Exercise 9.2

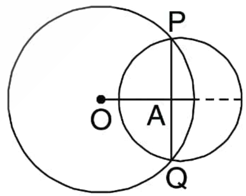
1. *Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.*

Sol. We know that line joining the centres is perpendicular bisector of the common chord.

The common chord passes through the centre of the smaller circle.

$\therefore$ $\angle PAO = 90°$, OA = 4 cm and OP = 5 cm.

$\therefore$ Applying Pythagoras theorem, we have



$$PA = \sqrt{(5)^2 - (4)^2} \text{ cm}$$

$$= \sqrt{25 - 16} \text{ cm}$$

$$= \sqrt{9} \text{ cm} = 3 \text{ cm.}$$

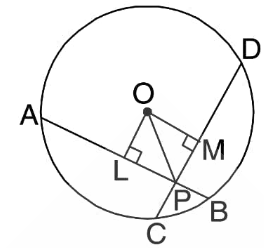
Further, PQ = 2PA = 2 × 3 = 6 cm.

2. *If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.*

Sol. **Construction:** Draw OL and OM perpendiculars to chords AB and CD respectively. Join OP.

To prove: AP = DP and PB = CP.

Proof: Consider triangles OLP and OMP,



OL = OM [Equal chords AB and CD are equidistant from the centre of the circle]

OP is common.

$\angle OLP = \angle OMP$ [90° each]

$\therefore \quad \Delta OLP \cong \Delta OMP$ [RHS]

$\therefore \quad LP = PM$ ...(*i*) [CPCT]

Also, $AL = LB$ ...(*ii*)

[Perpendicular from centre to the chord bisects the chord]

$CM = DM$ ...(*iii*) [Reason same as above]

$AL + LP = DM + MP$ [From (*i*), (*ii*), (*iii*)]

$AP = DP$ ...(*iv*)

Now, $AB = CD$

$\Rightarrow \quad AP + PB = CP + PD$

$\Rightarrow \quad AP + PB = CP + AP$ [From (*iv*)]

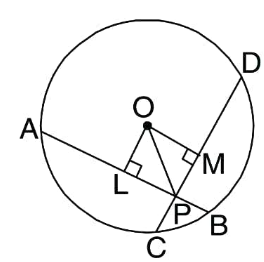
$\Rightarrow \quad PB = CP.$

3. *If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.*

Sol. Construction: Draw OL and OM perpendiculars to chords AB and CD respectively. Join OP.

To prove: $\angle OPL = \angle OPM$

Proof: Consider triangles OLP and OMP,



OL = OM [Equal chords AB and CD are equidistant from the centre of the circle]

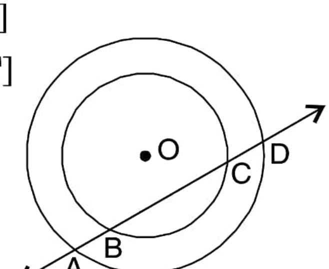
OP is common.

$\angle OLP = \angle OMP$ [90° each]

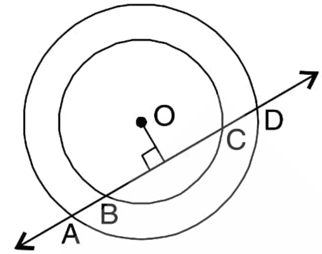
$\Delta OLP \cong \Delta OMP$ [RHS]

$\Rightarrow \quad \angle OPL = \angle OPM.$ [CPCT]

4. *If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see figure).*



Sol. Construction: Draw perpendicular OL, from centre O, to the line l.



Proof: AD is the chord of a bigger circle and OL ⊥ AD.

$\therefore$ AL = DL ...(*i*)

[Perpendicular from centre of the circle to the chord bisects the chord]

Also, BC is the chord of a smaller circle and OL ⊥ BC.

BL = CL ...(*ii*) [Reason same as above]

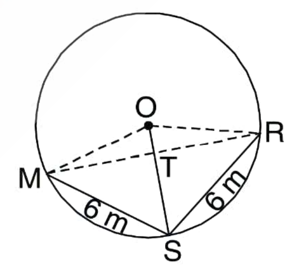
$\Rightarrow$ AL – BL = DL – CL [From (*i*) and (*ii*)]

$\Rightarrow$ AB = CD.

5. *Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5 m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip?*

Sol. $\angle$MOS = $\angle$ROS

[Angles subtended by equal chords are equal]



OM = OR [Radii]

OT is common.

$\therefore$ ΔOMT ≅ ΔORT. [SAS]

$\therefore$ MT = TR ...(*i*)

$\angle$OTM = $\angle$OTR = 90°

Let OT = x

In right-angled triangle OTM,

$$MT = \sqrt{25 - x^2} \quad ...(ii)$$

In right-angled triangle MTS,

$$MT = \sqrt{36 - (5 - x)^2} \quad ...(iii)$$

From (*ii*) and (*iii*), we get

$$\sqrt{25 - x^2} = \sqrt{36 - 25 - x^2 + 10x}$$

$\Rightarrow \quad 25 - x^2 = 11 - x^2 + 10x$

$\Rightarrow \quad 10x = 14 \qquad \Rightarrow \quad x = 1.4$

Substituting this value of x in (ii), we get

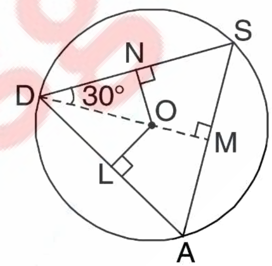
$$MT = \sqrt{25-(1.4)^2} = \sqrt{25-1.96} = \sqrt{23.04} = 4.8 \text{ m.}$$

From (i), MR = 2MT = 2 × 4.8 m = 9.6 m.

6. *A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.*

Sol. Let Ankur, Syed and David are sitting at A, S and D respectively and so, DAS is an equilateral triangle, as if arc are equal then corresponding chords are equal.

$\therefore \quad \angle ADS = 60°.$



Also, $\quad \angle NDO = \frac{1}{2} \angle ADS = 30°$

$$\frac{DN}{OD} = \cos 30°$$

$\Rightarrow \quad \frac{DN}{20} = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad DN = 10 \times 1.73 = 17.3 \text{ m}$

$\therefore \quad DS = 2DN = 2 \times 17.3 = 34.6 \text{ m.}$