Miscellaneous Example

Q1. The mean and variance of eight observations are 9 and 9.25, respectively. If six of the observations are 6, 7, 10, 12, 12 and 13, find the remaining two observations.

A.1. Let the other two observation be x and y.

Therefore, the series is 6, 7, 10, 12, 12, 13, x, y.

So, Mean $\bar{x} = 9$

$$\Rightarrow \frac{6+7+10+12+12+13+x+y}{8} = 9$$

$$\Rightarrow x + y = 72 - 60$$

$$\Rightarrow x + y = 12 \quad \ldots\ldots (1)$$

And, variance = 9.25

$$\Rightarrow \frac{1}{8}\sum_{i=1}^{8}\left(x_i - \bar{x}\right)^2 = 9.25$$

$$\Rightarrow (-3)^2 + (-2)^2 + (1)^2 + (3)^2 + (3)^2 + (4)^2 + (x-9)^2 + (y-9)^2 = 9.25 \times 8$$

$$\Rightarrow 9 + 4 + 1 + 9 + 9 + 16 + x^2 + 81 - 18x + y^2 + 81 - 18y = 74$$

$$\Rightarrow 210 + x^2 + y^2 - 18(x+y) = 74$$

$$\Rightarrow x^2 + y^2 = 74 - 210 + 18(12) \qquad [\therefore \text{equation (1)}]$$

$$\Rightarrow x^2 + y^2 = 80 \qquad \ldots\ldots(2)$$

Squaring Equation (1) we get,

$$x^2 + y^2 + 2xy = 144$$

$$\Rightarrow 80 + 2xy = 144$$

$$\Rightarrow 2xy = 64 \qquad (3)$$

Subtracting (3) from (2) we get,

$x^2 + y^2 - 2xy = 16$

$\Rightarrow (x - y)^2 = 4^2$

$\Rightarrow x - y = \pm 4 \quad \ldots\ldots(4)$

From (1) and (4)

Case I, $x + y = 12$ and $x - y = 4$

$\Rightarrow x = 8$ and $y = 4$

Case II, $x + y = 12$ and $x - y = -4$

$\Rightarrow x = 4$ and $y = 8$.

Hence, the remaining observations are 8 and 4.

Q2. The mean and variance of 7 observations are 8 and 16, respectively. If five of the observations are 2, 4, 10, 12, 14. Find the remaining two observations.

A.2. Let the other two observation be x and y. Then, the series is 2, 4, 10, 12, 14, x, y.

So, mean, $\bar{x} = \dfrac{2 + 4 + 10 + 12 + 14 + x + y}{7} = 8.$

$\Rightarrow 8 \times 7 = 42 + x + y.$

$\Rightarrow x + y = 14 \quad \ldots.(1)$

And variance = $\dfrac{1}{n}\sum_{i=1}^{7}(x_i - \bar{x})^2$

$\Rightarrow 16 = \dfrac{1}{7}\left[(-6)^2 + (-4)^2 + (-2)^2 + (4)^2 + (6)^2 + (x-8)^2 + (y-8)^2\right]$

$\Rightarrow 36 + 16 + 4 + 36 + x^2 64 - 16x + y^2 + 64 - 16y = 112.$

$\Rightarrow 236 + x^2 y^2 - 16(x + y) = 112$

$\Rightarrow x^2 + y^2 = 112 - 236 + 16(14) \qquad$ [Using eqn (1)]

$\Rightarrow x^2 + y^2 = 100 \quad \ldots.(2)$

Squaring eqn (1) we get,

$x^2 + y^2 + 2xy = 196$

$\Rightarrow 2xy = 96 - 100$

$\Rightarrow 2xy = 96 \quad \ldots..(3)$

Subtracting eqn (3) from (2) we get,

$x^2 + y^2 - 2xy = 4$

$\Rightarrow (x - y)^2 = 2^2$

$\Rightarrow x - y = \pm 2. \quad \ldots.(4)$

Using eqn (1) and (2) we get,

Case I. $x + y = 14$ and $x - y = 2$

$\Rightarrow x = 8$ and $y = 6$

Case II. $x + y = 14$ and $x - y = -2$

$\Rightarrow x = 6$ and $y = 8$

Hence, the remaining observations are 6 and 8.

