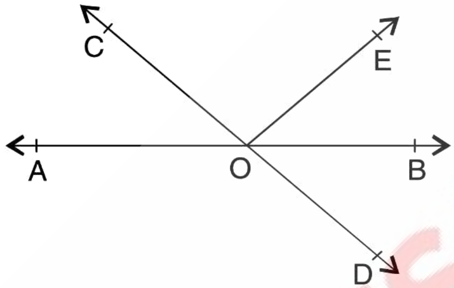
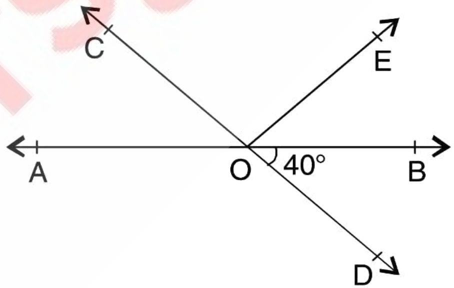
Exerise 6.1

1. *In figure given below, lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70°$ and $\angle BOD = 40°$, find $\angle BOE$ and reflex $\angle COE$.*



Sol. Ray OE stands on line AB.



$\therefore \quad \angle AOE + \angle EOB = 180°$ [Linear pair]

$\Rightarrow \quad (\angle AOC + \angle COE) + \angle EOB = 180°$

$\Rightarrow \quad (\angle AOC + \angle EOB) + \angle COE = 180°$ ...(*i*)

$\Rightarrow \quad 70° + \angle COE = 180°$

[$\because \quad \angle AOC + \angle BOE = 70°$ (given)]

$\Rightarrow \quad \angle COE = 110°$ ...(*ii*)

$\therefore$ Reflex $\angle COE = 360° - \angle COE = 360° - 110° = 250°$.

Also, $\angle AOC = \angle BOD = 40°$ ...(*iii*)

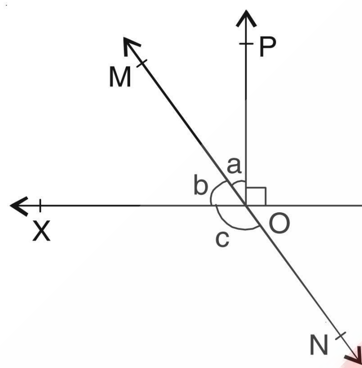
[Vertically opposite angles]

From (i), (ii), (iii), we get

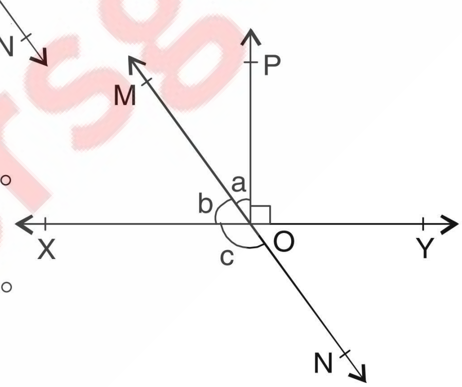
$$40^\circ + 110^\circ + \angle BOE = 180^\circ$$

$$\Rightarrow \quad \angle BOE = 180^\circ - 150^\circ = 30^\circ.$$

2. *In figure given below, lines XY and MN intersect at O. If $\angle POY = 90^\circ$ and $a : b = 2 : 3$, find c.*



Sol. Ray OP stands on line XY.



$\therefore \quad \angle XOP + \angle POY = 180^\circ$ [Linear pair]

$\Rightarrow \quad \angle XOP + 90^\circ = 180^\circ$

$\Rightarrow \quad \angle XOP = 90^\circ$

$\Rightarrow \quad \angle XOM + \angle MOP + = 90^\circ$

$\Rightarrow \quad b + a = 90^\circ$ ...(i)

Also, $a : b = 2 : 3 \Rightarrow \dfrac{a}{b} = \dfrac{2}{3} \Rightarrow a = \dfrac{2b}{3}$ ...(ii)

$\Rightarrow \quad b + \dfrac{2b}{3} = 90^\circ \Rightarrow \dfrac{5b}{3} = 90^\circ$ [From (i), (ii)]

$\Rightarrow \quad b = 54^\circ$ ...(iii)

From (i), we get

$54^\circ + a = 90^\circ \Rightarrow a = 36^\circ$

$\angle NOY = \angle XOM$ [Vertically opposite angles]

$\Rightarrow \quad \angle NOY = b = 54^\circ$ ...(iv) [From (iii)]

Ray NO stands on line XY.

