Ex - 8.2

Q1. Evaluate :

(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

(iii) $\dfrac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

(iv) $\dfrac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

(v) $\dfrac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

Sol. (i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$= \frac{3}{4} + \frac{1}{4} = 1$$

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

$$= 2 \times (1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2 + \frac{3}{4} - \frac{3}{4} = 2$$

(iii) $\dfrac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

$$= \frac{1/\sqrt{2}}{\frac{2}{\sqrt{3}} + 2} = \frac{1/\sqrt{2}}{2\left(\frac{1+\sqrt{3}}{\sqrt{3}}\right)} = \frac{1(\sqrt{3})}{2\sqrt{2}(1+\sqrt{3})}$$

$$= \frac{\sqrt{3}}{2(\sqrt{2})} \times \frac{(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{\sqrt{3}(\sqrt{3}-1)}{2\sqrt{2} \times 2}$$

$$= \frac{(3-\sqrt{3})}{4\sqrt{2}}$$

(iv) $\dfrac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ - \cos 60^\circ + \cot 45^\circ}$

$$= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \frac{\frac{\sqrt{3} + 2\sqrt{3} - 4}{2\sqrt{3}}}{\frac{4 + \sqrt{3} + 2\sqrt{3}}{2\sqrt{3}}}$$

$$= \frac{(3\sqrt{3}-4)}{(4+3\sqrt{3})} \frac{(4-3\sqrt{3})}{(4-3\sqrt{3})}$$

$$= \frac{12\sqrt{3} - 27 - 16 + 12\sqrt{3}}{16 - 9 \times 3} = \frac{24\sqrt{3} - 43}{-11} = \frac{43 - 24\sqrt{3}}{11}$$

(v) $\dfrac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

$= \dfrac{5(\cos 60^\circ)^2 + 4(\sec 30^\circ)^2 - (\tan 45^\circ)^2}{(\sin 30^\circ)^2 + (\cos 30^\circ)^2}$

$= \dfrac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \dfrac{\frac{5}{4} + 4 \times \frac{4}{3} - 1}{\frac{1}{4} + \frac{3}{4}}$

$= \dfrac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{1}{4} + \frac{3}{4}} = \frac{5}{4} + \frac{16}{3} - 1$

$= \dfrac{15 + 64 - 12}{12} = \dfrac{67}{12}$

Q2. Choose the correct option and justify your choice:

(i) $\dfrac{2\tan 30^\circ}{1 + \tan^2 30^\circ} =$

(A) $\sin 60^\circ$ (B) $\cos 60^\circ$ (C) $\tan 60^\circ$ (D) $\sin 30^\circ$

(ii) $\dfrac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} =$

(A) $\tan 90^\circ$ (B) 1 (C) $\sin 45^\circ$ (D) 0

(iii) $\sin 2A = 2\sin A$ is true when A =

(A) 0° (B) 30° (C) 45° (D) 60°

(iv) $\dfrac{2\tan 30^\circ}{1 - \tan^2 30^\circ} =$

(A) $\cos 60^\circ$ (B) $\sin 60$ (C) $\tan 60^\circ$ (D) $\sin 30^\circ$

Sol. (i) Option (A) is correct.

$\dfrac{2\tan 30^\circ}{1 + \tan^2 30^\circ} = \dfrac{2\left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \dfrac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}}$

$= \dfrac{2}{\sqrt{3}} \times \dfrac{3}{4} = \dfrac{\sqrt{3}}{2} = \sin 60^\circ$

(ii) Option (D) is correct.

$\dfrac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} = \dfrac{1 - 1}{1 + 1} = 0$

(iii) Option (A) is correct.

$\sin 2A = 2\sin A$

$\Rightarrow 2\sin A \cdot \cos A = 2\sin A$

$\Rightarrow \cos A = 1$

$\Rightarrow A = 0^\circ$

(iv) Option (C) is correct.

$$\frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$$

$$= \frac{2\times\frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$= \tan 60^\circ$

Q3. If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$;

$0^\circ < A + B \leq 90^\circ$; $A > B$, find A and B.

Sol. $\tan(A + B) = \sqrt{3} \quad \Rightarrow A + B = 60^\circ \quad ...(1)$

$\tan(A - B) = \frac{1}{\sqrt{3}} \quad \Rightarrow A - B = 30^\circ \quad ...(2)$

Adding (1) and (2),

$2A = 90^\circ \Rightarrow A = 45^\circ$

Then from (1), $45^\circ + B = 60^\circ \Rightarrow B = 15^\circ$

Q4. State whether the following are true or false. Justify your answer.

(i) $\sin(A + B) = \sin A + \sin B$

(ii) The value of $\sin\theta$ increases as θ increases.

(iii) The value of $\cos\theta$ increases as θ increases.

(iv) $\sin\theta = \cos\theta$ for all values of θ.

(v) cot A is not defined for $A = 0^\circ$.

Sol. (i) False.

When $A = 60^\circ$, $B = 30^\circ$

LHS $= \sin(A + B) = \sin(60^\circ + 30^\circ)$

$= \sin 90^\circ = 1$

RHS $= \sin A + \sin B$

$= \sin 60^\circ + \sin 30^\circ$

$= \frac{\sqrt{3}}{2} + \frac{1}{2} \neq 1$

i.e., LHS $\neq$ RHS

(ii) True.

Note that $\sin 0^\circ = 0$, $\quad \sin 30^\circ = \frac{1}{2} = 0.5$,

$\sin 45^\circ = \frac{1}{\sqrt{2}} = 0.7$ (approx.),

$\sin 60^\circ = \frac{\sqrt{3}}{2} = 0.87$ (approx.)

and $\sin 90^\circ = 1$

i.e., value of $\sin\theta$ increases as θ increases from 0° to 90°.

(iii) False.

Note that $\cos 0° = 1$,

$\cos 30° = \frac{\sqrt{3}}{2} = 0.87$ (approx.)

$\cos 45° = \frac{1}{\sqrt{2}} = 0.7$ (approx.),

$\cos 60° = \frac{1}{2} = 0.5$ and $\cos 90° = 0$

i.e., value of $\cos \theta$ decreases as θ increases from 0° to 90°.

(iv) False, it is true for only $\theta = 45°$

(v) True, $\cot A = \frac{1}{0}$ = not defined.