Q3. The mean and standard deviation of six observations are 8 and 4, respectively. If each observation is multiplied by 3, find the new mean and new standard deviation of the resulting observations.

A.3. Let 'x' be the given observations with $n = 6$.

Then, Σ mean, $\bar{x} = \frac{\Sigma x_i}{n}$

$\Rightarrow 8 = \frac{\Sigma x_i}{6}$

$\Rightarrow \Sigma x_i = 48$ …..(1)

And $\sigma_x = \sqrt{\frac{1}{n}\Sigma\left(x_i - \bar{x}\right)^2}$

$\Rightarrow 4^2 = \frac{1}{6}\Sigma\left(x_i - \bar{x}\right)^2$

$\Rightarrow \Sigma\left(x_i - \bar{x}\right)^2 = 96$ ….(2)

Let '*y*' be the new observations with same *n*.

So, new mean,

$\bar{y} = 3\frac{\Sigma x_i}{n}$

$= \frac{3 \times 48}{6}$ [using(1)]

= 24

or $\bar{y} = 3\bar{x}$ (3)

and $y_i = 3x_i$ (4)

Putting (3) and (4) in eqn (2) we get,

$\sum\left(\frac{y_i}{3} - \frac{\bar{y}}{3}\right)^2 = 96$

$\sum(yi - \bar{y})^2 = 96 \times 9$

$\Rightarrow \frac{\sum\left(y_i - \bar{y}\right)^2}{n} = \frac{864}{6}$ [dividing both sider by $n = 6$]

$\Rightarrow \sqrt{\frac{1}{n}\sum\left(y_i - \bar{y}\right)^2} = \sqrt{144}$ [Taking undermost on both sides]

$\Rightarrow \sigma_y = 12$

Hence the new mean and standard deviation is 24 and 12.

Q4. Given that $\bar{x}$ is the mean and σ2 is the variance of n observations x_1, x_2, ...,xn. Prove that the mean and variance of the observations ax_1, ax_2, ax_3, ..., ax_n are a $\bar{x}$ and a_2 σ_2, respectively, (a ≠ 0).

A.4. For *n* observations $x_1, x_2, \ldots, x_n$.

We have mean = $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ (1)

and variance = $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$ (2)

Let y_i be the new observations with same *n*.

So, $y_i = ax_i$ (3)

Now mean, $\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n} = \frac{\sum_{i=1}^{n} ax_i}{n} = \frac{a\sum_{i=1}^{n} y_i}{n} = a\bar{x}$ [Using (1)]

So $\bar{y} = a\bar{x} \quad (4)$

And, putting (3) and (4) in (2) we get,

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_i}{a} - \frac{\bar{y}}{a}\right)^2$$

$$\Rightarrow \sigma^2 = \frac{1}{a^2}\left[\frac{1}{n}\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2\right]$$

$$\Rightarrow \left(\sigma'\right)^2 = \sigma^2 \alpha^2.$$

Hence, the mean and variance of $ax_1, ax_2, \ldots\ldots, ax_n$ are $a\bar{x}$ and $a^2\sigma^2$.

Q5. The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On rechecking, it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation in each of the following cases:

(i) If wrong item is omitted. (ii) If it is replaced by 12.

A.5. (i) Given, $n = 20$.

Incorrect mean $(\bar{x}) = 10$

Incorrect standard deviation $(\sigma) = 2$

We know that,

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\Rightarrow 10 = \frac{1}{20}\sum_{i=1}^{20} x_i$$

$$\Rightarrow \sum_{i=1}^{20} x_i = 200.$$

So, incorrect sum of observation = 200.

correct sum of observation =200 – 8 = 192

And correct mean $= \frac{192}{19} = 10.1$

And Standard deviation, $\sigma = \sqrt{\frac{1}{n}\sum_{i}^{n} x_i^2 - \frac{1}{n^2}\left(\sum_{i=1}^{n} x_i\right)^2} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - (\bar{x})^2}$

$$\Rightarrow 2 = \sqrt{\frac{1}{20} x \text{ incorrect } \sum_{i=1}^{n} x_i^2 - (10)^2}$$

$$\Rightarrow 4 = \frac{1}{20} \times \text{ incorrect } \sum_{i=1}^{n} x_i^2 - 100$$

$$\Rightarrow \text{ incorrect } \sum_{i=1}^{n} x_i^2 = 2080.$$

Therefore, correct $\sum_{i=1}^{n} x_i^2 =$ incorrect $\sum_{i=1}^{n} x_i^2 - 8^2$

$= 2000 - 64$

$= 2016$

Hence, correct standard deviation $= \sqrt{\frac{\text{correct } \sum x_i^2}{n-1} - (\text{ correct mean })^2}$

$$= \sqrt{\frac{2016}{19} - (10.1)^2}$$

$= \sqrt{106.1 - 102.01}$
$= \sqrt{4.09}$
= 2.02.

