Exercise 7.2

Question 1:

Find the coordinates of the point which divides the join of (− 1, 7) and (4, − 3) in the ratio 2:3.

Answer:

Let P(x, y) be the required point. Using the section formula, we obtain

$$x=\frac{2\times 4+3\times(-1)}{2+3}=\frac{8-3}{5}=\frac{5}{5}=1$$

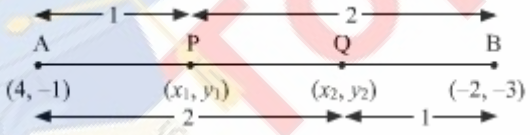
$$y=\frac{2\times(-3)+3\times 7}{2+3}=\frac{-6+21}{5}=\frac{15}{5}=3$$

Therefore, the point is (1, 3).

Question 2:

Find the coordinates of the points of trisection of the line segment joining (4, − 1) and (− 2, − 3).

Answer:



Let P (x_1, y_1) and Q (x_2, y_2) are the points of trisection of the line segment joining the given points i.e., AP = PQ = QB

Therefore, point P divides AB internally in the ratio 1:2.

$$x_1 = \frac{1\times(-2)+2\times 4}{1+2}, \quad y_1 = \frac{1\times(-3)+2\times(-1)}{1+2}$$

$$x_1 = \frac{-2+8}{3} = \frac{6}{3} = 2, \quad y_1 = \frac{-3-2}{3} = \frac{-5}{3}$$

Therefore, $P(x_1, y_1) = \left(2, -\frac{5}{3}\right)$

Point Q divides AB internally in the ratio 2:1.

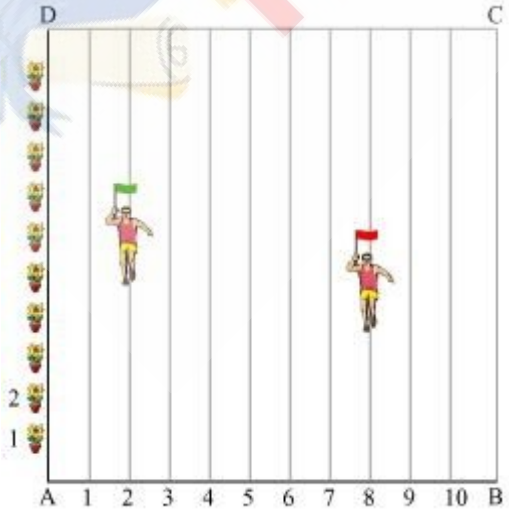
$$x_2 = \frac{2\times(-2)+1\times 4}{2+1}, \quad y_2 = \frac{2\times(-3)+1\times(-1)}{2+1}$$

$$x_2 = \frac{-4+4}{3} = 0, \quad y_2 = \frac{-6-1}{3} = \frac{-7}{3}$$

$$Q(x_2, y_2) = \left(0, -\frac{7}{3}\right)$$

Question 3:

To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in the following figure. Niharika runs $\frac{1}{4}$th the distance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$th the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?



Answer:

It can be observed that Niharika posted the green flag at $\frac{1}{4}$ of the distance AD i.e., $\left(\frac{1}{4}\times 100\right)\text{m} = 25$ m from the starting point of 2nd line. Therefore, the coordinates of this point G is (2, 25).

Similarly, Preet posted red flag at $\frac{1}{5}$ of the distance AD i.e., $\left(\frac{1}{5}\times 100\right)\text{m} = 20$ m from the starting point of 8th line. Therefore, the coordinates of this point R are (8, 20).

Distance between these flags by using distance formula = GR

$= \sqrt{(8-2)^2 + (25-20)^2} = \sqrt{36+25} = \sqrt{61}\,\text{m}$

The point at which Rashmi should post her blue flag is the mid-point of the line joining these points. Let this point be A (x, y).

$$x = \frac{2+8}{2}, \quad y = \frac{25+20}{2}$$

$$x = \frac{10}{2} = 5, \; y = \frac{45}{2} = 22.5$$

$$\text{Hence, } A(x, y) = (5, 22.5)$$

Therefore, Rashmi should post her blue flag at 22.5m on 5th line.

Question 4:

Find the ratio in which the line segment joining the points (− 3, 10) and (6, − 8) is divided by (− 1, 6).

Answer:

Let the ratio in which the line segment joining (−3, 10) and (6, −8) is divided by point (−1, 6) be k : 1.

Therefore, $-1=\frac{6k-3}{k+1}$

$-k-1=6k-3$

$7k=2$

$k=\frac{2}{7}$

Therefore, the required ratio is 2 : 7.

Question 5:

Find the ratio in which the line segment joining A (1, − 5) and B (− 4, 5) is divided by the x-axis. Also find the coordinates of the point of division.

Answer:

Let the ratio in which the line segment joining A (1, −5) and B (−4, 5) is divided by x-axisbe $k:1$.

Therefore, the coordinates of the point of division is $\left(\frac{-4k+1}{k+1},\frac{5k-5}{k+1}\right)$.

We know that y-coordinate of any point on x-axis is 0.

$\therefore \frac{5k-5}{k+1}=0$

$k=1$

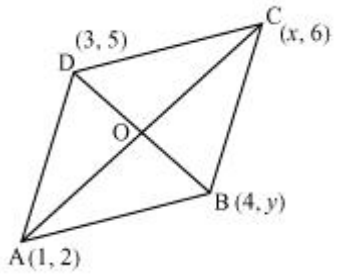
Therefore, x-axis divides it in the ratio 1:1.

