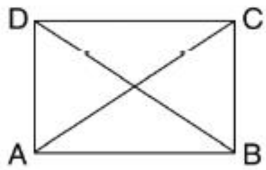
Exercise 8.1

2. Show that the diagonals of a square are equal and bisect each other at right angles.

Sol. Consider triangles DAB and CBA,

AD = BC [Opposite sides of a parallelogram]

AB is common.



AC = BD [Given]

∴ Δ DAB ≅ Δ CBA [SSS]

⇒ ∠ DAB = ∠ CBA ...(*i*) [CPCT]

As ABCD is a parallelogram. AD || BC and AB is transversal.

∴ ∠ DAB + ∠ CBA = 180° [Sum of interior angles on the same side of transversal is 180°.]

⇒ 2∠ DAB = 180° [From (*i*)]

⇒ ∠ DAB = 90°

As in a parallelogram, ∠DAB = 90°. Hence, the parallelogram is a rectangle.

3. Diagonal AC of a parallelogram ABCD bisects ∠ A (see Fig. 8.11). Show that

(i) it bisects ∠ C also,

(ii) ABCD is a rhombus.

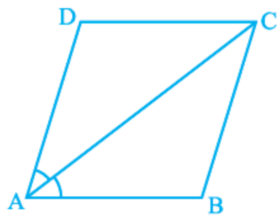


Fig. 8.11

Sol. (*i*) Consider triangles ABC and ADC,

$AB = CD$ [Opposite sides of parallelogram]

AC is common.

$AD = BC$ [Opposite sides of parallelogram]

$\therefore \quad \Delta DAC \cong \Delta BCA$ [SSS]

$\Rightarrow \quad \angle DAC = \angle BCA$ ...(*i*) [CPCT]

$\Rightarrow \quad \angle DCA = \angle BAC$ ...(*ii*) (CPCT)

Also, $\angle DAC = \angle BAC$ ...(*iii*) [Given]

$\Rightarrow \quad \angle DCA = \angle BCA$ [From (*i*), (*ii*), (*iii*)]

$\therefore$ AC bisects $\angle C$ also.

(*ii*) In parallelogram $\angle DAB = \angle DCB$,

[Opposite angles of a parallelogram are equal.]

$$\Rightarrow \quad \frac{1}{2}\angle DAB = \frac{1}{2}\angle DCB.$$

$\Rightarrow \quad \angle DAC = \angle DCA$ [$\because$ AC is bisector of $\angle A$ and $\angle C$.]

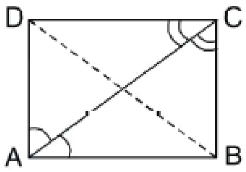
$\therefore \quad CD = AD$ [Sides opposite to equal angles are equal.]

In parallelogram, as adjacent sides are equal, hence ABCD is a rhombus.

4. ABCD is a rectangle in which diagonal AC bisects $\angle A$ as well as $\angle C$. Show that: (i) ABCD is a square (ii) diagonal BD bisects $\angle B$ as well as $\angle D$.

(*i*) *ABCD is a square*

(*ii*) *diagonal BD bisects ∠B as well as ∠D.*



Sol. (*i*) Consider triangles ADC and ABC,

∠DAC = ∠BAC [AC is bisector of ∠A]

∠DCA = ∠BCA [AC is bisector of ∠C]

AC is common.

∴ Δ ADC ≅ ΔABC [ASA]

AD = AB. [CPCT]

As in rectangle ABCD, adjacent sides are equal. Hence ABCD is a square.

(*ii*) Consider triangles DAB and BCD,

AB = BC = CD = DA [Sides of a square]

BD is common.

∴ Δ DAB ≅ Δ DCB [SSS]

∴ ∠ADB = ∠CDB [CPCT]

and ∠ABD = ∠CBD [CPCT]

∴ BD bisects ∠B and ∠D. [Using above results]

5. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see Fig. 8.12). Show that:

(i) Δ APD ≅ Δ CQB

(ii) AP = CQ

(iii) Δ AQB ≅ Δ CPD

(iv) AQ = CP

(v) APCQ is a parallelogram

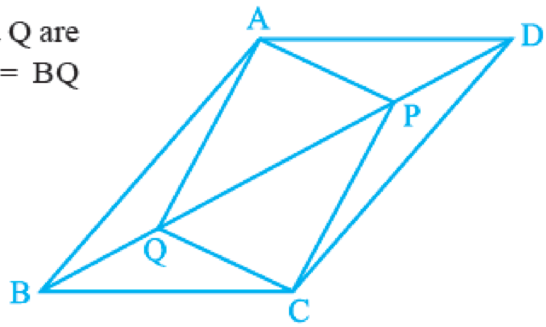
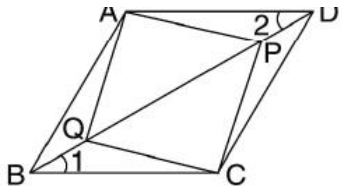


Fig. 8.12



Sol. (*i*) Consider triangles APD and CQB,

AD ∥ BC and BD is transversal.