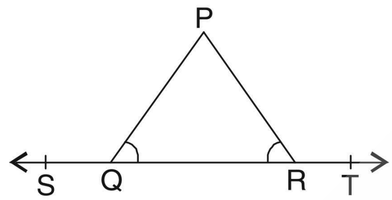
$\therefore \quad \angle XON + \angle NOY = 180^\circ$ [Linear pair]

$\Rightarrow \quad c + 54^\circ = 180^\circ$ [From (iv)]

$\Rightarrow \quad c = 180^\circ - 54^\circ = 126^\circ$

$\therefore \quad a = 36^\circ, b = 54^\circ, c = 126^\circ.$

3. *In the given figure, $\angle PQR = \angle PRQ$, then prove that $\angle PQS = \angle PRT$.*



Sol. $\angle PQR = \angle PRQ$ ...(*i*) [Given]

Line segment PQ stands on ST.

$\therefore \quad \angle PQS + \angle PQR = 180°$ ...(*ii*) [Linear pair]

Line segment PR stands on ST.

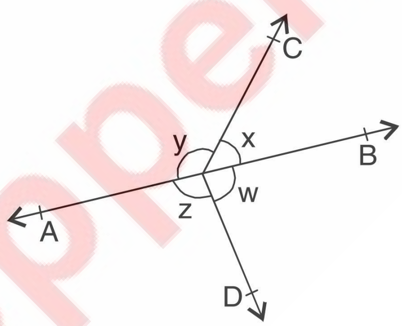
$\therefore \quad \angle PRQ + \angle PRT = 180°$ ...(*iii*) [Linear pair]

From (*ii*) and (*iii*), we get

$\angle PQS + \angle PQR = \angle PRQ + \angle PRT$

$\Rightarrow \quad \angle PQS = \angle PRT$. [Using (*i*)]

4. *In figure given below, if $x + y = w + z$, then prove that AOB is a line.*



Sol. We have $x + y + z + w = 360°$ ...(*i*)

Also $x + y = z + w$ ...(*ii*) [Given]

$\therefore \quad (x + y) + (x + y) = 360°$ [From (*i*), (*ii*)]

$\Rightarrow \quad 2(x + y) = 360°$

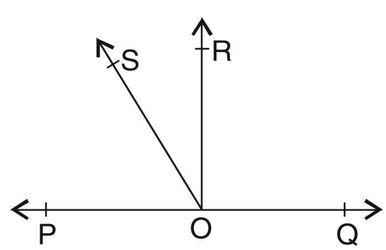
$\Rightarrow \quad x + y = 180°$ ...(*iii*)

As ray CO stands on line AB, such that

$x + y = 180°$ [From (*iii*)]

Hence, AOB is a straight line.

5. *In the adjoining figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that*



$$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$$

Sol. Ray OR is perpendicular to line PQ.

$\therefore \quad \angle POR = \angle QOR$ ...(*i*)

and $\angle POR + \angle QOR = 180^\circ$ [Linear pair]

$\Rightarrow \quad 2\angle POR = 180^\circ$ ...(*ii*) [Using (*i*)]

$\because$ Also ray OS stands on line PQ.

$\therefore \quad \angle POS + \angle QOS = 180^\circ$ [Linear pair]

$\Rightarrow \quad \angle POS + \angle QOS = 2\angle POR$ [From (*ii*)]

$\Rightarrow \quad \angle POS + \angle QOS = 2(\angle POS + \angle ROS)$

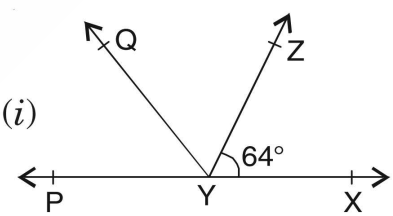
$\Rightarrow \quad \angle POS + \angle QOS = 2\angle POS + 2\angle ROS$

$\Rightarrow \quad 2\angle ROS = \angle QOS - \angle POS$

$\Rightarrow \quad \angle ROS = \frac{1}{2}(\angle QOS - \angle POS).$

6. *It is given that $\angle XYZ = 64^\circ$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$.*

Sol. Ray YQ bisects $\angle PYZ$.



$\therefore \quad \frac{1}{2}\angle PYZ = \angle PYQ = \angle QYZ$ ...(*i*)

Ray YZ stands on line PX.

$\therefore \quad \angle PYZ + \angle ZYX = 180^\circ$ [Linear pair]

$\Rightarrow \quad 2\angle PYQ + 64^\circ = 180^\circ$ [From (*i*)]

$\Rightarrow \quad 2\angle PYQ = 180^\circ - 64^\circ = 116^\circ$

$\Rightarrow \quad \angle PYQ = 58^\circ$ ...(*ii*)

$\therefore$ Reflex $\angle QYP = 360^\circ - \angle PYQ = 360^\circ - 58^\circ = 302^\circ.$

Also $\angle XYQ = \angle XYZ + \angle QYZ = 64^\circ + 58^\circ = 122^\circ.$

[From (*i*), (*ii*)]