

Exercise 5.1

1. Which of the following statements are true and which are false? Give reasons for your answers.

- (i) Only one line can pass through a single point.
- (ii) There are an infinite number of lines which pass through two distinct points.
- (iii) A terminated line can be produced indefinitely on both the sides.
- (iv) If two circles are equal, then their radii are equal.
- (v) In figure given below, if $AB = PQ$ and $PQ = XY$, then $AB = XY$.



Sol. (i) False, through one point, we can draw lines in different directions, so infinite lines can be drawn.

(ii) False, only one line passes through two distinct points.

(iii) True, according to Euclid's postulate.

(iv) True, if we superimpose one circle on the other by coinciding their centres, then their boundaries coincide. So their radii are equal.

(v) True, things which are equal to the same things are equal to one another.

2. Give a definition for each of the following terms. Are there other terms that need to be defined first? What are they, and how might you define them?

- (i) Parallel lines
- (ii) Perpendicular lines

- (iii) *Line segment* (iv) *Radius of a circle* (v) *Square*

- Sol.** (i) Lines which do not meet at a point when produced both sides; lines, intersecting lines.
 (ii) If angle between two given lines is 90° ; lines, angle between lines.
 (iii) A part of a line; two given points, line.
 (iv) Constant distance from any point on the boundary of a circle to the centre of the circle; centre, boundary.
 (v) A closed figure of four sides where all the sides are equal and diagonals are equal; points, line segments.

3. Consider two 'postulates' given below:

- (i) *Given any two distinct points A and B, there exists a third point C which is in between A and B.*
 (ii) *There exist at least three points that are not on the same line.*

Do these postulates contain any undefined terms? Are these postulates consistent? Do they follow from Euclid's postulates? Explain.

Sol. There are several undefined terms which the student should list. They are consistent, because they deal with two different situations:

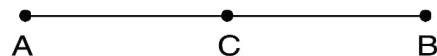
- (i) Says that given two points A and B, there is a point C lying on the line in between them.
 (ii) Says that given A and B, you can take C not lying on the line through A and B.

These 'postulates' do not follow from Euclid's postulates.

4. *If a point C lies between two points A and B such that*

$AC = BC$, then prove that $AC = \frac{1}{2}AB$. Explain by drawing the figure.

Sol. Given: $AC = BC$



Adding AC to both sides, we get

$$AC + AC = AC + BC$$

$$\Rightarrow 2AC = AB \Rightarrow AC = \frac{1}{2}AB.$$

- 5.** In Question 4, point C is called a mid-point of line segment AB. Prove that every line segment has one and only one mid-point.

Sol. Let D is mid-point of AB.

$$\therefore AD = BD \Rightarrow AD + AD = AD + BD$$

$$\Rightarrow 2AD = AB \Rightarrow AD = \frac{1}{2}AB$$

From Ans. 4 and 5, we have $AC = AD$

\Rightarrow Points C and D must coincide. Hence mid-point is unique.

- 6.** In figure given below, if $AC = BD$, then prove that

$$AB = CD.$$

Sol. Given: $AC = BD$

$$\Rightarrow AB + BC = BC + CD$$

Subtracting BC from both sides, we get

$$AB + BC - BC = BC + CD - BC$$

(Subtracting equals from equals)

$$\therefore AB = CD.$$

Hence proved.

- 7.** Why is Axiom 5, in the list of Euclid's axioms, considered a 'universal truth'? (Note that the question is not about the fifth postulate).

Sol. As statement is true in all the situations. Hence, it is considered a 'universal truth'.

