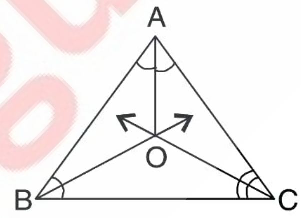
Exerise 7.2

1. *In an isosceles triangle ABC, with AB = AC, the bisectors of ∠B and ∠C intersect each other at O. Join A to O. Show that:*

(*i*) *OB = OC* (*ii*) *AO bisects ∠A.*

Sol. (*i*) In Δ ABC, AB = AC [Given]



⇒ ∠ABC = ∠ACB

[Angles opposite to equal sides are equal.]

$$\Rightarrow \quad \frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB$$

⇒ ∠ OBC = ∠ OCB

[∵ OB and OC are bisectors of ∠ABC and ∠ACB respectively.]

⇒ OC = OB ...(*i*)

[Sides opposite to equal angles of a triangle are equal.]

(*ii*) Consider triangles AOB and AOC,

We have OC = OB [From (*i*)]

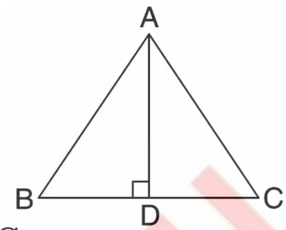
AO = OA [Common]

and $AB = AC$ [Given]

$\therefore \quad \Delta AOB \cong \Delta AOC$ [SSS rule]

$\Rightarrow \quad \angle OAB = \angle OAC$ [CPCT]

$\Rightarrow$ AO bisects $\angle BAC$.



2. *In ΔABC, AD is the perpendicular bisector of BC (see figure). Show that ΔABC is an isosceles triangle in which $AB = AC$.*

Sol. Since AD is perpendicular bisector of BC.

$\therefore \quad BD = CD$ and $\angle ADB = \angle ADC = 90°$ ...(*i*)

Consider triangles ADB and ADC,

We have $BD = DC$

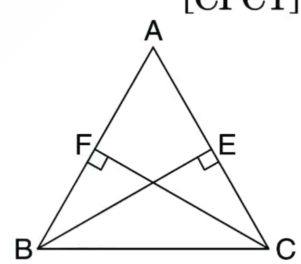
and $\angle ADB = \angle ADC$ [From (*i*)]

$AD = AD$ [Common]

$\therefore \quad \Delta ADB \cong \Delta ADC$ [SAS rule]

$\therefore \quad AB = AC.$

Therefore, ΔABC is an isosceles triangle with $AB = AC$. [CPCT]



3. *ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (see figure). Show that these altitudes are equal.*

Sol. In ΔABC, $AB = AC$ [Given]

$\Rightarrow \quad \angle ACB = \angle ABC$ ...(*i*)

[Angles opposite to equal sides of a triangle are equal.]

Consider triangles BFC and BCE,

We have $\angle FBC = \angle ECB$ [From (*i*)]

$BC = CB$ [Common]

$\angle BFC = \angle CEB$ [90° each]

$\therefore \quad \Delta BCF \cong \Delta CBE$ [AAS rule]

$\Rightarrow \quad CF = BE.$

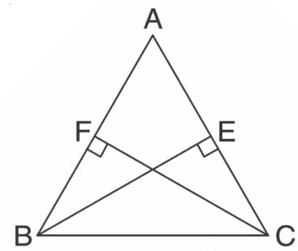
Hence, altitudes to the equal sides of a triangle are equal.

4. *ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (see figure). Show that:*

(*i*) $\Delta ABE \cong \Delta ACF$

(*ii*) *AB = AC, i.e., ABC is an isosceles triangle.*

Sol. (*i*) Consider triangles ABE and ACF,



We have BE = CF [Given]

$\angle$A = $\angle$A [Common]

$\angle$AEB = $\angle$AFC [90° each]

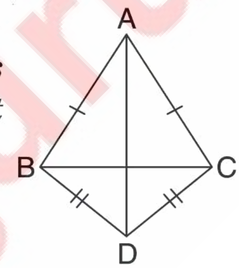
$\therefore$ Δ ABE $\cong$ Δ ACF [AAS rule]

(*ii*) Since Δ ABE $\cong$ Δ ACF [Proved above]

Hence, AB = AC [CPCT]

i.e., Δ ABC is isosceles.

