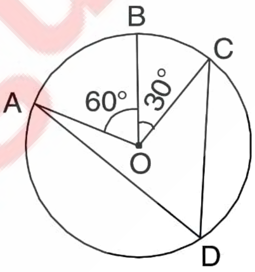
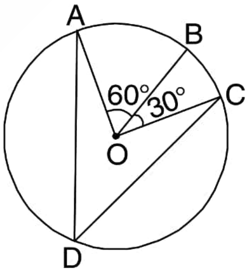
Exercise 9.3

1. *In the adjoining figure, A, B and C are three points on a circle with centre O such that ∠BOC = 30° and ∠AOB = 60°. If D is a point on the circle other than the arc ABC, find ∠ADC.*



Sol. $\angle AOC = 60° + 30° = 90°$

$$\angle ADC = \frac{1}{2} \angle AOC$$

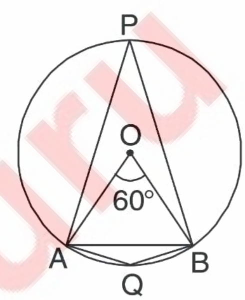


[Angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.]

$\angle ADC = \frac{1}{2} \times 90° = 45°.$

2. *A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.*

Sol. We have, OA = OB = AB Given]



$\therefore$ ΔOAB is equilateral triangle.

$\angle AOB = 60°$

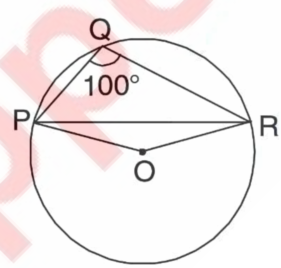
$\angle APB = \frac{1}{2} \angle AOB = \frac{1}{2} \times 60° = 30°.$

Also APBQ is a cyclic quadrilateral.

$\therefore$ $\angle P + \angle Q = 180°$ [Sum of opposite angles of a cyclic quadrilateral is 180°.]

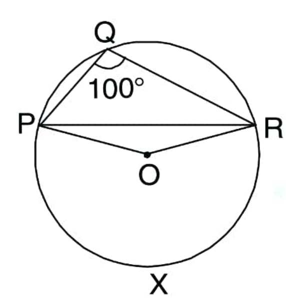
$\Rightarrow$ $30° + \angle Q = 180°$ $\Rightarrow$ $\angle Q = 150°.$

3. *In the figure given below, $\angle PQR = 100°$, where P, Q and R are points on a circle with centre O. Find $\angle OPR$.*



Sol. Let $\angle OPR = x$, then $\angle ORP = x$

and $\angle POR = 180° - 2x$.



$\therefore$ Angle formed by arc PXR at the centre

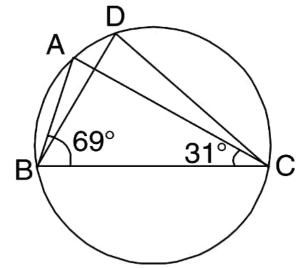
$= 360° - (180° - 2x)$

$= 180° + 2x.$

Also, $\angle PQR = \frac{1}{2}(180° + 2x)$

$\Rightarrow$ $100° = 90° + x$ $\Rightarrow$ $x = 10°.$

4. *In figure, ∠ABC = 69°, ∠ACB = 31°, find ∠BDC.*



Sol. In triangle ABC,

$\angle A + 69^\circ + 31^\circ = 180^\circ$

[Sum of angles of a triangle is 180°]

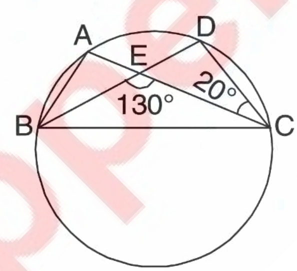
$\Rightarrow \quad \angle A = 180^\circ - 100^\circ = 80^\circ.$

Also, $\angle D = \angle A = 80^\circ$

[Angles in the same segment of a circle]

i.e., $\quad \angle BDC = 80^\circ.$

5. *In figure, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that ∠BEC = 130° and ∠ECD = 20°. Find ∠BAC.*



Sol. Given $\quad \angle BEC = 130^\circ, \angle ECD = 20^\circ.$

$\angle DEC + \angle BEC = 180^\circ$ [Linear pair]

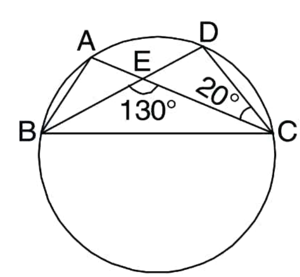
$\therefore \quad \angle DEC = 180^\circ - 130^\circ = 50^\circ.$

In triangle DEC,

$\angle D + 50^\circ + 20^\circ = 180^\circ$

[Sum of angles of a triangle is 180°]

$\Rightarrow \quad \angle D = 110^\circ \quad ...(i)$



Also, $\quad \angle BAC = \angle D$

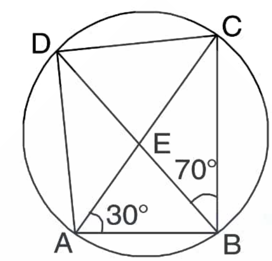
[Angles in the same segment of a circle are equal]

$\therefore \quad \angle BAC = 110^\circ.$ [From (*i*)]

6. *ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If $\angle DBC = 70°$, $\angle BAC$ is $30°$, find $\angle BCD$. Further, if AB = BC, find $\angle ECD$.*

Sol. $\angle CDB = \angle BAC = 30°$ ...(*i*)

[Angles in the same segment]