Division point = $\left(\frac{-4(1)+1}{1+1},\frac{5(1)-5}{1+1}\right)=\left(\frac{-4+1}{2},\frac{5-5}{2}\right)=\left(\frac{-3}{2},0\right)$

Question 6:

If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

Answer:



Let (1, 2), (4, y), (x, 6), and (3, 5) are the coordinates of A, B, C, D vertices of a parallelogram ABCD. Intersection point O of diagonal AC and BD also divides these diagonals.

Therefore, O is the mid-point of AC and BD.

If O is the mid-point of AC, then the coordinates of O are

$$\left(\frac{1+x}{2}, \frac{2+6}{2}\right) \Rightarrow \left(\frac{x+1}{2}, 4\right)$$

If O is the mid-point of BD, then the coordinates of O are

$$\left(\frac{4+3}{2}, \frac{5+y}{2}\right) \Rightarrow \left(\frac{7}{2}, \frac{5+y}{2}\right)$$

Since both the coordinates are of the same point O,

$$\therefore \frac{x+1}{2} = \frac{7}{2} \text{ and } 4 = \frac{5+y}{2}$$
$$\Rightarrow x+1 = 7 \text{ and } 5+y = 8$$
$$\Rightarrow x = 6 \text{ and } y = 3$$

Question 7:

Find the coordinates of a point A, where AB is the diameter of circle whose centre is (2, − 3) and B is (1, 4)

Answer:

Let the coordinates of point A be (x, y).

Mid-point of AB is (2, −3), which is the center of the circle.

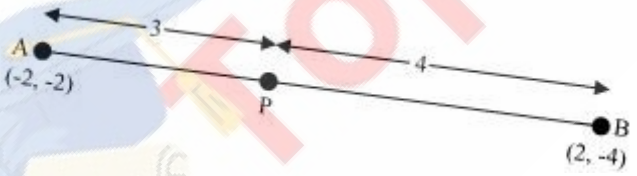
$$\therefore (2,-3)=\left(\frac{x+1}{2},\frac{y+4}{2}\right)$$
$$\Rightarrow \frac{x+1}{2}=2 \text{ and } \frac{y+4}{2}=-3$$
$$\Rightarrow x+1=4 \text{ and } y+4=-6$$
$$\Rightarrow x=3 \text{ and } y=-10$$

Therefore, the coordinates of A are $(3,-10)$.

Question 8:

If A and B are (− 2, − 2) and (2, − 4), respectively, find the coordinates of P such that $AP=\frac{3}{7}AB$ **and P lies on the line segment AB.**

Answer:



The coordinates of point A and B are (−2, −2) and (2, −4) respectively.

Since $AP=\frac{3}{7}AB$,

Therefore, AP: PB = 3:4

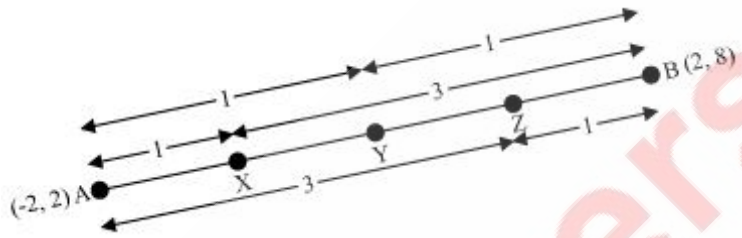
Point P divides the line segment AB in the ratio 3:4.

$$\text{Coordinates of P} = \left(\frac{3\times 2+4\times(-2)}{3+4}, \frac{3\times(-4)+4\times(-2)}{3+4}\right)$$
$$= \left(\frac{6-8}{7}, \frac{-12-8}{7}\right)$$
$$= \left(-\frac{2}{7}, -\frac{20}{7}\right)$$

Question 9:

Find the coordinates of the points which divide the line segment joining A (− 2, 2) and B (2, 8) into four equal parts.

Answer:



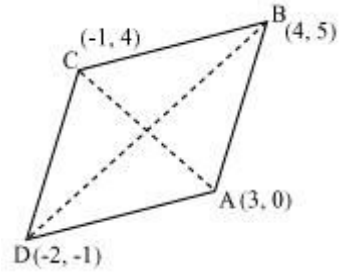
From the figure, it can be observed that points P, Q, R are dividing the line segment in a ratio 1:3, 1:1, 3:1 respectively.

$$\text{Coordinates of P} = \left(\frac{1\times 2+3\times(-2)}{1+3}, \frac{1\times 8+3\times 2}{1+3}\right)$$
$$= \left(-1, \frac{7}{2}\right)$$
$$\text{Coordinates of Q} = \left(\frac{2+(-2)}{2}, \frac{2+8}{2}\right)$$
$$= (0, 5)$$
$$\text{Coordinates of R} = \left(\frac{3\times 2+1\times(-2)}{3+1}, \frac{3\times 8+1\times 2}{3+1}\right)$$
$$= \left(1, \frac{13}{2}\right)$$

Question 10:

Find the area of a rhombus if its vertices are (3, 0), (4, 5), (− 1, 4) and (− 2, −1) taken in order. [Hint: Area of a rhombus = $\frac{1}{2}$ (product of its diagonals)]

Answer:



Let (3, 0), (4, 5), (−1, 4) and (−2, −1) are the vertices A, B, C, D of a rhombus ABCD.

$$\text{Length of diagonal AC} = \sqrt{\left[3-(-1)\right]^2+(0-4)^2}$$
$$= \sqrt{16+16} = 4\sqrt{2}$$
$$\text{Length of diagonal BD} = \sqrt{\left[4-(-2)\right]^2+\left[5-(-1)\right]^2}$$
$$= \sqrt{36+36} = 6\sqrt{2}$$
$$\text{Therefore, area of rhombus ABCD} = \frac{1}{2}\times 4\sqrt{2}\times 6\sqrt{2}$$
$$= 24 \text{ square units}$$