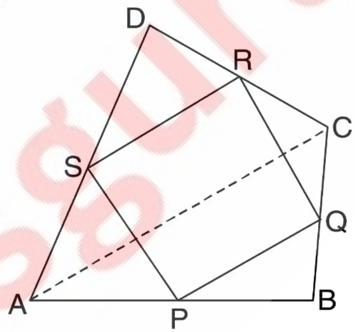
Exerise 8.2

1. *ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see figure). AC is a diagonal. Show that:*



(*i*) *SR* $\parallel$ *AC and SR* $= \frac{1}{2}AC$

(*ii*) *PQ = SR*

(*iii*) *PQRS is a parallelogram.*

Sol. (*i*) Consider triangle ACD,

S and R are mid-points of sides AD and DC respectively.

$\therefore$ SR $\parallel$ AC and SR $= \frac{1}{2}$ AC ...(*i*)

[Line segment joining mid-points of two sides of a triangle is parallel to the third and half of it.]

(*ii*) Consider triangle ABC, P and Q are mid-points of sides AB and BC respectively.

$\therefore$ PQ $\parallel$ AC and PQ $= \frac{1}{2}$ AC ...(*ii*)

[Reason same as above]

From (*i*) and (*ii*),

SR || AC and PQ || AC $\Rightarrow$ SR || PQ ...(*iii*)

and SR = $\frac{1}{2}$AC and PQ = $\frac{1}{2}$AC $\Rightarrow$ SR = PQ. ...(*iv*)

(*iii*) SR || PQ and SR = PQ. [From (*iii*) and (*iv*)]

$\Rightarrow$ PQRS is a parallelogram.

2. *ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.*

Sol. First prove that PQRS is a parallelogram.

(*i*) Consider triangle ACD,

S and R are mid-points of sides AD and DC respectively.

$\therefore$ SR || AC and SR = $\frac{1}{2}$AC ...(*i*)

[Line segment joining mid-points of two sides of a triangle is parallel to the third and half of it.]

(*ii*) Consider triangle ABC, P and Q are mid-points of sides AB and BC respectively.

$\therefore$ PQ || AC and PQ = $\frac{1}{2}$AC ...(*ii*)

[Reason same as above]

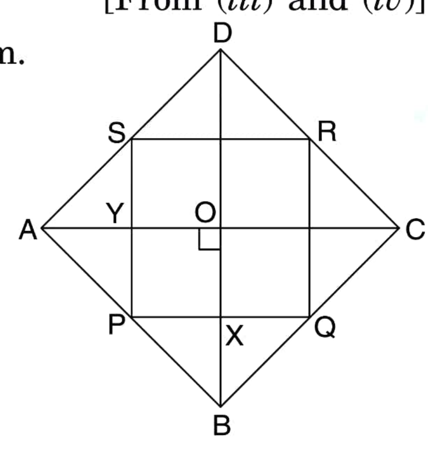
From (*i*) and (*ii*),

SR || AC and PQ || AC $\Rightarrow$ SR || PQ ...(*iii*)

and SR = $\frac{1}{2}$AC and PQ = $\frac{1}{2}$AC $\Rightarrow$ SR = PQ. ...(*iv*)

(*iii*) SR || PQ and SR = PQ. [From (*iii*) and (*iv*)]

$\Rightarrow$ PQRS is a parallelogram.



As PX || YO and PY || OX, PXOY is a parallelogram.

$\Rightarrow$ $\angle$YPX = $\angle$YOX = 90°

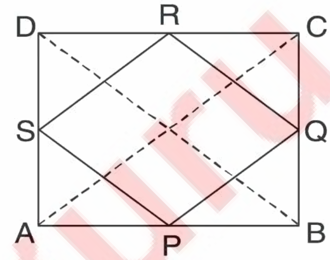
[$\because$ Diagonals of a rhombus bisect each other and are at right angles.]

As in parallelogram PQRS,
∠SPQ is 90°.

∴ PQRS is a rectangle.

3. *ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.*

Sol. **Construction:** Join AC and BD.



As ABCD is a rectangle.

∴ AC = BD ...(*i*)

Consider ΔABC, P and Q are mid-points of sides AB and BC respectively.

$$\therefore \quad PQ \parallel AC \text{ and } PQ = \frac{1}{2}AC \quad ...(ii)$$

Similarly, consider ΔADC, S and R are mid-points of sides AD and DC respectively.

$$\therefore \quad SR \parallel AC \text{ and } SR = \frac{1}{2}AC \quad ...(iii)$$

From (*ii*) and (*iii*),

$$PQ = SR = \frac{1}{2}AC \quad ...(iv)$$

Similarly, we can show

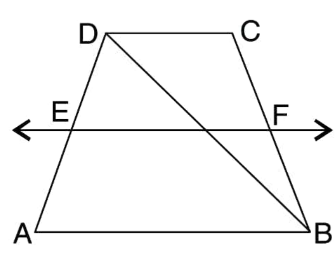
$$PS = QR = \frac{1}{2}BD \quad ...(v)$$

From (*i*), (*iv*) and (*v*), we have

$$PQ = QR = RS = SP$$

∴ PQRS is a rhombus.

