Exerise 7.3

1. ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see

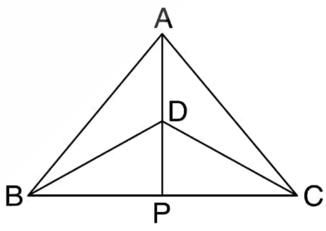


figure). If AD is extended to intersect BC at P, show that

(*i*) $\Delta ABD \cong \Delta ACD$

(*ii*) $\Delta ABP \cong \Delta ACP$

(*iii*) *AP bisects $\angle A$ as well as $\angle D$.*

(*iv*) *AP is the perpendicular bisector of BC.*

Sol. (*i*) Consider triangles ABD and ACD,

We have $AB = AC$ [Given]

$BD = CD$ [Given]

$AD = DA$ [Common]

So, $\Delta ABD \cong \Delta ACD$ [SSS rule]

$\therefore$ $\angle BAD = \angle CAD$ and $\angle ABD = \angle ACD$ ...(*i*) [CPCT]

(*ii*) Consider triangles ABP and ACP,

We have $AB = AC$ [Given]

$AP = PA$ [Common]

and $\angle BAP = \angle CAP$ [From (*i*)]

$\therefore$ $\Delta ABP \cong \Delta ACP$ [SAS rule]

$\Rightarrow$ $BP = PC$, $\angle BPA = \angle CPA$

(*iii*) $\angle BAP = \angle CAP$ [From result (*ii*)]

$\Rightarrow$ AP bisects $\angle A$.

Also, $\angle BAD + \angle ABD = \angle CAD = \angle ACD$ [From (*i*)]

$\Rightarrow$ $\angle BDP = \angle CDP$ [Exterior angle property]

So, DP bisects $\angle D$.

Hence, AP bisects $\angle A$ as well as $\angle D$.

(*iv*) Also, $\angle BPA + \angle CPA = 180°$ [Linear pair]

$\Rightarrow$ $2\angle BPA = 180°$

$\Rightarrow$ $\angle BPA = 90°$ [From result (*ii*)]

As $BP = CP$ [From result (*ii*)]

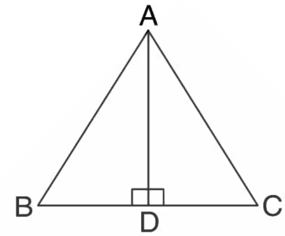
and $AP \perp BC$ [Proved above]

$\Rightarrow$ AP is perpendicular bisector of BC.

2. *AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that*

(*i*) *AD bisects BC* (*ii*) *AD bisects $\angle A$.*

Sol. Consider triangles ABD and ACD,



We have AB = AC [Given]

AD = AD [Common]

$\angle$ADB = $\angle$ADC [90° each]

$\therefore$ ΔABD $\cong$ ΔACD [RHS rule]

(*i*) $\therefore$ BD = CD [CPCT]

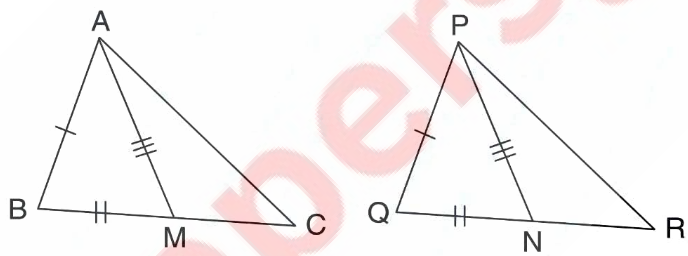
$\Rightarrow$ AD bisects BC.

(*ii*) and $\angle$BAD = $\angle$CAD [CPCT]

AD bisects $\angle$BAC, *i.e.*, $\angle$A.

3. *Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of ΔPQR (see figure). Show that:*

(*i*) $\Delta ABM \cong \Delta PQN$ (*ii*) $\Delta ABC \cong \Delta PQR$.



Sol. (*i*) M and N are mid-points of BC and QR respectively, as AM and PN are medians.

$\therefore$ BM = $\frac{1}{2}$BC and QN = $\frac{1}{2}$QR ...(*i*)

Also, BC = QR [Given]

$\Rightarrow$ $\frac{1}{2}$BC = $\frac{1}{2}$QR $\Rightarrow$ BM = QN ...(*ii*) [From (*i*)]

Consider triangles ABM and PQN,

We have AB = PQ [Given]

AM = PN [Given]

and BM = QN [From (*ii*)]

$\therefore$ Δ ABM $\cong$ Δ PQN [SSS rule]

(*ii*) From result (*i*),

$\angle$ABM = $\angle$PQN $\Rightarrow$ $\angle$ABC = $\angle$PQR ...(*iii*)

Consider triangles ABC and PQR,

We have AB = PQ [Given]

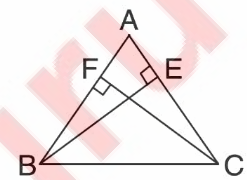
BC = QR [Given]

and $\angle$ ABC = $\angle$ PQR [From (*iii*)]

$\therefore$ Δ ABC $\cong$ Δ PQR. [SAS rule]

4. *BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.*

Sol. Consider triangles BFC and CEB,



We have BE = CF [Given]

BC = CB [Common]

and $\angle$BFC = $\angle$BEC [90° each]

$\therefore$ ΔBFC $\cong$ ΔCEB [RHS rule]

$\Rightarrow$ $\angle$FBC = $\angle$ECB [CPCT]

i.e., $\angle$ABC = $\angle$ACB

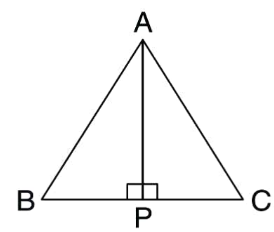
$\Rightarrow$ AC = AB

[Sides opposite to equal angles of a triangles are equal.]

$\Rightarrow$ Δ ABC is an isosceles triangle.

5. *ABC is an isosceles triangle with AB = AC. Draw AP $\perp$ BC to show that $\angle$B = $\angle$C.*

Sol. Consider triangles APB and APC,



We have AB = AC [Given]

AP = PA [Common]

$\angle$ APB = $\angle$ APC [90° each]

$\therefore$ Δ ABP $\cong$ Δ ACP [RHS rule]

$\Rightarrow$ $\angle$ B = $\angle$ C. [CPCT]