Exercise 2.4

1. *Use suitable identities to find the following products:*

(*i*) $(x + 4)(x + 10)$ (*ii*) $(x + 8)(x - 10)$

(*iii*) $(3x + 4)(3x - 5)$ (*iv*) $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$

(*v*) $(3 - 2x)(3 + 2x)$.

Sol. (*i*) $(x + 4)(x + 10) = x^2 + (4 + 10)x + (4 \times 10)$

[Using identity: $(x + a)(x + b) = x^2 + (a + b)x + ab$]

$= x^2 + 14x + 40.$

(*ii*) $(x + 8)(x - 10) = x^2 + (8 - 10)x + 8(-10)$

[Using identity: $(x + a)(x + b) = x^2 + (a + b)x + ab$]

$= x^2 - 2x - 80.$

(*iii*) $(3x + 4)(3x - 5) = (3x)^2 + (4 - 5)(3x) + 4 \times (-5)$

[Using identity: $(x + a)(x + b) = x^2 + (a + b)x + ab$]

$= 9x^2 - 3x - 20.$

(*iv*) $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right) = (y^2)^2 - \left(\frac{3}{2}\right)^2$

[Using identity: $(x + a)(x - a) = x^2 - a^2$]

$= y^4 - \frac{9}{4}.$

(*v*) $(3 - 2x)(3 + 2x) = (3)^2 - (2x)^2$

[Using identity: $(x + a)(x - a) = x^2 - a^2$]

$= 9 - 4x^2.$

2. *Evaluate the following products without multiplying directly*:

(*i*) *103 × 107* (*ii*) *95 × 96* (*iii*) *104 × 96.*

Sol. (*i*) $103 \times 107 = (100 + 3)(100 + 7)$

$= (100)^2 + (3 + 7) \times 100 + 3 \times 7$

[Using identity: $(x + a)(x + b) = x^2 + (a + b)x + ab$]

$= 10000 + 1000 + 21 = 11021.$

(*ii*) $95 \times 96 = (90 + 5)(90 + 6)$

$= (90)^2 + (5 + 6) \times 90 + 5 \times 6$

[Using identity: $(x + a)(x + b) = x^2 + (a + b)x + ab$]

$= 8100 + 990 + 30 = 9120.$

(*iii*) $104 \times 96 = (100 + 4)(100 - 4) = (100)^2 - (4)^2$

[Using identity: $(x + y)(x - y) = x^2 - y^2$]

$= 10000 - 16 = 9984.$

3. *Factorise the following using appropriate identities*:

(*i*) $9x^2 + 6xy + y^2$ (*ii*) $4y^2 - 4y + 1$ (*iii*) $x^2 - \frac{y^2}{100}.$

Sol. (*i*) $9x^2 + 6xy + y^2 = (3x)^2 + 2 \times 3x \times y + (y)^2 = (3x + y)^2$

[Using identity: $x^2 + 2xy + y^2 = (x + y)^2$]

$= (3x + y)(3x + y)$

(ii) $4y^2 - 4y + 1 = (2y)^2 - 2 \times 2y \times 1 + (1)^2 = (2y - 1)^2$

[Using identity: $x^2 - 2xy + y^2 = (x - y)^2$]

$= (2y - 1)(2y - 1)$.

(iii) $x^2 - \frac{y^2}{100} = (x)^2 - \left(\frac{y}{10}\right)^2 = \left(x - \frac{y}{10}\right)\left(x + \frac{y}{10}\right)$.

[Using identity: $a^2 - b^2 = (a - b)(a + b)$]

4. *Expand each of the following, using suitable identities:*

(i) $(x + 2y + 4z)^2$ (ii) $(2x - y + z)^2$

(iii) $(-2x + 3y + 2z)^2$ (iv) $(3a - 7b - c)^2$

(v) $(-2x + 5y - 3z)^2$ (vi) $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2$.

Sol. (i) $(x + 2y + 4z)^2 = (x)^2 + (2y)^2 + (4z)^2 + 2(x)(2y) + 2(2y)(4z) + 2(x)(4z)$

[Using identity: $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$]

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8xz$.

