{y^2}{5} - \frac{x^2}{5} = 1$