(ii) Correct rum of observation = Incorrect sum $-8+12$

$= 200 - 8 + 12$
= 204.

$\therefore$ Correct mean $= \dfrac{\text{correct sum}}{n} = \dfrac{204}{20} = 10.2$

And

$$\text{correct} \sum_{i=1}^{n} x_i^2 = \text{ incorrect} \sum_{i=1}^{n} x_i^2 - 8^2 + 12^2$$
$$= 2080 - 64 + 144$$
$$= 2160.$$

So, correct standard deviation $= \sqrt{\dfrac{\text{correct} \sum 3_i^2}{x} - (\text{ correct mean })^2}$

$$= \sqrt{\frac{2160}{20} - (10.2)^2}$$
$$= \sqrt{108 - 104.04}$$
$$= \sqrt{3.96}$$
$$= 1.98$$

Q6. The mean and standard deviation of marks obtained by 50 students of a class in three subjects, Mathematics, Physics and Chemistry are given below:

Subject	**Mathematics**	**Physics**	**Chemistry**
Mean	**42**	**32**	**40.9**
Standard deviation	**12**	**15**	**20**

Which of the three subjects shows the highest variability in marks and which shows the lowest?

A.6. C.V in mathematics $= \dfrac{12}{42} \times 100 = 28.57.$

C.V in Physics = $\dfrac{15}{32} \times 100 = 46.87.$

C.V in chemistry $= \dfrac{20}{40.9} \times 100 = 48.89$

$\therefore$ Chemistry has highest variability and mathematics has lowest variability.

Q7. The mean and standard deviation of a group of 100 observations were found to be 20 and 3, respectively. Later on it was found that three observations were incorrect, which were recorded as 21, 21 and 18. Find the mean and standard deviation if the incorrect observations are omitted.

A.7. Given, n = 100.

incorrect mean ($\bar{x}$) = 20.

incorrect standard deviation (σ) = 3

We know that,

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} n_i$$

$$\Rightarrow 20 = \frac{1}{100}\sum_{i=1}^{n} n_i$$

$$\Rightarrow \sum_{i=1}^{n} n_i = 2000.$$

So, incorrect sum of observation = 2000

Correct sum of observation $= 2000 - 21 - 21 - 18$

$= 1940$

$\therefore$ Correct mean $= \dfrac{\text{correct sum}}{100-3} = \dfrac{1940}{97} = 20.$

Standard deviation, $\sigma = \sqrt{\dfrac{1}{n}\sum_{i=1}^{n} x_i^2 = \left(\overline{x}\right)^2}$

$$\Rightarrow 3 = \sqrt{\frac{1}{100}\times \text{incorrect} \sum_{i=1}^{n} x_i^2 - (20)^2}$$

$$\Rightarrow 3^2 = \frac{1}{100}\times \text{incorrect} \sum_{i=1}^{n} x_i^2 - 400$$

$$\Rightarrow \text{incorrect} \sum_{i=1}^{n} x_i^2 - 40900.$$

So, correct

$$\sum_{i=1}^{n} x_i^2 = \text{incorrect} \sum_{i=1}^{n} x_i^2 - (21)^2 - (21)^2 - (18)^2$$

$$= 40900 - 441 - 441 - 324$$

$$= 39694$$

$\therefore$ Correct standard deviation $= \sqrt{\dfrac{\text{correct} \sum_{i=1}^{n} x_i^2}{n-3} - (\text{correct mean})^2}$

$$= \sqrt{\frac{39694}{97} - (20)^2}$$

$$= \sqrt{409.216 - 400}$$

$$= \sqrt{9.216}$$

$$= 3.036.$$