(ii) $(2x - y + z)^2 = (2x)^2 + (-y)^2 + (z)^2 + 2(2x)(-y) + 2(-y)(z) + 2(2x)(z)$

[Using identity: $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$]

$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$.

(iii) $(-2x + 3y + 2z)^2 = (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) + 2(3y)(2z) + 2(-2x)(2z)$

[Using identity: $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$]

$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8xz$.

(iv) $(3a - 7b - c)^2 = (3a)^2 + (-7b)^2 + (-c)^2 + 2(3a)(-7b) + 2(-7b)(-c) + 2(3a)(-c)$

[Using identity: $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$]

$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ac$.

(v) $(-2x + 5y - 3z)^2 = (-2x)^2 + (5y)^2 + (-3z)^2$
$+ 2(-2x)(5y) + 2(5y)(-3z) + 2(-2x)(-3z)$
[Using identity: $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$]
$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12xz.$

(vi) $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2 = \left(\frac{1}{4}a\right)^2 + \left(-\frac{1}{2}b\right)^2 + (1)^2$
$+ 2\left(\frac{1}{4}a\right)\left(-\frac{1}{2}b\right) + 2\left(-\frac{1}{2}b\right)(1) + 2\left(\frac{1}{4}a\right)(1)$
[Using identity: $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$]
$= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{1}{4}ab - b + \frac{1}{2}a.$

5. *Factorise:*

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}\,xy + 4\sqrt{2}\,yz - 8xz.$

Sol. *(i)* $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz = (2x)^2$
$+ (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(2x)(-4z)$
[Using identity: $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$]
$= (2x + 3y - 4z)^2.$
$= (2x + 3y - 4z)(2x + 3y - 4z).$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}\,xy + 4\sqrt{2}\,yz - 8xz$
$= (-\sqrt{2}\,x)^2 + (y)^2 + (2\sqrt{2}\,z)^2 + 2(-\sqrt{2}\,x)(y)$
$+ 2(y)(2\sqrt{2}\,z) + 2(-\sqrt{2}\,x)(2\sqrt{2}\,z)$
[Using identity: $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$]
$= (-\sqrt{2}\,x + y + 2\sqrt{2}\,z)^2$
$= (-\sqrt{2}\,x + y + 2\sqrt{2}\,z)(-\sqrt{2}\,x + y + 2\sqrt{2}\,z).$

6. *Write the following cubes in expanded form:*

(i) $(2x + 1)^3$ *(ii)* $(2a - 3b)^3$ *(iii)* $\left[\frac{3}{2}x + 1\right]^3$ *(iv)* $\left[x - \frac{2}{3}y\right]^3.$

Sol. (*i*) $(2x + 1)^3 = (2x)^3 + (1)^3 + 3(2x)(1)(2x + 1)$

[Using identity: $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$]

$= 8x^3 + 1 + 12x^2 + 6x = 8x^3 + 12x^2 + 6x + 1.$

(*ii*) $(2a - 3b)^3 = (2a)^3 - (3b)^3 - 3(2a)(3b)(2a - 3b)$

[Using identity: $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$]

$= 8a^3 - 27b^3 - 36a^2b + 54ab^2.$

(*iii*) $\left(\frac{3}{2}x+1\right)^3 = \left(\frac{3}{2}x\right)^3 + (1)^3 + 3 \times \frac{3}{2}x \times 1 \left(\frac{3}{2}x+1\right)$

[Using identity: $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$]

$= \frac{27}{8}x^3 + 1 + \frac{27}{4}x^2 + \frac{9}{2}x = \frac{27}{8}x^3 + \frac{27}{4}x^2$

$+ \frac{9}{2}x + 1.$

(*iv*) $\left(x - \frac{2}{3}y\right)^3 = (x)^3 - \left(\frac{2}{3}y\right)^3 - 3(x)\left(\frac{2}{3}y\right)\left(x - \frac{2}{3}y\right)$

[Using identity: $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$]

$= x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2.$

7. *Evaluate the following using suitable identities*:

(*i*) *(99)*^{3} (*ii*) *(102)*^{3} (*iii*) *(998)*^{3}.

