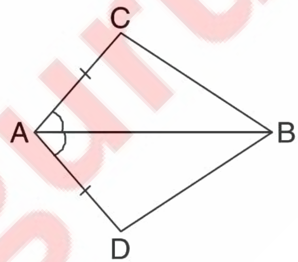
Exerise 7.1



1. *In quadrilateral ABCD, AC = AD and AB bisects ∠A (see figure). Show that ΔABC ≅ ΔABD.*

What can you say about BC and BD?

Sol. Consider triangles ABC and ABD,

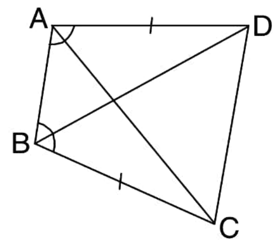
We have AC = AD [Given]

AB = AB [Common]

and ∠CAB = ∠DAB [∵ AB bisects ∠CAD]

∴ Δ ABC ≅ Δ ABD [SAS rule]

Therefore, BC = BD. [CPCT]



2. *ABCD is a quadrilateral in which AD = BC and ∠DAB = ∠CBA (see figure). Prove that*

(*i*) *Δ ABD ≅ Δ BAC*

(*ii*) *BD = AC*

(*iii*) *∠ABD = ∠BAC.*

Sol. (*i*) Consider triangles ABD and ABC, We have AD = BC [Given]

AB = BA [Common]

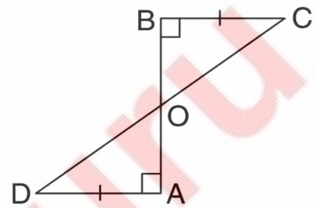
and ∠DAB = ∠CBA [Given]

∴ ΔABD ≅ ΔBAC [SAS rule]

(*ii*) BD = AC. [CPCT]

(*iii*) ∠ABD = ∠BAC. [CPCT]

3. *AD and BC are equal perpendiculars to a line segment AB (see figure). Show that CD bisects AB.*



Sol. AD and BC are perpendiculars to AB.

⇒ AD ∥ BC and CD is transversal.

∴ ∠BCD = ∠ADC, *i.e.*, ∠BCO = ∠ADO [Alternate angles]

Consider triangles BCO and ADO,

We have BC = AD [Given]

∠ OBC = ∠ OAD [90° each]

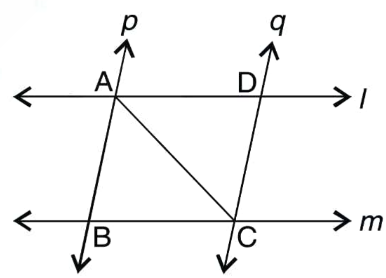
and ∠ BCO = ∠ ADO [Proved above]

∴ ΔOBC ≅ ΔOAD [ASA rule]

Therefore, OB = OA [CPCT]

Hence, CD bisects AB.

4. *l and m are two parallel lines intersected by another pair of parallel lines p and q (see figure). Show that Δ ABC ≅ Δ CDA.*



Sol. Since *l* ∥ *m* and AC is transversal.

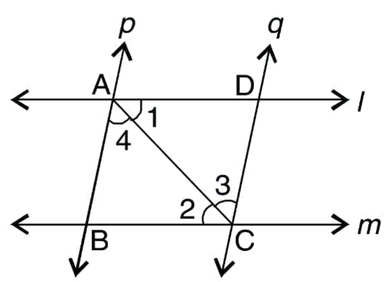
So, ∠ 1 = ∠ 2 ...(*i*)

[Alternate angles]

Again, *p* ∥ *q* and AC is transversal.

So, ∠ 3 = ∠ 4 ...(*ii*)

[Alternate angles]



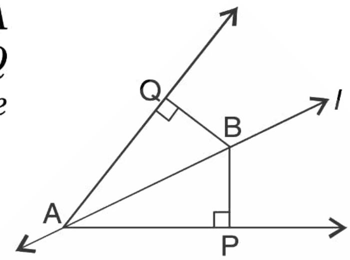
Now, in Δ ADC and Δ ABC,

AC = CA [Common]

∠ 1 = ∠ 2 and ∠ 3 = ∠ 4 [From (*i*) and (*ii*)]

∴ Δ ABC ≅ Δ CDA. [ASA rule]

5. *Line l is the bisector of an angle ∠A and B is any point on l. BP and BQ are perpendiculars from B to the arms of ∠A (see figure). Show that:*



(*i*) *Δ APB ≅ Δ AQB*

(*ii*) *BP = BQ or B is equidistant from the arms of ∠A.*

Sol. ∵ Line *l* is the bisector of ∠ QAP.