∴ $\angle 1 = \angle 2$ [Alternate angles]

AD = BC [Opposite sides of a parallelogram]

DP = BQ [Given]

∴ Δ APD $\cong$ Δ CQB. [SAS]

(*ii*) AP = CQ [CPCT] [From result (*i*)]

(*iii*) Consider triangles AQB and CPD,

AB = CD [Opposite sides of a parallelogram]

$\angle$ ABQ = $\angle$ CDP [Alternate interior angles as AB ∥ CD and BD is transversal]

BQ = DP [Given]

∴ Δ AQB $\cong$ Δ CPD [SAS]

(*iv*) From result (*iii*),

Δ AQB $\cong$ Δ CPD

∴ AQ = CP [Corresponding sides]

(*v*) In quadrilateral APCQ,

AP = CQ [From result (*ii*)]

AQ = CP [From result (*iv*)]

Thus, opposite sides of quadrilateral APCQ are equal. Hence, APCQ is a parallelogram.

6. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see Fig. 8.13). Show that

(i) Δ APB $\cong$ Δ CQD

(ii) AP = CQ

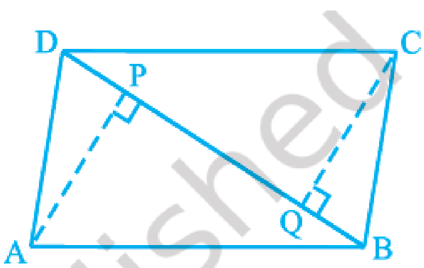
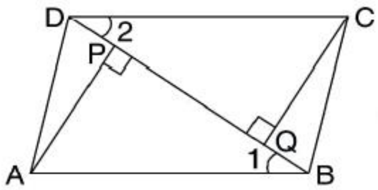


Fig. 8.13

Sol. Consider triangles APB and CQD,



$\angle 1 = \angle 2$ [Alternate angles, AB || CD, BD is transversa]

$\angle APB = \angle DQC$ [90° each]

AB = CD [Opposite sides of a parallelogram]

(*i*) $\therefore$ $\Delta APB \cong \Delta CQD$ [AAS]

(*ii*) AP = CQ. [CPCT]

7. ABCD is a trapezium in which AB || CD and AD = BC (see Fig. 8.14). Show that

(i) $\angle A = \angle B$

(ii) $\angle C = \angle D$

(iii) $\Delta ABC \cong \Delta BAD$

(iv) diagonal AC = diagonal BD

[*Hint*: Extend AB and draw a line through C parallel to DA intersecting AB produced at E.]

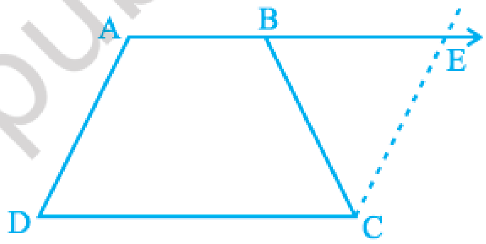
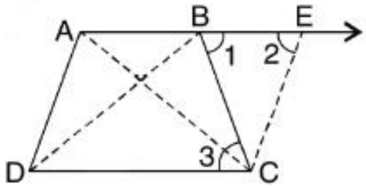


Fig. 8.14

Sol. (*i*) **Construction:** Draw CE || AD, meeting AB produced at E.



Proof: AB || CD [Given]

and AD || CE [Construction]

$\therefore$ AECD is a parallelogram.

$\Rightarrow$ AD = CE [Opposite sides of a arallelogram]

Also, AD = BC [Given]

$\therefore$ CE = BC.

$\Rightarrow$ $\angle 1 = \angle 2$ ...(*i*)

[Angles opposite to equal sides are equal]

Also, $\angle D = \angle 2$ ...(*ii*)

[Opposite angles of a parallelogram]

and $\angle 1 = \angle 3$ ...(*iii*) [Alternate angles]

$\Rightarrow$ $\angle D = \angle 3$ [From equations (*i*), (*ii*), (*iii*)]

$\Rightarrow$ $\angle D = \angle C$

As AB || CD and AD, BC are transversals.

$\therefore \quad \angle A + \angle D = 180°$ ...(*iv*) [Sum of interior angles on the same side of transversal is 180°.]

$\angle B + \angle C = 180°$ ...(*v*) [Reason same as above]

Also, $\angle C = \angle D$ ...(*vi*) [Proved above]

$\therefore \quad \angle A = \angle B$ [From equations (*iv*), (*v*), (*vi*)]

(*iii*) **Construction:** Draw AC and BD.

Proof: Consider triangles DAB and CBA.

AD = BC [Given]

AB is common.

$\angle DAB = \angle CBA$ [Proved above]

$\therefore \quad \Delta ABC \cong \Delta BAD$ [SAS]

(*iv*) AC = BD. [CPCT]