Exercise 8.3

Q1. Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

Sol. We have $\text{cosec}^2 A - \cot^2 A = 1$

$\Rightarrow \text{cosec}^2 A = 1 + \cot^2 A$

$\Rightarrow (\text{cosec } A)^2 = \cot^2 A + 1$

$$\Rightarrow \left(\frac{1}{\sin A}\right)^2 = \cot^2 A + 1$$

$$\Rightarrow (\sin A)^2 = \frac{1}{\cot^2 A + 1}$$

$$\Rightarrow \sin A = \pm \frac{1}{\sqrt{\cot^2 A + 1}}$$

We reject negative value of sin A for acute angle

A. Therefore, $\sin A = \dfrac{1}{\sqrt{\cot^2 A + 1}}$ $\tan A = \dfrac{1}{\cot A}$

We have $\sec^2 A - \tan^2 A = 1$

$\Rightarrow \sec^2 A = 1 + \tan^2 A$

$$= 1 + \frac{1}{\cot^2 A} = \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\Rightarrow \sec A = \frac{\sqrt{\cot^2 A + 1}}{\cot A}$$

Q2. Write all the other trigonometric ratios of ∠A in terms of sec A.

Sol. (i) $\sin A = \sqrt{1 - \cos^2 A}$

$$= \sqrt{1 - \frac{1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

(ii) $\cos A = \dfrac{1}{\sec A}$

(iii) $\tan A = \sqrt{\sec^2 A - 1}$

(iv) $\cot A = \dfrac{1}{\tan A} = \dfrac{1}{\sqrt{\sec^2 A - 1}}$

(v) $\text{cosec } A = \dfrac{1}{\sin A} = \dfrac{\sec A}{\sqrt{\sec^2 A - 1}}$

Q3. Evaluate

(i) $\dfrac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

(ii) $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

Sol. (i) $\dfrac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

$$= \frac{\{\sin(90^\circ - 27^\circ)\}^2 + \sin^2 27^\circ}{\cos^2 17^\circ + \{\cos(90^\circ - 17^\circ)\}^2}$$

$$= \frac{\{\cos 27^\circ\}^2 + \sin^2 27^\circ}{\cos^2 17^\circ + \{\sin 17^\circ\}^2}$$

$$= \frac{\cos^2 27^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \sin^2 17^\circ} = \frac{1}{1} = 1$$

(ii) $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

$= \sin(90^\circ - 65^\circ) \cos 65^\circ + \cos(90^\circ - 65^\circ) \sin 65^\circ$

$= \cos 65^\circ \cos 65^\circ + \sin 65^\circ \sin 65^\circ$

$= \cos^2 65^\circ + \sin^2 65^\circ = 1$

Q4. Choose the correct option. Justify your choice :

(i) $9 \sec^2 A - 9 \tan^2 A =$

(A) 1 (B) 9 (C) 8 (D) 0

(ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta) =$

(A) 0 (B) 1 (C) 2 (D) – 1

(iii) $(\sec A + \tan A)\ (1 - \sin A) =$

(A) sec A (B) sin A (C) cosec A (D) cos A

(iv) $\dfrac{1 + \tan^2 A}{1 + \cot^2 A} =$

(A) $\sec^2 A$ (B) – 1 (C) $\cot^2 A$ (D) $\tan^2 A$

Sol. (i) Correct option is (B).

$9 \sec^2 A - 9 \tan^2 A = 9(\sec^2 A - \tan^2 A)$

$= 9 \times 1 = 9.$

(ii) Correct option is (C).

$(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$

$$= \left\{1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right\} \times \left\{1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right\}$$

$$= \left\{\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right\} \times \left\{\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right\}$$

$$= \frac{\{(\cos\theta + \sin\theta) + 1\} \times \{(\cos\theta + \sin\theta) - 1\}}{\cos\theta \times \sin\theta}$$

$$= \frac{(\cos\theta + \sin\theta)^2 - (1)^2}{\cos\theta \times \sin\theta}$$

$\{\because (a + b)(a - b) = a^2 - b^2\}$

$$= \frac{\cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta - 1}{\cos\theta \times \sin\theta}$$

$$= \frac{1 + 2\cos\theta\sin\theta - 1}{\cos\theta\sin\theta} = 2.$$

(iii) Correct option is (D).

