Square root spiral

Exerise 1.3

1. *Write the following in decimal form and say what kind of decimal expansion each has:*

(i) $\frac{36}{100}$ (ii) $\frac{1}{11}$ (iii) $4\frac{1}{8}$

(iv) $\frac{3}{13}$ (v) $\frac{2}{11}$ (vi) $\frac{329}{400}$.

Sol. (i) $\frac{36}{100} = 0.36$, terminating decimal expansion.

(ii) $\frac{1}{11} = 0.090909..... = 0.\overline{09}$, non-terminating repeating decimal expansion.

(iii) $4\frac{1}{8} = \frac{33}{8} = 4.125$, terminating decimal expansion.

(iv) $\frac{3}{13} = 0.230769230769..... = 0.\overline{230769}$, non-terminating repeating decimal expansion.

(*v*) $\frac{2}{11}$ = 0.181818 ..... = $0.\overline{18}$, non-terminating repeating decimal expansion.

(*vi*) $\frac{329}{400}$ = 0.8225, terminating decimal expansion.

2. *You know that* $\frac{1}{7} = 0.\overline{142857}$. *Can you predict what the decimal expansions of* $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ *are without actually doing the long division? If so, how?*

[**Hint:** *Study the remainders while finding the value of* $\frac{1}{7}$ *carefully.*]

Sol. Yes, we can predict the required decimal expansions.

We are given, $\frac{1}{7} = 0.\overline{142857}$

On dividing 1 by 7, we find that the remainders repeat after six divisions, therefore, the quotient has a repeating block of six digits in the decimal expansion of $\frac{1}{7}$. So, to obtain decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}$ and $\frac{6}{7}$; we multiply 142857 by 2, 3, 4, 5 and 6 respectively, to get the integral part and in the decimal part, we take block of six repeating digits in each case. Hence, we get

$$\frac{2}{7} = 2 \times \frac{1}{7} = 0.\overline{285714}$$

$$\frac{3}{7} = 3 \times \frac{1}{7} = 0.\overline{428571}$$

$$\frac{4}{7} = 4 \times \frac{1}{7} = 0.\overline{571428}$$

$$\frac{5}{7} = 5 \times \frac{1}{7} = 0.\overline{714285}$$

and $\frac{6}{7} = 6 \times \frac{1}{7} = 0.\overline{857142}$.

3. *Express the following in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$:*

(i) $0.\overline{6}$ *(ii)* $0.4\overline{7}$ *(iii)* $0.\overline{001}$.

Sol. (*i*) Let $x = 0.\overline{6}$

or $x = 0.666...$ ...(*i*)

$10x = 6.666...$ ...(*ii*) [On multiplying (*i*) by 10]

$\Rightarrow 9x = 6$ [Subtracting (*i*) from (*ii*)]

$\therefore x = \frac{2}{3}$.

(*ii*) Let $x = 0.4\overline{7}$

or $x = 0.4777...$ ...(*i*)

Multiplying (*i*) by 10, we get

$10x = 4.777...$ ...(*ii*)

Again multiplying (*ii*) by 10, we get

$100x = 47.777...$ ...(*iii*)

Subtracting equation (*ii*) from equation (*iii*), we get

$90x = 43$

$\therefore x = \frac{43}{90}$.

(*iii*) Let $x = 0.\overline{001}$

or $x = 0.001001...$ ...(*i*)

$\Rightarrow 1000x = 1.001001...$ ...(*ii*)

[On multiplying (*i*) by 1000]

$\Rightarrow 999x = 1$ [On subtracting (*i*) from (*ii*)]

$\therefore x = \frac{1}{999}$.

4. *Express 0.99999... in the form $\frac{p}{q}$. Are you surprised by your answer? With your teacher and classmates, discuss why the answer makes sense.*

Sol. Let $x = 0.99999...$ ...(*i*)

$\Rightarrow 10x = 9.99999...$ ...(*ii*) [On multiplying (*i*) by 10]

$\Rightarrow 9x = 9$ [On subtracting (*i*) from (*ii*)]

$\therefore x = 1$.

Yes, we are surprised by our answer.

5. *What can the maximum number of digits be in the repeating block of digits in the decimal expansion of* $\frac{1}{17}$*? Perform the division to check your answer.*

Sol.

```
          0.0588235294117647...
     17) 1.0000000000000000
          00
          100  <--
           85
           150
           136
            140
            136
              40
              34
               60
               51
                90
                85
                 50
                 34
                 160
                 153
                   70
                   68
                    20
                    17
                     30
                     17
                     130
                     119
                      110
                      102
                        80
                        68
                        120
                        119
                          1  <-- Repeating
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Thus, $\frac{1}{17} = 0.\overline{0588235294117647}$

Hence, the required number of digits in the repeating block is 16.

6. *Look at several examples of rational numbers in the form $\frac{p}{q}$ $(q \neq 0)$, where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?*

Sol. Examples are $\frac{3}{4}, \frac{4}{5}, \frac{7}{8}, \frac{9}{10}$, etc.

q have only powers of 2 or powers of 5 or both.

7. *Write three numbers whose decimal expansions are non-terminating non-recurring.*

Sol. (*i*) 2.01011011101111011111....... (*ii*) 0.03003000300003.....

(*iii*) 4.12112111211112.......

8. *Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.*

Sol. Given: $\frac{5}{7} = 0.\overline{714285}$ and $\frac{9}{11} = 0.\overline{81}$

We can have irrational numbers as 0.72072007200072.......; 0.801001800018.......; 0.74301010010001......;

9. *Classify the following numbers as rational or irrational:*

(*i*) $\sqrt{23}$ (*ii*) $\sqrt{225}$ (*iii*) *0.3796*

(*iv*) *7.478478.....* (*v*) *1.101001000100001....*

Sol. (*i*) $\sqrt{23}$. As it is square root of a prime number, so, irrational number.

(*ii*) $\sqrt{225} = 15$, rational number.

(*iii*) 0.3796, terminating decimal, so rational number.

(*iv*) 7.478478..... $= 7.\overline{478}$, non-terminating repeating (recurring), so rational number.

(*v*) 1.101001000100001....... non-terminating non-repeating, so irrational number.