∴ ∠ QAB = ∠ PAB ...(*i*)

AB = BA ...(*ii*) [Common]

and ∠ BQA = ∠ BPA. ...(*iii*) [90° each]

(*i*) In triangles AQB and APB,

∠QAB = ∠PAB; AB = BA and ∠BQA = ∠BPA.

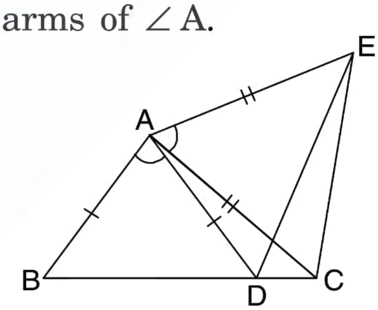
[From (*i*), (*ii*), (*iii*)]

∴ Δ APB ≅ Δ AQB. [AAS rule]

(*ii*) Therefore, BP = BQ [CPCT]

i.e., B is equidistant from the arms of ∠ A.

6. *In figure, AC = AE, AB = AD and ∠BAD = ∠EAC. Show that BC = DE.*



Sol. As ∠BAD = ∠CAE [Given]

⇒ ∠BAD + ∠DAC

= ∠DAC + ∠CAE

[∠ DAC is added to both sides]

⇒ ∠ BAC = ∠ DAE ...(*i*)

Consider triangles BAC and DAE,

We have AB = AD [Given]

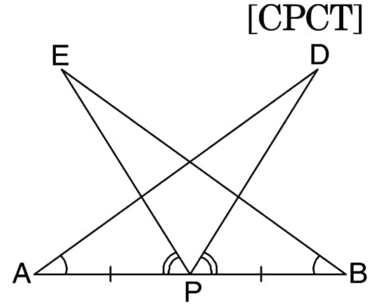
AC = AE [Given]

and ∠BAC = ∠DAE [From (*i*)]

∴ ΔBAC ≅ ΔDAC [SAS rule]

⇒ BC = DE. [CPCT]

7. *AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that ∠BAD = ∠ABE and ∠EPA = ∠DPB (see figure). Show that*



(*i*) $\Delta DAP \cong \Delta EBP$

(*ii*) $AD = BE$.

Sol. Since $\angle EPA = \angle DPB$ [Given]

$\Rightarrow$ $\angle EPA + \angle EPD = \angle EPD + \angle DPB$

[Adding $\angle EPD$ to both sides]

$\Rightarrow$ $\angle APD = \angle BPE$ ...(*i*)

Also, $AP = BP$ ...(*ii*) [Given]

and $\angle DAP = \angle EBP$ ...(*iii*) [Given]

(*i*) Consider triangles DAP and EBP,

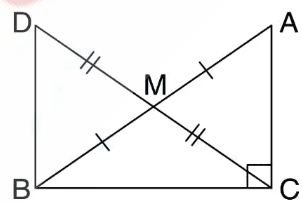
$\angle APD = \angle BPE$, $AP = BP$ and $\angle DAP = \angle EBP$.

[From (*i*), (*ii*), (*iii*)]

$\Delta DAP \cong \Delta EBP$. [SAS rule]

(*ii*) $AD = BE$. [CPCT]

8. *In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see figure). Show that:*



(*i*) $\Delta AMC \cong \Delta BMD$

(*ii*) $\angle DBC$ *is a right angle.*

(*iii*) $\Delta DBC \cong \Delta ACB$

(*iv*) $CM = \frac{1}{2}AB$.

Sol. (*i*) Consider triangles AMC and DMB,

We have $AM = BM$ [Given]

$CM = DM$ [Given]

and $\angle AMC = \angle BMD$ [Vertically opposite angles]

$\therefore$ $\Delta AMC \cong \Delta BMD$ [SAS rule]

(*ii*) As $\angle BAC = \angle DBA$ [CPCT, from part (*i*)]

and AB is the transversal.

So, $DB \parallel AC$ ...(*i*)

We have $AC \perp BC$ ...(*ii*) [Given]

So, $DB \perp BC$ [From (*i*) and (*ii*)]

i.e., $\angle DBC$ is a right angle, *i.e.,* $\angle DBC = 90°$.

(*iii*) Consider triangles ABC and DCB.

We have $AC = DB$ [Since $\Delta AMC \cong \Delta BMD$; result (*i*)]

$BC = CB$ [Common]

and $\angle ACB = \angle DBC$ [Each 90°]

$\therefore$ $\Delta DBC \cong \Delta ACB$ [SAS rule]

(*iv*) As $DC = AB$ [CPCT from part (*iii*)]

$\Rightarrow$ $2CM = AB$ [$\because$ M is mid-point of DC]

$\Rightarrow$ $CM = \frac{1}{2}AB$.