$(\sec A + \tan A)(1 - \sin A)$

$$= \sec A - \tan A + \tan A - \frac{\sin^2 A}{\cos A}$$

$$= \frac{1}{\cos A} - \frac{\sin^2 A}{\cos A} = \frac{1 - \sin^2 A}{\cos A}$$

$$= \frac{\cos^2 A}{\cos A} = \cos A$$

(iv) Correct option is (D).

$$\frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{\sec^2 A}{\operatorname{cosec}^2 A} = \tan^2 A$$

Q5. Prove the following identities, where the angles involved are acute angles for which the following expressions are defined.

(i) $(\operatorname{cosec}\theta - \cot\theta)^2 = \dfrac{1 - \cos\theta}{1 + \cos\theta}$.

(ii) $\dfrac{\cos A}{1 + \sin A} + \dfrac{1 + \sin A}{\cos A} = 2\sec A$.

(iii) $\dfrac{\tan\theta}{1 - \cot\theta} + \dfrac{\cot\theta}{1 - \tan\theta} = 1 + \sec\theta\operatorname{cosec}\theta$.

(iv) $\dfrac{1 + \sec A}{\sec A} = \dfrac{\sin^2 A}{1 - \cos A}$.

(v) $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$, using the identity $\operatorname{cosec}^2 A = 1 + \cot^2 A$.

(vi) $\sqrt{\dfrac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$.

(vii) $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$.

(viii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= 7 + \tan^2 A + \cot^2 A.$

(ix) $(\operatorname{cosec} A - \sin A)(\sec A - \cos A)$

$$= \frac{1}{\tan A + \cot A}.$$

(x) $$\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A.$$

Sol. (i) LHS $= (\operatorname{cosec}\theta - \cot\theta)^2$

$$= \left\{\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right\}^2 = \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta} = \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{(1-\cos\theta)\times(1+\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta}$$

$\therefore$ LHS = RHS.

(ii) LHS $= \dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A}$

$$= \frac{\cos^2 A + (1+\sin A)^2}{\cos A(1+\sin A)}$$

$$= \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{\cos A(1+\sin A)}$$

$$= \frac{2(1+\sin A)}{\cos A(1+\sin A)}$$

$= 2\sec A =$ R.H.S.

(iii) LHS $= \dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta}$

$$= \frac{\left(\frac{\sin\theta}{\cos\theta}\right)}{\left(1-\frac{\cos\theta}{\sin\theta}\right)} + \frac{\left(\frac{\cos\theta}{\sin\theta}\right)}{\left(1-\frac{\sin\theta}{\cos\theta}\right)}$$

$$= \frac{\left(\frac{\sin\theta}{\cos\theta}\right)}{\left(\frac{\sin\theta-\cos\theta}{\sin\theta}\right)} + \frac{\left(\frac{\cos\theta}{\sin\theta}\right)}{\left(\frac{\cos\theta-\sin\theta}{\cos\theta}\right)}$$

$$= \frac{\sin\theta\times\sin\theta}{\cos\theta\times(\sin\theta-\cos\theta)} + \frac{\cos\theta\times\cos\theta}{\sin\theta\times(\cos\theta-\sin\theta)}$$

$$= \frac{\sin^2\theta}{\cos\theta \times (\sin\theta - \cos\theta)} - \frac{\cos^2\theta}{\sin\theta \times (\sin\theta - \cos\theta)}$$

$$= \frac{\sin\theta \times \sin^2\theta - \cos\theta \times \cos^2\theta}{\cos\theta \times \sin\theta \times (\sin\theta - \cos\theta)}$$

$$= \frac{\sin^3\theta - \cos^3\theta}{\cos\theta \times \sin\theta \times (\sin\theta - \cos\theta)}$$

$$= \frac{(\sin\theta - \cos\theta) \times (\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta}{\cos\theta \times \sin\theta \times (\sin\theta - \cos\theta)}$$

$\{\because a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$$= \frac{\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta}{\cos\theta \times \sin\theta}$$

$$= \frac{1 + \sin\theta\cos\theta}{\cos\theta\sin\theta} = \frac{1}{\cos\theta\sin\theta} + 1$$

$$= 1 + \left(\frac{1}{\cos\theta}\right)\left(\frac{1}{\sin\theta}\right)$$

$= 1 + \sec\theta \operatorname{cosec}\theta$

$\therefore$ LHS = RHS

(iv) $\text{L.H.S.} = \frac{1 + \sec A}{\sec A} = \frac{1 + \frac{1}{\cos A}}{\frac{1}{\cos A}} = \frac{\cos A + 1}{1}$

$\text{R.H.S.} = \frac{\sin^2 A}{1 - \cos A} = \frac{1 - \cos^2 A}{1 - \cos A} = 1 + \cos A$

$\therefore$ L.H.S. = R.H.S.