In triangle BCD,

$$\angle CBD + \angle BCD + \angle CDB = 180°$$

[Sum of angles of a triangle is 180]

$$70° + \angle BCD + 30° = 180°$$

$\Rightarrow$ $\angle BCD = 80°$ ...(*ii*)

Now, in ΔABC,

if AB = BC, then $\angle BCA = \angle BAC = 30°$ ...(*iii*)

[Angles opposite to equal sides are equal]

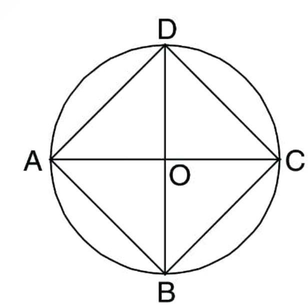
Now, $\angle BCD = \angle BCA + \angle ACD$

$\Rightarrow$ $80° = 30° + \angle ECD$ [$\because$ $\angle ACD = \angle ECD$]

$\Rightarrow$ $\angle ECD = 50°$. [From (*ii*) and (*iii*)]

7. *If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.*

Sol. As AC and BD are the diagonals of a cyclic quadrilateral.

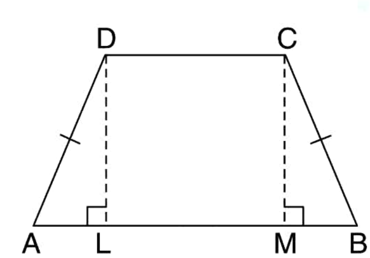


$\therefore$ $\angle ADC$, $\angle BAD$, $\angle ABC$ and $\angle BCD$ are angles in a semicircle. Hence, each angle is $90°$.

As in a quadrilateral each angle is $90°$, hence quadrilateral is a rectangle.

8. *If the non-parallel sides of a trapezium are equal, prove that it is cyclic.*

Sol. **Construction:** Draw DL and CM perpendi-culars to AB.



Proof: In ΔDLA and ΔCMB,

DL = CM

[Distance between parallel lines]

AD = BC [Given]

$\angle DLA = \angle CMB$ [90° each] [Construction]

$\therefore$ $\Delta DLA \cong \Delta CMB$ [RHS]

$\therefore$ $\angle DAL = \angle CBM$ ...(*i*) [CPCT]

Now, AB || CD and AD is transversal

$\therefore$ $\angle CDA + \angle DAL = 180°$

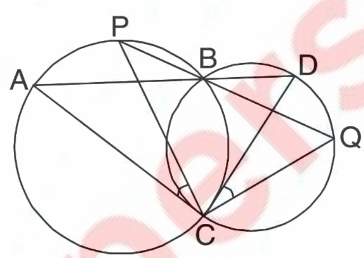
$\Rightarrow$ $\angle CDA + \angle CBM = 180°$ [From (*i*)]

$\Rightarrow$ $\angle CDB + \angle CBA = 180°$ [$\because$ $\angle CBM = \angle CBA$]

As sum of opposite angles of a quadrilateral is 180°, then it is cyclic.

Hence, ABCD is a cyclic quadrilateral.

9. *Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see figure). Prove that $\angle ACP = \angle QCD$.*



Sol. $\angle ACP = \angle ABP$ ...(*i*)

[Angles in the same segment of a circle are equal]

$\angle QCD = \angle QBD$ ...(*ii*) [Reason same as above]

$\angle ABP = \angle QBD$ ...(*iii*) [Vertically opposite angles]

From (*i*), (*ii*) and (*iii*), we get

$\angle ACP = \angle QCD$.

10. *If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.*

Sol. Construction: Join AD.

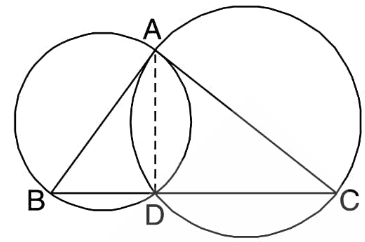
Proof: Let circle with AB as diameter meets BC at D.

Then $\angle ADB = 90°$. [Angle in a semicircle]

Now $\angle ADB + \angle ADC = 180°$ [Linear pair]

$\therefore$ $\angle ADC = 90°$

As we know angle in a semicircle is 90°, therefore, a circle with AC as diameter passes through D.



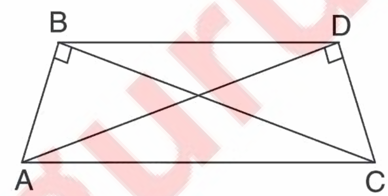
Hence both the circles meet the third side at D.

11. *ABC and ADC are two right triangles with common hypotenuse AC. Prove that ∠CAD = ∠CBD.*

Sol. ∠ABC = ∠ADC = 90°

∴ ACDB is a cyclic quadrilateral.

[As if a line segment subtends equal angles at two other points on the same side of the segment, then the four points are concyclic.]



∴ ∠CAD = ∠CBD.

[Angles in the same segment of a circle are equal.]

12. *Prove that a cyclic parallelogram is a rectangle.*

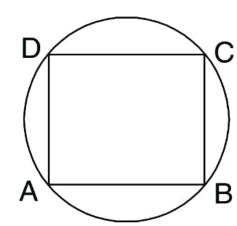
Sol. ABCD is a cyclic parallelogram.

∴ ∠A + ∠C = 180° ...(*i*)

[Sum of opposite angles of a cyclic quadrilateral is 180°.]

Also, ∠A = ∠C ...(*ii*)

[Opposite angles of a parallelogram]



From (*i*) and (*ii*), we have

2∠A = 180° ⇒ ∠A = 90°

As in a parallelogram one angle is 90°, hence it is a rectangle.