Exercise 1.4

1. *Classify the following numbers as rational or irrational:*

(*i*) $2 - \sqrt{5}$ (*ii*) $(3 + \sqrt{23}) - \sqrt{23}$

(*iii*) $\dfrac{2\sqrt{7}}{7\sqrt{7}}$ (*iv*) $\dfrac{1}{\sqrt{2}}$ (*v*) 2π.

Sol. (*i*) $2 - \sqrt{5}$ is an irrational number, as difference of a rational and an irrational number is irrational.

(*ii*) $(3 + \sqrt{23}) - \sqrt{23} = 3 + \sqrt{23} - \sqrt{23} = 3$, is a rational number.

(*iii*) $\dfrac{2\sqrt{7}}{7\sqrt{7}} = \dfrac{2}{7}$, is a rational number.

(*iv*) $\dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$ is an irrational number, as divisors of an irrational number by a non-zero rational number is irrational.

(*v*) 2π, irrational number, as π is an irrational number and multiplication of a rational and an irrational number is irrational.

2. *Simplify each of the following expressions:*

(*i*) $(3 + \sqrt{3})(2 + \sqrt{2})$ (*ii*) $(3 + \sqrt{3})(3 - \sqrt{3})$

(*iii*) $(\sqrt{5} + \sqrt{2})^2$ (*iv*) $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$

Sol. (*i*) $(3 + \sqrt{3})(2 + \sqrt{2}) = 6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6}$

(*ii*) $(3 + \sqrt{3})(3 - \sqrt{3}) = (3)^2 - (\sqrt{3})^2 = 9 - 3 = 6.$

(*iii*) $(\sqrt{5} + \sqrt{2})^2 = (\sqrt{5})^2 + 2.\sqrt{5}.\sqrt{2} + (\sqrt{2})^2$

$= 5 + 2\sqrt{10} + 2 = 7 + 2\sqrt{10}.$

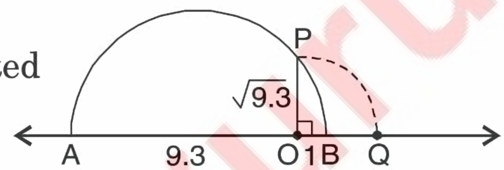
(*iv*) $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2}) = (\sqrt{5})^2 - (\sqrt{2})^2 = 5 - 2 = 3.$

3. *Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, $\pi = \frac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?*

Sol. On measuring c with any device, we get only approximate measurement. Therefore, π is an irrational.

4. *Represent $\sqrt{9.3}$ on the number line.*

Sol. $\sqrt{9.3} = \sqrt{9.3 \times 1}$



Let position 0 be represented by O on the number line.

Let OA = 9.3 and OB = 1.

With AB as diameter draw a semicircle. Draw OP perpendicular to AB, meeting the semicircle at P. Then OP = $\sqrt{9.3}$. With O as centre and OP as radius draw an arc to meet the number line at Q on the positive side. Then, OQ = $\sqrt{9.3}$ and the point Q thus obtained represents $\sqrt{9.3}$.

5. *Rationalise the denominators of the following:*

(i) $\frac{1}{\sqrt{7}}$ (ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$ (iv) $\frac{1}{\sqrt{7}-2}$.

Sol. (i) $\frac{1}{\sqrt{7}} = \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}$.

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7}-\sqrt{6})(\sqrt{7}+\sqrt{6})} = \frac{\sqrt{7}+\sqrt{6}}{7-6} = \sqrt{7}+\sqrt{6}$.

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}} = \frac{\sqrt{5}-\sqrt{2}}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})} = \frac{\sqrt{5}-\sqrt{2}}{5-2} = \frac{\sqrt{5}-\sqrt{2}}{3}$.

(iv) $\frac{1}{\sqrt{7}-2} = \frac{\sqrt{7}+2}{(\sqrt{7}-2)(\sqrt{7}+2)} = \frac{\sqrt{7}+2}{7-4} = \frac{\sqrt{7}+2}{3}$.