(v) $\text{LHS} = \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

$$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}}$$

(Dividing the numerator and denominator by sin A)

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A} = \frac{(\operatorname{cosec} A + \cot A) - 1}{\{1 + \cot A - \operatorname{cosec} A\}}$$

$$= \frac{(\operatorname{cosec} A + \cot A) - (\operatorname{cosec}^2 A - \cot^2 A)}{\{1 + \cot A - \operatorname{cosec} A\}}$$

$(\because \operatorname{cosec}^2 A = 1 + \cot^2 A$, i.e., $\operatorname{cosec}^2 A - \cot^2 A = 1)$

$$= \frac{(\text{cosec}A + \cot A) - (\text{cosec}A + \cot A) \times (\text{cosec}A - \cot A)}{(1 + \cot A - \text{cosec}A)}$$

$\{\because (a + b)(a - b) = a^2 - b^2\}$

$$= \frac{(\text{cosec}A + \cot A) \times \{1 - (\text{cosec}A - \cot A)\}}{\{1 + \cot A - \text{cosec}A\}}$$

$$= \frac{(\text{cosec}A + \cot A) \times (1 + \cot A - \text{cosec}A)}{\{1 + \cot A - \text{cosec}A\}}$$

$= \text{cosec}\,A + \cot A$

$= \text{RHS}$

(vi) $\text{LHS} = \sqrt{\dfrac{1+\sin A}{1-\sin A}} = \sqrt{\dfrac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}}$

$$= \sqrt{\frac{(1+\sin A)^2}{(1)^2 - (\sin A)^2}} = \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}} = \frac{1+\sin A}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A$$

$\therefore$ LHS = RHS.

(vii) $\text{L.H.S.} = \dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$$= \frac{\sin\theta(1 - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$$

$$= \frac{\sin\theta(\sin^2\theta + \cos^2\theta - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - \sin^2\theta - \cos^2\theta)}$$

$$= \frac{\tan\theta(\cos^2\theta - \sin^2\theta)}{(\cos^2\theta - \sin^2\theta)}$$

$= \tan\theta = \text{R.H.S.}$

(viii) $\text{L.H.S.} = (\sin A + \text{cosec}A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \text{cosec}^2 A + 2 + \cos^2 A + \sec^2 A + 2$

$= 4 + 1 + 1 + \cot^2 A + 1 + \tan^2 A$

$= 7 + \tan^2 A + \cot^2 A = \text{R.H.S.}$

(ix) LHS = (cosec A – sin A) (sec A – cos A)

$$= \left(\frac{1}{\sin A} - \sin A\right) \times \left(\frac{1}{\cos A} - \cos A\right)$$

$$= \frac{1-\sin^2 A}{\sin A} \times \frac{1-\cos^2 A}{\cos A}$$

$$= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A} = \sin A \cos A$$

Now, RHS $= \dfrac{1}{\tan A + \cot A} = \dfrac{1}{\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}}$

$$= \frac{1}{\dfrac{\sin^2 A + \cos^2 A}{\sin A \cos A}} = \frac{\sin A \cos A}{\sin^2 A + \cos^2 A}$$

$$= \frac{\sin A \cos A}{1}$$

∴ LHS = RHS.

(x) L.H.S. $= \left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right) = \dfrac{\sec^2 A}{\operatorname{cosec}^2 A}$

$= \tan^2 A$ = R.H.S.

& $\left(\dfrac{1-\tan A}{1-\cot A}\right)^2 = \left(\dfrac{\dfrac{\cos A - \sin A}{\cos A}}{\dfrac{\sin A - \cos A}{\sin A}}\right)^2$

$= \dfrac{\sin^2 A}{\cos^2 A} = \tan^2 A$ = R.H.S.