5. *ABC and DBC are two isosceles triangles on same base BC (see figure). Show that $\angle ABD = \angle ACD$.*



Sol. Construction: Join A and D.

Proof: Consider triangles ABD and ACD,

We have AB = AC [Given]

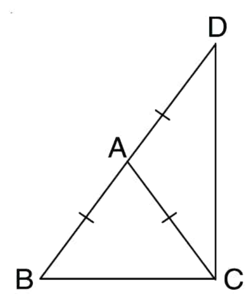
BD = CD [Given]

AD = AD [Common]

$\therefore$ Δ ABD $\cong$ Δ ACD [SSS rule]

$\therefore$ $\angle$ ABD = $\angle$ ACD. [CPCT]

6. *ΔABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB (see figure). Show that $\angle$BCD is a right angle.*



Sol. As AB = AC [Given]

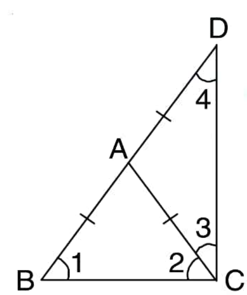
$\Rightarrow$ $\angle$ 1 = $\angle$ 2 ...(*i*)

[Angles opposite to equal sides of a triangle are equal.]

and AC = AD ($\because$ AB = AD)[Given]

$\Rightarrow$ $\angle$ 3 = $\angle$ 4 ...(*ii*)

[Angles opposite to equal sides of a triangle are equal.]



Also, in Δ DBC,

$\angle$ DBC + $\angle$ BCD + $\angle$ CDB = 180°

[Sum of angles of a triangle is 180°.]

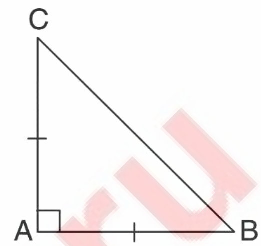
$\Rightarrow$ $\angle$ 1 + ($\angle$ 2 + $\angle$ 3) + $\angle$ 4 = 180°

$\Rightarrow \quad \angle 2 + \angle 2 + \angle 3 + \angle 3 = 180°$ [Using (*i*), (*ii*)]

$\Rightarrow \quad 2(\angle 2 + \angle 3) = 180°$

$\Rightarrow \quad \angle 2 + \angle 3 = 90° \quad \Rightarrow \quad \angle BCD = 90°.$

7. *ABC is a right angled triangle in which ∠A = 90° and AB = AC. Find ∠B and ∠C.*



Sol. As $AB = AC$ [Given]

$\Rightarrow \quad \angle B = \angle C$ ...(*i*) [Angles opposite to equal sides of a triangle are equal.]

In ΔABC, $\angle A + \angle B + \angle C = 180°$

[Sum of angles of a triangle is 180°.]

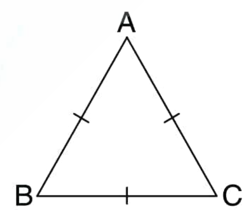
$\Rightarrow \quad 90° + \angle B + \angle B = 180°$

$\Rightarrow \quad 2\angle B = 90°$ [From (*i*)]

$\Rightarrow \quad \angle B = 45°$

$\therefore \quad \angle B = \angle C = 45°.$

8. *Show that the angles of an equilateral triangle are 60° each.*



Sol. As ΔABC is equilateral.

$\therefore \quad AB = BC = CA$

Now $AB = BC \Rightarrow \angle C = \angle A$ ...(*i*)

[Angles opposite to equal sides of a triangle are equal.]

Similarly, $BC = AC \Rightarrow \angle A = \angle B$ ...(*ii*)

[Reason same as above]

$\Rightarrow \quad \angle A = \angle B = \angle C$ ...(*iii*) [From (*i*) and (*ii*)]

In ΔABC, $\angle A + \angle B + \angle C = 180°$

[Sum of angles of a triangle is 180°.]

$\Rightarrow \quad \angle A + \angle A + \angle A = 180°$ [From (*iii*)]

$\Rightarrow \quad 3\angle A = 180°$

$\Rightarrow \quad \angle A = 60°$

$\therefore \quad \angle A = \angle B = \angle C = 60°.$