Sol. (*i*) $(99)^3 = (100 - 1)^3$

$= (100)^3 - (1)^3 - 3 \times 100 \times 1\ (100 - 1)$

[Using identity: $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$]

$= 1000000 - 1 - 30000 + 300 = 970299.$

(*ii*) $(102)^3 = (100 + 2)^3$

$= (100)^3 + (2)^3 + 3 \times 100 \times 2\ (100 + 2)$

[Using identity: $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$]

$= 1000000 + 8 + 60000 + 1200 = 1061208.$

(*iii*) $(998)^3 = (1000 - 2)^3$

$= (1000)^3 - (2)^3 - 3 \times 1000 \times 2(1000 - 2)$

[Using identity: $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$]

$= 1000000000 - 8 - 6000000 + 12000 = 994011992.$

8. *Factorise each of the following*:

(*i*) $8a^3 + b^3 + 12a^2b + 6ab^2$

(*ii*) $8a^3 - b^3 - 12a^2b + 6ab^2$

(*iii*) $27 - 125a^3 - 135a + 225a^2$

(*iv*) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

(*v*) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p.$

Sol. (*i*) $8a^3 + b^3 + 12a^2b + 6ab^2$

$$= (2a)^3 + (b)^3 + 3(2a)(b)(2a + b)$$
$$= (2a + b)^3$$

[Using identity: $x^3 + y^3 + 3xy(x + y) = (x + y)^3$]

$$= (2a + b)(2a + b)(2a + b).$$

(*ii*) $8a^3 - b^3 - 12a^2b + 6ab^2$

$$= (2a)^3 - (b)^3 - 3(2a)^2b + 3(2a)b^2$$
$$= (2a - b)^3$$

[Using identity: $x^3 - y^3 - 3x^2y + 3xy^2 = (x - y)^3$]

$$= (2a - b)(2a - b)(2a - b).$$

(*iii*) $27 - 125a^3 - 135a + 225a^2$

$$= (3)^3 - (5a)^3 - 3 \times (3)^2 \times (5a) + 3(3)(5a)^2$$
$$= (3 - 5a)^3$$

[Using identity: $x^3 - y^3 - 3x^2y + 3xy^2 = (x - y)^3$]

$$= (3 - 5a)(3 - 5a)(3 - 5a).$$

(*iv*) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

$$= (4a)^3 - (3b)^3 - 3(4a)^2(3b) + 3(4a)(3b)^2$$
$$= (4a - 3b)^3$$

[Using identity: $x^3 - y^3 - 3x^2y + 3xy^2 = (x - y)^3$]

$$= (4a - 3b)(4a - 3b)(4a - 3b).$$

(*v*) $$27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p = (3p)^3 - \left(\frac{1}{6}\right)^3 - 3 \times (3p)^2 \times \frac{1}{6} + 3 \times (3p) \times \left(\frac{1}{6}\right)^2$$

$$= \left(3p - \frac{1}{6}\right)^3$$

[Using identity: $x^3 - y^3 - 3x^2y + 3xy^2 = (x - y)^3$]

$$= \left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right).$$

9. *Verify*:

(*i*) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

(*ii*) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$.

Sol. (*i*) Consider the identity $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$\Rightarrow \quad x^3 + y^3 = (x + y)^3 - 3xy(x + y)$

$= (x + y)\{(x + y)^2 - 3xy\}$

$\Rightarrow \quad x^3 + y^3 = (x + y)\ \{x^2 + y^2 + 2xy - 3xy\}$

$= (x + y)(x^2 - xy + y^2)$.

(*ii*) Consider the identity $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$\Rightarrow \quad x^3 - y^3 = (x - y)^3 + 3xy(x - y)$

$= (x - y)\{(x - y)^2 + 3xy\}$

$\Rightarrow \quad x^3 - y^3 = (x - y)\ \{x^2 + y^2 - 2xy + 3xy\}$

$= (x - y)(x^2 + xy + y^2)$.

10. *Factorise each of the following*:

(*i*) $27y^3 + 125z^3$ (*ii*) $64m^3 - 343n^3$.