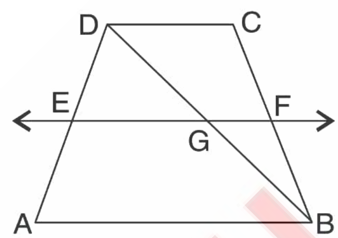
4. *ABCD is a trapezium in which AB ∥ DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (see figure). Show that F is the mid-point of BC.*



Sol. Consider Δ ADB, AB ∥ EF ⇒ AB ∥ EG.

⇒ G is mid-point of BD. ...(*i*)

[∵ A line drawn through mid-point of one side, parallel to other bisects the third side.]



Consider triangle BCD,

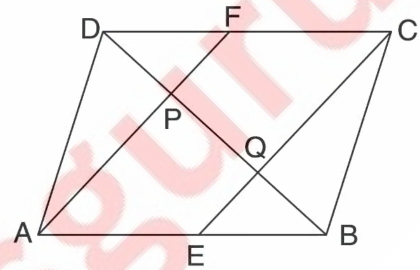
AB ∥ CD and EF ∥ AB

⇒ EF ∥ CD ⇒ GF ∥ CD

⇒ F is mid-point of BC.

[Reason same as above]

5. *In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see figure). Show that the line segments AF and EC trisect the diagonal BD.*



Sol. AB = CD ⇒ $\frac{1}{2}$AB = $\frac{1}{2}$CD

⇒ AE = CF

As AE = CF and AE ∥ CF [∵ AB ∥ CD]

⇒ AECF is a parallelogram.

⇒ AP ∥ CE ...(*i*)

Consider triangle ABP,

E is mid-point of AB and EQ ∥ AP [From (*i*)]

⇒ Q is mid-point of BP [A line segment drawn through mid-point of one side of a triangle and parallel to other, bisects the third side.]

BQ = PQ ...(*ii*)

Similarly, by considering triangle DCQ and proceeding as above, we can show that

DP = PQ ...(*iii*)

⇒ BQ = PQ = DP [From (*ii*) and (*iii*)]

⇒ P and Q trisect BD.

6. *Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.*

Sol. (*i*) Consider triangle ACD,

S and R are mid-points of sides AD and DC respectively.

$\therefore$ SR $\parallel$ AC and SR = $\frac{1}{2}$AC ...(*i*)

[Line segment joining mid-points of two sides of a triangle is parallel to the third and half of it.]

(*ii*) Consider triangle ABC, P and Q are mid-points of sides AB and BC respectively.

$\therefore$ PQ $\parallel$ AC and PQ = $\frac{1}{2}$AC ...(*ii*)

[Reason same as above]

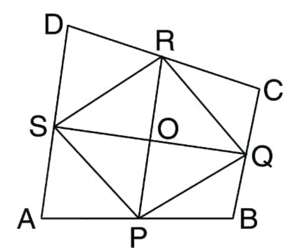
From (*i*) and (*ii*),

SR $\parallel$ AC and PQ $\parallel$ AC $\Rightarrow$ SR $\parallel$ PQ ...(*iii*)

and SR = $\frac{1}{2}$AC and PQ = $\frac{1}{2}$AC $\Rightarrow$ SR = PQ. ...(*iv*)

(*iii*) SR $\parallel$ PQ and SR = PQ. [From (*iii*) and (*iv*)]

$\Rightarrow$ PQRS is a parallelogram.



We know that diagonals of a parallelogram bisect each other, *i.e.*, OP = OR and OQ = OS.

Hence, line segments joining mid-points of opposite sides of a quadrilateral bisect each other.

7. *ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that*

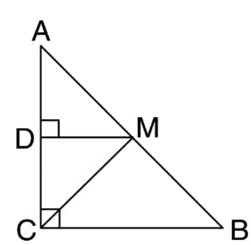
(*i*) *D is the mid-point of AC*

(*ii*) *MD $\perp$ AC*

(*iii*) *CM = MA = $\frac{1}{2}$AB.*

Sol. (*i*) MD $\parallel$ BC, meets AC at D.

$\therefore$ D is mid-point of AC.



[A line through the mid-point of a side of a triangle parallel to other bisects and third side.]

(*ii*) MD || BC and AC is transversal.

$\therefore$ $\angle ADM = \angle ACB$ [Corresponding angles]

$\Rightarrow$ $\angle ADM = 90°$ [$\because$ $\angle ACB = 90°$]

$\Rightarrow$ $MD \perp AC$.

(*iii*) Consider triangles ADM and CDM,

AD = DC [From result (*i*)]

MD is common.

$\angle ADM = \angle CDM$ [90° each] [From result (*ii*)]

$\therefore$ $\Delta ADM \cong \Delta CDM$ [SAS]

$\therefore$ $MA = CM = \frac{1}{2}AB.$

[$\because$ M is mid-point of AB)]