Sol. (*i*) Consider $27y^3 + 125z^3 = (3y)^3 + (5z)^3$

$= (3y + 5z)\ \{(3y)^2 - (3y)(5z) + (5z)^2\}$

[Using identity: $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$]

$= (3y + 5z)(9y^2 - 15yz + 25z^2)$

(*ii*) Consider $64m^3 - 343n^3 = (4m)^3 - (7n)^3$

$= (4m - 7n)\ \{(4m)^2 + (4m)(7n) + (7n)^2\}$

[Using identity: $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$]

$= (4m - 7n)(16m^2 + 28mn + 49n^2)$.

11. *Factorise*: $27x^3 + y^3 + z^3 - 9xyz$.

Sol. Consider $27x^3 + y^3 + z^3 - 9xyz$

$= (3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z)$

[Using identity: $x^3 + y^3 + z^3 - 3xyz$
$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$]

$= (3x + y + z)\{(3x)^2 + y^2 + z^2 - (3x)y - yz - (3x)z\}$
$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3xz).$

12. *Verify that:*

$$x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$$

Sol. Consider identity $x^3 + y^3 + z^3 - 3xyz$

$$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$
$$= \frac{1}{2}(x + y + z)(2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx)$$
$$= \frac{1}{2}(x + y + z)\{(x^2 + y^2 - 2xy) + (y^2 + z^2 - 2yz) + (z^2 + x^2 - 2xz)\}$$
$$= \frac{1}{2}(x + y + z)\{(x - y)^2 + (y - z)^2 + (z - x)^2\}.$$

13. *If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$.*

Sol. Consider the identity $x^3 + y^3 + z^3 - 3xyz$

$$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) \quad ...(i)$$

If $x + y + z = 0$, then

$$(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) = 0$$

$\therefore \quad x^3 + y^3 + z^3 - 3xyz = 0$ [From (*i*)]

$\Rightarrow \quad x^3 + y^3 + z^3 = 3xyz.$

14. *Without actually calculating the cubes, find the value of each of the following:*

(*i*) $(-12)^3 + (7)^3 + (5)^3$ (*ii*) $(28)^3 + (-15)^3 + (-13)^3$.

Sol. (*i*) Consider $(-12)^3 + (7)^3 + (5)^3$

Let $x = -12$, $y = 7$, $z = 5$

Now, $x + y + z = -12 + 7 + 5 = 0$

We know, if $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$.

$\therefore \quad (-12)^3 + (7)^3 + (5)^3 = 3(-12)(7)(5)$
$= -21 \times 60 = -1260.$

(*ii*) Consider $(28)^3 + (-15)^3 + (-13)^3$

Let $x = 28, y = -15, z = -13$

Now, $x + y + z = 28 - 15 - 13 = 0$

We know, if $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$.

$\therefore \quad (28)^3 + (-15)^3 + (-13)^3$

$$= 3(28)(-15)(-13) = 16380.$$

15. *Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:*

Area: $25a^2 - 35a + 12$	Area: $35y^2 + 13y - 12$
(*i*)	(*ii*)

Sol. (*i*) Area $= 25a^2 - 35a + 12 = 25a^2 - 20a - 15a + 12$

$$= 5a(5a - 4) - 3(5a - 4) = (5a - 3)(5a - 4).$$

Possible expressions for length and breadth are $(5a - 3)$ and $(5a - 4)$ respectively.

(*ii*) Area $= 35y^2 + 13y - 12 = 35y^2 + 28y - 15y - 12$

$$= 7y(5y + 4) - 3(5y + 4) = (7y - 3)(5y + 4).$$

Possible expressions for length and breadth are

$(7y - 3)$ and $(5y + 4)$.

16. *What are the possible expressions for the dimensions of the cuboids whose volumes are given below?*

Volume: $3x^2 - 12x$	Volume: $12ky^2 + 8ky - 20k$
(*i*)	(*ii*)

Sol. (*i*) Volume $= 3x^2 - 12x = 3x(x - 4)$

Possible dimensions are 3, x and $(x - 4)$.

(*ii*) Volume $= 12ky^2 + 8ky - 20k = 4k(3y^2 + 2y - 5)$

$$= 4k(3y^2 + 5y - 3y - 5)$$

$$= 4k\{y(3y + 5) - 1(3y + 5)\}$$

$$= 4k(y - 1)(3y + 5).$$

Possible dimensions are $4k$, $y - 1$ and $3y + 5$.